Statistical Mechanics

Problem Assignment # 2

1. **Random numbers** (2 points)

   A number is chosen at random between 0 and 1. What is the probability that exactly 5 of its first 10 decimal places consist of digits less than 5?

2. **Russian roulette** (5 points)

   In a game of Russian roulette, one inserts a single cartridge into the drum of a revolver, leaving the other five chambers of the drum empty. One then spins the drum, aims at one's head, and pulls the trigger.

   (a) What is the probability of still being alive after 7 turns of the game? What is the probability, $P(n)$ of still being alive after n tries?

   (b) What is the mean lifetime of a player, measured in the number of times he pulls the trigger?

   (c) What is the probability of surviving the first N tries, and shooting oneself on the $(N+1)$st try?

3. **Health hazards** (3 points)

   A prospective buyer of a Socorro county home has the well checked for coliform bacteria. Suppose that the well is contaminated with 10 bacteria per 100 ml, and that the analytical method requires 2 bacteria per 100 ml for a positive result.

   (a) What is the probability that the test comes back negative?

   (b) The buyer is suspicious, and has the test repeated. What is the probability that both results are negative? What is the probability that the first test is negative, while the second one finds 20 bacteria in a 100 ml sample?

4. **Random walk** (8 points)

   A man walks along a straight line in steps of equal lengths, forward or backward, starting from the origin. The probability of taking a forward step is p, and the probability of taking a backward step is $1 - p$.

   (a) Find the probability of the man being m steps away from the origin after he has made s steps.

   (b) Find the average, the mean-square deviation, and the relative fluctuation of his displacement from the origin.

5. **Central Limit Theorem** (4 points)

   Consider a binomial distribution $\rho_b(n, s)$ with $s = 2500$ and probability $p = 0.02$. Let $\rho_G(n)$ be the Gaussian distribution that is approached by ρ_b for $s \to \infty$. Compile a table comparing the two distribution function, and plot them over a reasonable range of n-values.