

# 1 Diagnosing Convection with Weak Temperature 2 Gradient Simulations of DYNAMO

Stipo Sentić<sup>1</sup>, Sharon L. Sessions<sup>1</sup>, and Željka Fuchs<sup>1,2</sup>

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Corresponding author: S. Sentić, Physics Department and Geophysical Research Center, New Mexico Institute of Mining and Technology, 801 Leroy Place, Socorro, NM 87801, USA. (ssentic@nmt.edu)

<sup>1</sup>Physics Department and Geophysical Research Center, New Mexico Institute of Mining and Technology, 801 Leroy Place, Socorro, NM 87801, USA.

<sup>2</sup>Department of Physics, Faculty of Science, Teslina 12, 21000 Split, Croatia.

**3 Abstract.**

4 Determining relationships between convective and environmental diagnos-  
5 tics can improve our understanding of mechanisms controlling tropical con-  
6 vection, and consequently, result in better representations of convection in  
7 coarsely resolved models. We identify important diagnostic relationships in  
8 observations taken during the Dynamics of the Madden-Julian Oscillation  
9 (MJO) campaign, and perform weak temperature gradient (WTG) simula-  
10 tions of DYNAMO convection to determine if the observed relationships are  
11 reproduced in our model.

12 We find that the WTG approximation models local changes in the diag-  
13 nostics used in the study—precipitation rate, atmospheric stability, moisture,  
14 and gross moist stability (GMS)—and reproduces diagnostic relationships sug-  
15 gested in previous studies; an increase in precipitation rate is correlated with  
16 increased atmospheric moisture content, which, in turn, is correlated with  
17 greater atmospheric stability. Large scale atmospheric stability—changes of  
18 which might be related to balanced dynamics, we speculate—seems to be a  
19 candidate for a convective controlling mechanism.

20 Observed and modeled interactions of local convection with the large scale  
21 environment—quantified by the GMS—are in agreement with the theory of In-  
22 oue and Back (2015b); the GMS increases from small, positive or negative,  
23 values during developing convection, and further increases for decaying con-  
24 vection past a characteristic GMS found at peak precipitation rates, atmo-  
25 spheric stability, and moisture content. Understanding the link between the

26 characteristic GMS and the diagnostics—still a standing problem—could fur-  
27 ther our understanding of interactions between local convection and the large  
28 scale environment.

29 Three key point statements:

30 Relationships between convective diagnostics characterize DYNAMO con-  
31 vection.

32 WTG simulations reproduce observed diagnostic relationships.

33 GMS characterizes convective development and decay.

## 1. Introduction

34 The Madden Julian Oscillation [MJO, *Madden and Julian*, 1972] ties weather and cli-  
35 mate [*Zhang*, 2013]: it regulates meteorology from the shortest time scales (precipitation,  
36 tornadoes, floods, fires, lightning, tropical cyclones) to the longest time scales (surface  
37 temperatures, extratropical climate modes, ENSO, Indian ocean dipole). Current weather  
38 and climate prediction models do not capture the MJO adequately [*Kim et al.*, 2014; *Bene-*  
39 *dict et al.*, 2014], in part because convective parameterizations are not properly modeling  
40 relationships between convection and the large scale environment in which it is embed-  
41 ded. In order to improve our ability to model the MJO, and consequently the climate,  
42 we need to improve convective parameterizations. One way to to do this is is to im-  
43 prove our understanding of the relationships between convective processes and large scale  
44 conditions.

