Statistical Mechanics

PHYS 508 Spring 2015

Problem Assignment # 4

due 03-13-15

1. **Ensembles** (3 points)

   We have learned about three types of statistical ensembles: microcanonical, canonical, and grand canonical. Describe the physical situation which applies to each type of ensemble, and *in words* discuss the distribution function associated with each type of ensemble (briefly, what does it physically relate to?).

2. **Grand canonical ensemble** (1 point)

   Consider a grand canonical ensemble with partition function Z, grand canonical thermodynamic potential J, and chemical potential μ. Show, by means of a direct calculation, that the mean particle number is given by

   $$\langle N \rangle = -\frac{\partial J}{\partial \mu} = k_B T \frac{\partial \ln Z}{\partial \mu} \quad .$$

3. **Homogeneous functions** (1 point)

   Show that the monomial $f(x) = c\, x^a$ is the most general homogeneous function of one variable of degree a.

4. **Harmonic oscillator** (9 points)

   A one-dimensional quantum mechanical harmonic oscillator is in thermal contact with a heat bath of temperature T.

   (a) Calculate the canonical partition function Z.

   (b) Calculate the mean energy $U = \langle E \rangle$ directly from its definition.

   (c) Calculate U from $U = -\partial \ln Z / \partial \beta$, and compare with the result of part **b**.

   (d) Calculate the root-mean-square energy fluctuation, (ΔE), the Helmholtz free energy, F, and the entropy, S, for the harmonic oscillator.

   (e) Determine the classical limit ($\hbar \to 0$), and the low-temperature limit ($T \to 0$), for U and (ΔE). Interpret your results.

   (f) Calculate, discuss, and plot the specific heat C_V as a function of the temperature.