Statistical Mechanics

PHYS 508 Spring 2015

Problem Assignment # 3

due 02-06-15

1. **Integrated density of states** (7 points)

   The number of accessible states $\Omega(E)$, is defined as the number of states with energies between $E - \Delta E$ and E. Alternatively, we can define $\tilde{\Omega}_x(E)$ as the number of states with energies between xE and E, where $0 \leq x \leq 1$. For a classical system,

   $$\tilde{\Omega}(E) = const. \times \int_{xE \leq H \leq E} d\Gamma \quad ,$$

   where $d\Gamma = d^3x_1 ... d^3x_N d^3p_1 ... d^3p_N$ is the phase space volume element, and H is the energy of a microstate. The normalization constant will be of no relevance for what follows. $\tilde{\Omega}_{x=0}(E)$ is called the *integrated density of states.*

   (a) For a classical ideal gas (N noninteracting point particles of mass m in a volume V), show that

   $$\tilde{\Omega}_{x=0}(E) = const. \times V^N (2mE)^{3N/2} C_{3N} \quad ,$$

   with C_d the volume of the d-dimensional unit sphere. Calculate C_d.

   (b) Show that

   $$\tilde{\Omega}_x(E) = f(x) \tilde{\Omega}_0(E) \quad ,$$

   and determine $f(x)$. How close to 1 do you have to choose x in order for $f(x)$ to be substaintially different from unity? Discuss the meaning of this result for the volumes of high-dimensional spheres, and for the physical significance of the arbitrary energy interval ΔE in the number of accessible states.

2. **Particle in a box** (4 points)

   Consider a quantum mechanical system consisting of one spinless particle in a 3-dimensional rectangular box with linear dimensions L_1, L_2, and L_3.

   (a) Suppose the system is in a particular microstate. From the change of the corresponding energy level under a quasi-static change fo the length L_i by dL_i, find the force exerted by the particle on the wall perpendicular to the i-axis.

   (b) For a cubic box, find the average pressure of the particle on a wall in terms of the average energy of the particle and the volume of the box.
   hint: You do not need to find the probability distribution explicitly.

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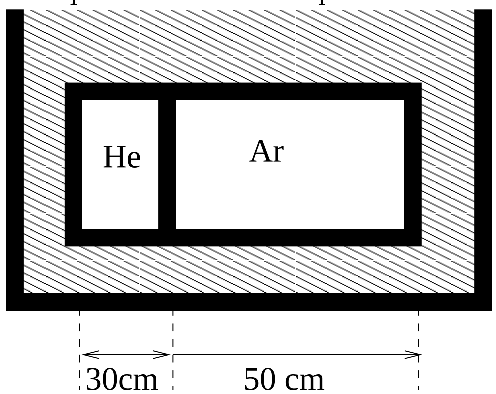
3. **Monatomic ideal gases** (5 points)

A cylindrical container 80 cm long is divided into two compartments by a piston of negligible thickness. The left compartment is filled with 1 mole of Helium gas, and the right compartment is filled with Argon gas. Both gases can be considered ideal. The cylinder is submerged in 1 liter of water. Initially, the piston is clamped in a position 30 cm from the left end of the cylinder, the pressure in the Helium chamber is 5 atm, the pressure in the Argon chamber is 1 atm, and the whole system is at a uniform temperature of 25° C. After releasing the piston, the system goes into a new equilibrium with the piston in a new position.



(a) What is the position of the piston in the final equilibrium state?

(b) What is the final equilibrium temperature?

(c) What is the total entropy change of the system?