45 Quantitative relationships between convective parameters and large scale environmen-  
46 tal conditions are a standing problem in modeling convection. For example, there is  
47 no consensus on the quantitative relationship between precipitation rate and moisture.  
48 Studies using satellite observations [*Back and Bretherton*, 2006; *Peters and Neelin*, 2006;  
49 *Masunaga*, 2012] and combined observation-numerical modeling studies [*Raymond et al.*,  
50 2007] have proposed several forms of the precipitation rate–moisture relationship in the  
51 tropics; though they differ in detail, they all suggest that precipitation is a sensitive  
52 function of moisture. Studies have shown that general circulation models improve when  
53 precipitation rate depends on moisture quantity [*Sherwood et al.*, 2010]. Further, both  
54 idealized numerical studies [*Sessions et al.*, 2010, 2015] and numerical case studies [*Wang*

55 *et al.*, 2014] find that there is a relationship between the precipitation rate and the import  
56 or export of large scale energy and moisture. Obtaining better diagnostic relationships  
57 between precipitation rate, moisture, and energy transport on the large scale—in the con-  
58 text of tropical convection—can improve parameterizations relevant to climate and weather  
59 predictions.

60 To understand how tropical convection interacts with the large scale environment, we  
61 model convection by parameterizing the large scale. One approach is to impose a mean,  
62 time dependent, vertical velocity profile that is consistent with large scale conditions in  
63 a model domain [*Fridlind et al.*, 2012; *Woolnough et al.*, 2010; *Wang et al.*, 2014] supple-  
64 mented by horizontal temperature and moisture tendencies. This approach constrains the  
65 convection in a manner that reproduces the observed precipitation very closely. Problems,  
66 however, can arise in interpretation of results because imposing vertical motion constrains  
67 the precipitation and obscures causal mechanisms. More recent approaches do not specify  
68 the vertical motion but utilize an alternate parameterization of the large scale. Two ex-  
69 amples are the weak temperature gradient [WTG, *Sobel and Bretherton*, 2000; *Raymond*  
70 *and Zeng*, 2005] and the weak pressure gradient [WPG, *Kuang*, 2008; *Romps*, 2012a, b]  
71 approximations. From these approximations the vertical velocity is calculated from con-  
72 vection responding to observed large scale conditions. The advantage of the WTG and  
73 WPG approximations is that the model convection is free to evolve, and is not constrained  
74 by imposing tendencies of moisture and temperature.

75 A number of recent studies have used the WTG and WPG approximations to study  
76 convection in cloud resolving models (CRMs). In idealized WTG numerical simulations,  
77 researchers studied how convection responds to localized changes in sea surface temper-

atures [SST, *Sobel and Bretherton*, 2000], surface fluxes and wind speed [*Raymond and Zeng*, 2005; *Sessions et al.*, 2010; *Anber et al.*, 2014], wind shear [*Anber et al.*, 2014], radiation [*Anber et al.*, 2015], and changes in atmospheric stability and moisture [*Raymond and Sessions*, 2007; *Sobel and Bellon*, 2009; *Sessions et al.*, 2015; *Daleu et al.*, 2015; *Raymond et al.*, 2015]. A recent study [*Wang et al.*, 2013] benchmarked how significantly the WTG and WPG approximations reproduced diagnostics in MJO convection of the Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment (TOGA-COARE). So far, no study has compared observations and modeling results to determine if observed relationships between convective and environmental diagnostics are preserved in models; that is the goal of this paper.

We compare diagnostic relationships hypothesized to be relevant to tropical convection in observations and WTG simulations. We use observations from the Dynamics of the Madden-Julian Oscillation (DYNAMO) field campaign [*Johnson and Ciesielski*, 2013; *Ciesielski et al.*, 2014a, b; *Johnson et al.*, 2015] to force and verify WTG simulations. This work is similar to a previous study that implemented observations into WTG and WPG simulations [*Wang et al.*, 2013], but there are some significant differences. For example, not only do we perform WTG simulations with domain averages from DYNAMO North and south Sounding Arrays, we also force WTG simulations with individual DYNAMO station soundings in order to benchmark the WTG approximation in more localized conditions. Probably the most important contribution of this work is a comparison between observed and modeled diagnostic relationships relevant to tropical dynamics. Identifying these relationships in observations is important for better understanding convection it-

100 self; determining whether these relationships are preserved in WTG experiments helps to  
101 validate the WTG approximation as a useful tool for understanding tropical convection.

