

Statistical Mechanics

PHYS 508

Spring 2015

Problem Assignment # 6

due 03-13-15

1. Energy averages (6 points)

A classical, monatomic ideal gas (N particles, each with mass m) fills a cubic container (linear dimension L). The container is placed in a gravitational field (acceleration g), so that the gravitational forces acts perpendicularly to a face of the cube.

- Find the average kinetic energy of a gas molecule.
- Find the average potential energy of a gas molecule.
- Find the average elevation of a gas molecule above the container floor. Discuss your result as a function of the temperature T , and in particular in the limit $T \rightarrow \infty$.

2. Spring Balance (5 points)

A spring balance consists of a harmonic spring (spring constant α) in an environment kept at temperature T . The gravitational acceleration is g .

- An object of mass m is suspended from the spring. What is the resulting mean elongation $\langle x \rangle$ of the spring?
- What is the rms fluctuation, $\langle (x - \langle x \rangle)^2 \rangle^{1/2}$, of the elongation due to thermal fluctuations?
- Suppose you have measured a mass that is so small that the thermal fluctuations just barely let you do the measurement. Now you want to measure a mass that is about ten times smaller. How much do you have to reduce the temperature?

3. Velocity averages (5 points)

A classical gas (molecular mass m) is in thermal equilibrium at temperature T . Let $\vec{v} = (v_x, v_y, v_z)$ be the velocity of a gas molecule, and $v = |\vec{v}|$ its speed.

- Find the following averages:

$$\langle v_x \rangle \quad , \quad \langle v_x^2 \rangle \quad , \quad \langle v^2 v_x \rangle \quad , \quad \langle v_x^2 v_y^2 \rangle \quad .$$

- Find $\langle 1/v \rangle$. Compare this with $1/\langle v \rangle$.
- What is the mean number of molecules per unit volume whose kinetic energy lies in the interval $[\epsilon, \epsilon + d\epsilon]$?