Statistical Mechanics

PHYS 508 Spring 2015

Problem Assignment # 7

due 03-30-15

1. **Paramagnets** (4 points)

   Consider a system of N noninteracting spins that can point either up or down in an external magnetic field H. Each up-spin contributes an energy $-\mu_B H$, and each down-spin contributes an energy $+\mu_B H$, so that the total energy of a state with n_+ up-spins and n_- down-spins is

   $$E_{n_+ n_-} = -(n_+ - n_-)\mu_B H \quad ,$$

   where μ_B is the Bohr magneton. Let the system be in contact with a heat bath of temperature T.

   (a) Calculate the system's partition function.

   (b) Calculate and discuss the internal energy and the specific heat.

2. **Two-dimensional quantum gases** (7 points)

   (a) Find the density of states per unit volume, $\omega(\epsilon)$, for a degenerate quantum gas in $d = 2$ dimensions.

   (b) Show that in a 2-d Bose gas there is no Bose-Einstein condensation.

   (c) Find an expression for the Fermi energy in terms of number density for a 2-d Fermi gas.

3. **Chemical potential** (9 points)

   Calculate and sketch the temperature dependence of the chemical potential of an ideal Fermi gas.

   (a) at low temperature, up to and including order $(T/\mu)^4$.
   hint: Use the Sommerfeld expansion to expand μ in powers of (T/μ), then solve for μ by iteration.

   (b) at high temperature ($T \gg \epsilon_F$), up to and including the next-to-leading term.