102 This paper is organized as follows. In Section 2, we describe the model, the WTG  
103 approximation, observations used in the study, and the methodology. In Section 3 we  
104 compare observed and modeled diagnostic time series and relationships. In section 4  
105 we present our conclusions, and in the appendix we present the details of verifying the  
106 modeled time series.

## 2. Model, observations and methodology

107 In this section we introduce the cloud resolving model (CRM), the WTG approxima-  
108 tion, observations, and the methodology used in this study. The version of the WTG  
109 approximation we use is called the spectral weak temperature gradient approximation  
110 [SWTG, *Herman and Raymond, 2014*], and is described in section 2.1.

### 2.1. Model and the weak temperature gradient (WTG) approximation

111 We use the CRM of *Herman and Raymond [2014]*. The model solves the fully com-  
112 pressible non-rotating momentum equations on a double periodic two or three dimensional  
113 domain. Prognostic equations govern specific moist entropy, the precipitation mixing ra-  
114 tio, and the total cloud water mixing ratio. The prognostic variable entropy is used  
115 because of improved conservation properties [*Raymond, 2013*] compared to the custom-  
116 ary moist static energy. The dry ( $s_d$ ), moist ( $s$ ), and saturated ( $s^*$ ) entropy are defined

117 approximately as:

$$118 \quad s_d = c_p \ln \left( \frac{\theta}{\theta_R} \right), \quad (1)$$

$$119 \quad s = c_p \ln \left( \frac{\theta_e}{\theta_R} \right), \quad (2)$$

$$120 \quad s^* = c_p \ln \left( \frac{\theta_e^*}{\theta_R} \right). \quad (3)$$

122 Here,  $c_p$  is the specific heat of dry air at constant pressure,  $\theta_R = 300$  K is a reference  
 123 temperature, and  $\theta$ ,  $\theta_e$ , and  $\theta_e^*$  are the potential, equivalent potential, and saturated equiv-  
 124 alent potential temperatures, respectively. The model uses simplified microphysics and  
 125 radiation [Raymond, 2001], and boundary layer turbulence is parameterized by Monin-  
 126 Obukhov similarity theory. Please refer to Herman and Raymond [2014] for a detailed  
 127 description of the model.

128 The CRM represents a limited region of the tropical atmosphere, and interacts with  
 129 a specified reference environment through the WTG approximation. The conventional  
 130 WTG approximation is based on the ideas from observations [Bretherton and Smo-  
 131 larkiewicz, 1989], and scaling arguments [Charney, 1963; Sobel and Bretherton, 2000]  
 132 that gravity waves quickly redistribute heating anomalies, and leave weak horizontal tem-  
 133 perature gradients in the tropics. Parcels with redistributed heat lift adiabatically and  
 134 cool, equilibrating the system. In a model with periodic boundary conditions, like the one  
 135 we are using, potential temperature anomalies due to latent heat release and radiative  
 136 cooling are trapped in the domain. In order to mimic the real world process, the model  
 137 generates a vertical velocity, called the WTG vertical velocity:

$$138 \quad w_{wtg}(z, t) = \frac{D_\theta(z, t)M(z)}{\tau}, \quad (4)$$



139 where  $D_\theta(z, t)$ —interpreted as the height a parcel needs to reach to remove a heating  
 140 anomaly—is given by:

$$141 \quad D_\theta(z, t) = \frac{\bar{\theta}(z, t) - \theta_{ref}(z, t)}{\frac{\partial \bar{\theta}(z, t)}{\partial z}}. \quad (5)$$

142 Here,  $\bar{\theta}(z, t)$  is the domain averaged potential temperature,  $\theta_{ref}(z, t)$  is the reference pro-  
 143 file of potential temperature,  $\tau$  is a relaxation timescale by which the model potential  
 144 temperature is relaxed to the reference potential temperature, and  $M(z) = \sin(\pi z/h)$   
 145 is a masking that modulates the enforcement of the WTG approximation to the region  
 146 between the surface and the tropopause ( $h$ , assumed 15 km). The WTG velocity is inter-  
 147 polated from the boundary layer height (assumed 1 km) to zero at the surface. Also, the  
 148 relaxation time scale,  $\tau$ , can be interpreted as the time it takes gravity waves to travel  
 149 the domain length and is taken to be 1 h in this study.

