Statistical Mechanics

PHYS 508 Spring 2015

Problem Assignment # 1

due 1-23-15

1. **Phase flow** (4 points)

   Consider a point mass in $d = 1$ subject to a constant force f, so that the equation of motion is

   $$\ddot{q}(t) = f/m \quad .$$

   (a) Determine the phase flow. Does Liouville's theorem hold?

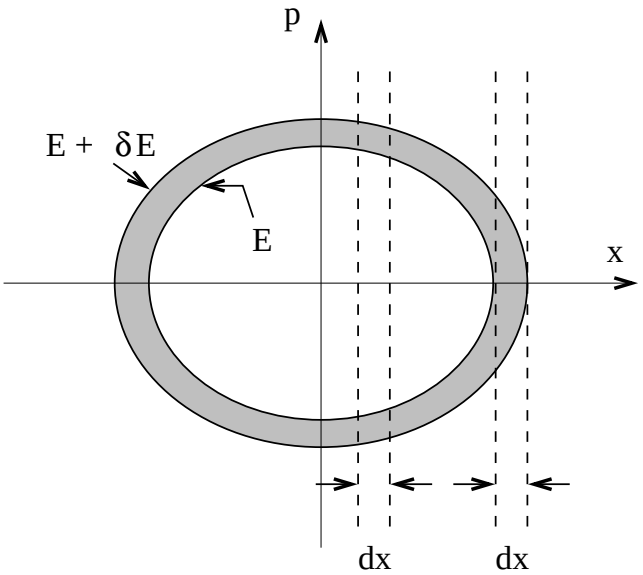
   (b) Does Poincaré's theorem hold? Justify your answer.

2. **HO phase space** (6 points)

   Consider an ensemble of classical one-dimensional harmonic oscillators.

   (a) Let the displacement x of an oscillator as a function of time t be given by $x = A\cos(\omega t + \phi)$. Assume that the phase angle ϕ is equally likely to assume any value in the range $0 < \phi < 2\pi$. The probability $w(\phi)d\phi$ that ϕ lies in the range between ϕ and $\phi + d\phi$ is then simply $w(\phi) = (2\pi)^{-1}d\phi$. For any fixed time t, find the probability $P(x)dx$ that x lies between x and $x + dx$ by summing $w(\phi)d\phi$ ovar all angles ϕ for which x lies in this range. Express $P(x)$ in terms of A and x.

   (b) Consider the classical phase space for such an ensemble of oscillators, their energy being known to lie in the small range between E and $E + \delta E$. Calculate $P(x)dx$ by taking the ratio of that volume of phase space lying in this energy range *and* in the range between x and $x + dx$ to the total volume of phase space lying in the energy range between E and $E + \delta E$ (see figure). Express $P(x)$ in terms of E and x. By relating E to the amplitude A, show that the result is the same as that obtained in part **a**.



3. **Rolling dice** (8 points)

   (a) Three fair dice are rolled once. What is the probability of throwing six points or less?

   (b) The Chevalier de Méré made bets on dice games that required a higher than 50% probability for

      i. throwing at least one six in rolling one die four times, and

      ii. throwing at least one double-six in rolling two dice 24 times.

      Where these winning bets?

   (c) What are the probabilities for throwing exactly one six and one double-six, respectively?