

# Statistical Mechanics

PHYS 508

Spring 2015

## Problem Assignment # 9

due 05-01-15

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### 1. 1-d Ising model (6 points)

A one-dimensional lattice consists of  $N + 1$  equally spaced Ising spins coupled by nearest-neighbor exchange interactions. The Hamiltonian for the system is

$$H = -J \sum_{j=1}^N S_j S_{j+1} \quad ,$$

where  $J$  is a positive constant.

For this problem, we define a “kink” as a place in the Ising chain where all spins to the right are pointed in a direction opposite those on the left (see LeBellac page 157 for an illustration). A configuration with  $m$  kinks has the energy

$$E_m = -NJ + m2J \quad .$$

- (a) Schematically represent the configurations corresponding to the states of minimum and maximum energy,  $E_{min}$  and  $E_{max}$ , respectively. Give the number of kinks and the degeneracies associated with these two states.

Now consider the Ising chain in contact with a heat bath at temperature  $T$ .

- (b) Show that the partition function for the 1-d Ising model may be written as

$$Z = 2^{N+1} (\cosh(\beta J))^N \quad .$$

- (c) Calculate the average energy  $E$  and the average number of kinks  $\langle m \rangle$ . Find an expression for the probability  $p$  that two neighboring spins are antiparallel. Sketch and discuss  $p$  as a function of  $T$ .