150 Recently, *Herman and Raymond* [2014] proposed a modification of this conventional  
 151 WTG approximation which accounts for the fact that different heating profiles result in  
 152 gravity waves of different speeds [*Bretherton and Smolarkiewicz*, 1989]. In contrast to the  
 153 conventional WTG approximation—which assumes one phase speed for all wave numbers of  
 154 buoyancy waves—the spectral WTG (SWTG) does a Fourier decomposition of the heating  
 155 anomaly and assigns each wave number a corresponding phase speed:

$$156 \quad c_j = N/m_j, \quad (6)$$

157 where  $N$  is the Brunt-Väisälä frequency, and  $m_j$  is the vertical wave number mode. In  
 158 SWTG, the equivalent height,  $D_\theta(z, t)$  can be written as:

$$159 \quad D_\theta(z, t) = \sum_j \Theta_j(t) \sin(m_j z), \quad (7)$$

160 where

$$161 \quad \Theta_j(t) = \frac{2}{h} \int_0^h D_\theta(z, t) \sin(m_j z) dz, \quad (8)$$

162 are the Fourier modes, and  $h$  is the tropopause height. The SWTG vertical velocity can  
163 then be written as:

$$164 \quad w_{swtg}(z, t) = \sum_j \frac{\Theta_j(t)}{\tau_j} \sin(m_j z), \quad (9)$$

165 where

$$166 \quad \tau_j = L/c_j = Lm_j/N = \pi Lj/hN, \quad (10)$$

167 is the relaxation time scale for each vertical wave number. In contrast to WTG where  
168 the user sets  $\tau$ , the primary length scale  $L$  is the free parameter in SWTG, related to  $\tau$   
169 approximately by  $L = \tau h N / \pi$ . In this study, we will present SWTG simulations with  $L =$   
170 171 km, which is roughly equivalent to  $\tau = 1$  h in the conventional SWTG. Henceforth,  
171 we will refer to the SWTG as WTG for convenience.

172 WTG sources of entropy and mixing ratio due to enforcing WTG are given by:

$$173 \quad S_{ent} = w_{wtg} \frac{\partial \bar{s}}{\partial z} + (\bar{s} - s_{ent}) \frac{1}{\rho_0} \frac{\partial \rho_0 w_{wtg}}{\partial z}, \quad (11)$$

$$174 \quad S_{r_t} = w_{wtg} \frac{\partial \bar{r}_t}{\partial z} + (\bar{r}_t - r_{t-ent}) \frac{1}{\rho_0} \frac{\partial \rho_0 w_{wtg}}{\partial z}, \quad (12)$$

176 where  $\rho_0$  is the domain average density,  $\bar{s}$  and  $\bar{r}_t$  are domain averaged moist entropy and  
177 total water vapor mixing ratio, and, depending on whether mass flux decreases with  
178 height (mass export) or increases with height (mass import):

$$179 \quad S_{ent}, r_{t-ent} = \begin{cases} \bar{s}, \bar{r}_t & \text{if } \frac{\partial \rho_0 w_{wtg}}{\partial z} < 0 \text{ (mass export),} \\ s_{ref}, r_{t-ref} & \text{if } \frac{\partial \rho_0 w_{wtg}}{\partial z} > 0 \text{ (mass import),} \end{cases} \quad (13)$$

180 where  $s_{ref}$  and  $r_{t-ref}$  are reference profiles of moist entropy and water vapor mixing ratio.

