1. **Potential problem** (7 points)

Consider a particle moving in a 1-dimensional potential consisting of an infinite barrier for \( x < 0 \), and for \( x > 0 \):

\[
V(x) = \begin{cases} 
V_I & 0 < x < a \\
V_{II} & a < x < b \\
0 & x > b 
\end{cases}
\]

where \( V_I < 0 < V_{II} \).

Solve the time-independent Shrödinger equation to determine the energy eigenvalues and eigenfunctions for all ranges of energy \( (E > V_I) \). If you cannot find closed form expressions, carry the calculations as far as possible and indicate how explicit solutions would be obtained (perhaps with help of a computer).

2. **Bound state energies** (6 points)

(Mahan chapter 2, problem 1) Derive the numerical value in eV for the bound-state energy of an electron in the one-dimensional square-well potential:

\[
V(x) = \begin{cases} 
-V_0 & |x| < b \\
0 & |x| > b 
\end{cases}
\]

where \( b = 1.0 \) and \( V_0 = 1.0 \) eV. What is the critical value of the coupling strength \( g_c \) for this potential?

Note: The coupling strength \( g \) is defined by \( g^2 = V_0 / E_b \) where \( E_b = \hbar^2 / 2mb^2 \). The critical coupling strength determines the energy above which there are no bound states (no bound states exist if \( g < g_c \)).

3. **Measuring a particle in a box** (8 points)

A particle moves in an infinite square-well potential confined between positions \( x = -L/2 \) and \( x = L/2 \). The particle is prepared at some time \( t < 0 \) in the lowest energy eigenstate. At time \( t = 0 \) and imprecise measurement of the particle position is made which is able to determine only that the particle lies to the right of the origin, i.e., in the range \( 0 < x < L/2 \).

(a) What is the normalized wavefunction of the particle immediately before the measurement?

(b) Show explicitly that the prepared state obeys the Heisenberg uncertainty principle.

(c) Determine the normalized wavefunctions of the particle immediately after the measurement.