181 The WTG sources are used in the moist entropy and total water vapor mixing ratio

182 equations to parameterize the effect of the environment on local modeled convection in  
 183 the prognostic equations:

$$184 \quad \frac{\partial \rho s}{\partial t} + \nabla \cdot (\rho \mathbf{v} s - K \nabla s) = \rho (S_{ss} + S_{sr} - S_{ent}), \quad (14)$$

$$185 \quad \frac{\partial \rho r_t}{\partial t} + \nabla \cdot (\rho \mathbf{v} r_t - K \nabla r_t) = \rho (S_{rs} + S_{rp} - S_{rt}), \quad (15)$$

187 where  $K$  is the eddy-mixing coefficient,  $\mathbf{v}$  is the velocity vector,  $S_{ss}$  and  $S_{sr}$  are the source  
 188 of moist entropy from surface fluxes and radiation respectively,  $S_{rs}$  is the total water vapor  
 189 mixing ratio source due to surface fluxes, and  $S_{rp}$  is minus the conversion rate of cloud  
 190 water to precipitation [details about these terms can be found in *Herman and Raymond,*  
 191 2014].

## 2.2. Observations and Diagnostic Variables

192 We use version 3a of the Colorado State University quality controlled observations from  
 193 the DYNAMO field campaign [*Johnson and Ciesielski, 2013; Ciesielski et al., 2014a, b;*  
 194 *Johnson et al., 2015*]. We also use high resolution radiosonde data from individual stations  
 195 in the DYNAMO array [ARM AMF Radiosonde L3 Data for stations Male, Colombo,  
 196 Gan, and Diego Garcia, and L3.1 for Research Vessels *Revelle* and *Mirai*; e.g. *Ciesielski*  
 197 *et al., 2014a*], positions of which are shown in figure 1. These observations are used for  
 198 obtaining time dependent reference profiles of potential temperature, mixing ratio, and  
 199 surface winds which are used to force the WTG simulations (see below).

200 The diagnostic variables we compare are precipitation rate, saturation fraction, instabil-  
 201 ity index and gross moist stability (GMS). These diagnostics provide important informa-  
 202 tion about the relationships between convection and the environment, and are significant  
 203 in the context of the tropical atmosphere [*Bretherton et al., 2004; Raymond et al., 2007;*

204 *Raymond and Sessions, 2007; Gjorgjievska and Raymond, 2014; Sessions et al., 2015;*  
 205 *Raymond et al., 2015*]. Precipitation rate is a direct measure of convective activity, while  
 206 saturation fraction—defined as precipitable water divided by saturated precipitable water—  
 207 diagnoses the moistening effects of convection, with larger values corresponding to moister  
 208 air columns. The instability index diagnoses the stability of air in the column. It is defined  
 209 as [e.g. *Raymond et al., 2011; Sessions et al., 2015*]:

$$210 \quad II = s_{low}^* - s_{high}^*, \quad (16)$$

211 where  $s_{low}^*$ , and  $s_{high}^*$ , are the horizontally averaged saturated moist entropy integrated  
 212 from 1-3 km, and 5-7 km, respectively. Smaller values of the instability index correspond  
 213 to a more stable column of air.

214 *Neelin and Held [1987]* introduced the GMS as a variable that quantifies the relationship  
 215 between convection and convective forcing in absence of the details of the nature of the  
 216 relationship. GMS is defined as a ratio of export of a quantity conserved in moist processes,  
 217 and import of a quantity that measures strength of convection [see *Raymond et al., 2009,*  
 218 for a comprehensive overview of GMS]. In this work we calculated GMS from observations,  
 219  $\Gamma_{obs}$ , analogous to *Sobel et al. [2014]*, as the ratio of moist and dry entropy divergence:

$$220 \quad \Gamma_{obs} = \frac{\left\langle \omega \frac{\partial s}{\partial p} + \mathbf{v} \cdot \nabla s \right\rangle}{\left\langle \omega \frac{\partial s_d}{\partial p} \right\rangle}, \quad (17)$$

221 where  $\omega$  is the vertical velocity in pressure coordinates, and the brackets denote vertical  
 222 pressure integral from surface to 100 hPa. Also, we neglect the horizontal part of dry  
 223 entropy divergence as in *Inoue and Back [2015a]*.

224 In calculating GMS from numerical output, the WTG velocity takes the place of large  
 225 scale vertical motion, and the lateral import or export of moist entropy and total mixing

ratio is dependent on the layers where mass flux is positive or negative respectively. The model GMS is defined as:

$$\Gamma_{mod} = \frac{\langle S_{ent} \rangle}{\langle w_{wtg} \frac{\partial s_d}{\partial z} \rangle}, \quad (18)$$

where  $S_{ent}$  is the source of moist entropy due to the WTG approximation (equation 11), and the brackets denote mass weighted height integral from the surface to the tropopause (15 km, around 100 hPa).

### 2.3. Methodology

We study the relationships between the observed and modeled DYNAMO diagnostics using the following procedure:

1. We run the model in non-WTG mode, with interactive radiation, until it reaches a state of radiative-convective equilibrium (RCE, i.e convective forcing balances radiative cooling). For the RCE simulation, we use an SST of 302 K, which is the average SST in the DYNAMO intensive observation region (see below). We set the surface wind speed to 2 m s<sup>-1</sup>, which is the average surface wind speed—here defined as the wind speed at the lowest level—during DYNAMO, and run the model on a 2D model domain 200 km long (1 km horizontal resolution) and 20 km high (250 m vertical resolution); the WTG simulations use the same model geometry as the RCE simulations. We obtain RCE potential temperature and total water mixing ratio profiles by averaging the last thirty days of a fifty day RCE simulation.

2. Observed potential temperature and mixing ratio anomalies, which represent changes in environmental conditions, are obtained by subtracting the time mean temperature and water vapor mixing ratio profile from the observational time series. We obtain anomalies for both DYNAMO array averaged profiles (North Sounding Array, NSA, and South

248 Sounding Array, SSA), and six individual sounding stations that compose NSA and SSA  
249 arrays (Male, Colombo, Gan, Diego Garcia, and Research Vessels Revelle and Mirai, see  
250 figure 1 for positions).

251 3. Finally, time dependent reference profiles for WTG simulations are obtained by  
252 adding the observed DYNAMO potential temperature and water vapor mixing ratio  
253 anomalies (interpolated on the model grid) to the RCE profiles of potential temperature  
254 and mixing ratio.

255 4. In addition to the WTG reference profiles, observed surface wind speed and SST  
256 variations are imposed in the WTG simulations. We do not consider the effects of shear  
257 and radiation in this study; we specify a non-interactive static cooling rate derived from  
258 the RCE simulation.

259 For the NSA and SSA sounding arrays, we imposed daily SST variations derived from  
260 ERA-I [*Dee et al.*, 2011] since we lacked complete and representative surface observations  
261 [*Wang et al.*, 2014]. Lacking SST observations for the single station simulations, we used  
262 an SST of 302 K (the average value from NSA and SSA derived from ERA-I). This  
263 treatment is justified since sensitivity studies (not shown) on NSA and SSA stations  
264 showed that removing or doubling SST variations did not have a significant impact on  
265 WTG results.

266 In addition to a qualitative comparison of observed and modeled diagnostic time series,  
267 we compare them quantitatively by computing statistical quantities such as correlation co-  
268 efficients (with t-student tests for significance levels), mean, and standard deviation; these  
269 are presented in the appendix. Since GMS is a sensitive variable, especially in numerical  
270 simulations, the time series data is smoothed with a five day moving average before daily

271 averaging the time series; global characteristics of the diagnostics are preserved despite  
272 the loss in detail.

### 3. Results and discussion

273 In this section we first compare time series in observed and modeled DYNAMO di-  
274 agnostics. The time series were smoothed with a five day moving average in order to  
275 focus on large time scales and processes that influence the development of convection in  
276 DYNAMO [*Sobel et al.*, 2014; *Inoue and Back*, 2015b]. Next, we compare observed and  
277 modeled diagnostic relationships and, where possible, compare them with past work. Here  
278 we do three sets of comparisons for each diagnostic relationship: observations with model,  
279 the NSA with the SSA, and arrays with the single stations. Details about time series for  
280 single station WTG simulations can be found in the appendix.

#### 3.1. Diagnostic time series

281 Figure 2 shows the NSA and the SSA observed (black) and modeled (red) time series of  
282 precipitation rate (panels a–b), saturation fraction (panels c–d), instability index (panels  
283 e–f), and GMS (panels g–h), smoothed with a five day moving average. Despite the  
284 applied smoothing, trends in both observed and modeled time series are preserved. Also,  
285 after smoothing the GMS denominator and numerator, time periods where denominator  
286 is less than one fifth of the numerator have been removed, similar to *Sobel et al.* [2014], to  
287 avoid division by zero. For the reader’s convenience, as rough guidelines, vertical dashed  
288 lines are aligned with precipitation maxima in the NSA, at days 25, 56, and 81.5. Despite  
289 the fact that precipitation maxima occur at different times for the NSA and the SSA, we  
290 use the NSA guidelines in the SSA plot for comparison.

### 291 3.1.1. Comparison of observations and simulations

292 Figure 2 shows three distinct precipitation intense events around 25, 56 and 81.5 days for  
293 the NSA, all of which are approximately captured by the model. Compared to the NSA,  
294 the SSA has less intense precipitation rates with maxima that lag the NSA maxima for the  
295 first two events. The precipitation rate increase prior to the NSA precipitation maxima  
296 is correlated with an increase in saturation fraction and a decrease in instability index,  
297 both in observations and simulations (though the relationship is not equally strong for  
298 every precipitation event, see e.g. first event in the SSA). The inverse holds for decrease of  
299 precipitation rate with time; after the precipitation maxima, saturation fraction decreases  
300 and instability index increases. This behavior is in agreement with a thermodynamic  
301 mechanism proposed in previous idealized WTG simulations and observations [*Raymond*  
302 *and Sessions, 2007; Gjorgjievska and Raymond, 2014; Sessions et al., 2015; Raymond*  
303 *et al., 2011, 2014, 2015*]. This mechanism posits that changes in the atmospheric stability  
304 (quantified here via the instability index) result in changes to the convective profile, which  
305 in turn adjusts the vertical advection and lateral entrainment of moisture (quantified by  
306 saturation fraction). A decrease in the instability index (i.e. a more stable environment)  
307 causes more bottom heavy mass flux profiles which entrain moister lower tropospheric  
308 air, and consequently increases the saturation fraction. An increased saturation fraction,  
309 in turn, results in higher precipitation rates, which has been confirmed in a number of  
310 studies [*Bretherton et al., 2004; Raymond et al., 2007; Masunaga, 2012*].

311 The saturation fraction and instability index peak and dip, respectively, in phase with  
312 peak precipitation rates, with significant differences in observed and modeled mean magni-  
313 tudes. The discrepancy between modeled and observed saturation fraction and instability



314 index could arise in part because of model parameters (e.g. dimensionality, microphysics,  
315 etc.), and the RCE profiles used in the WTG reference profiles. The parameters used in  
316 the model [*Raymond and Zeng, 2005*] are, approximately, standard values found in the  
317 tropics, which might vary with geographic location. Alternatively, the model might be  
318 missing an essential mechanism for modeling DYNAMO convection. Here, we restrict our  
319 discussion to the possible influence of the model setup to explain the discrepancy between  
320 observed and modeled saturation fraction and instability index, while possible mechanism  
321 deficiencies are beyond the scope of this study.

322 The discrepancy between observed and modeled saturation fraction and instability in-  
323 dex might arise because of the RCE profiles used in the WTG reference profiles—computed  
324 using ERA-I SST averaged over the NSA and the SSA domains. However, model param-  
325 eters and dimensionality of the computation domain [*Wang and Sobel, 2011*] influence  
326 significant differences between the RCE and observed mean potential temperature and  
327 moisture. Figure 3 shows the difference between the observed (black) and modeled (RCE,  
328 red) mean potential temperature (panel a) and water vapor mixing ratio (panel b). The  
329 RCE profile is cooler and shows a slightly smaller lapse rate than observations which re-  
330 sults in a smaller instability index compared to the observed mean profile. On the other  
331 hand, the RCE simulation is drier than the observed; the model, however, exhibits a  
332 higher saturation fraction because the more stable RCE environment more efficiently en-  
333 trains environmental air [*Raymond and Sessions, 2007; Sessions et al., 2015*]. Sensitivity  
334 studies (not shown) revealed that using RCE profiles of potential temperature and mixing  
335 ratio closer to the observed mean (e.g. using RCE profiles from SST of 304 K, shown  
336 with a red dashed line in figure 3), decreases the difference between observed and mod-

337 eled saturation fraction and instability index. However, the general results of this paper  
338 are not affected by this issue, and, for consistency, we choose to use the RCE profiles  
339 corresponding to the reanalysis SSTs.

340 Previous work [*Ciesielski et al.*, 2014b; *Sobel et al.*, 2014] has addressed the energy  
341 budget in DYNAMO, and proposed mechanisms that control the development of con-  
342 vection. In an analysis of the moist static energy budget, *Sobel et al.* [2014] concluded  
343 that the effects of radiation and surface fluxes exert the dominant control of convection  
344 during DYNAMO. However, convection may evolve as a result of changes to the atmo-  
345 spheric stability, which may be a consequence of changes in radiation and surface fluxes,  
346 and—by modification of the convective profile—would influence the vertical and horizontal  
347 advection of moist static energy. Alternatively, *Inoue and Back* [2015a] examined TOGA  
348 COARE data and found that vertical and horizontal advection of moist static energy is  
349 important for the development and decay of convection, and that radiation and surface  
350 fluxes act to destabilize convection throughout convective events. The effects of horizon-  
351 tal and vertical transport of convection can be summarized by the gross moist stability  
352 (GMS), and we address them next.

### 353 3.1.2. Gross Moist Stability (GMS)

354 GMS summarizes the effects of large scale interactions on convection; understanding  
355 what controls GMS can lead to a better representation of convection in coarsely resolved  
356 models.

357 The observed GMS (figure 2g–h) increases from small and even slightly negative values  
358 during developing convection (periods of increasing precipitation rate, prior to precip-  
359 itation maxima on days 25, 56 and 81.5), and it continues to increase for dissipating

360 convection (decreasing precipitation rate, after the precipitation maxima). The model  
 361 GMS behaves similarly, but with a magnitude that is smaller than the observed at peak  
 362 precipitation rates. Despite this difference in magnitude, the modeled GMS also shows an  
 363 increase similar to the observed GMS, relative to the GMS values at peak precipitation  
 364 rates; GMS increases throughout the life cycle of the convective event.

365 This GMS behavior has been addressed in a recent study. Based on TOGA-COARE  
 366 data, *Inoue and Back* [2015a] developed a theory for convective development and decay  
 367 relative to a characteristic GMS value:

$$368 \quad \Gamma_C = \frac{\langle Q_R \rangle + \langle SF \rangle}{\langle \nabla \cdot (s_d \vec{v}) \rangle}, \quad (19)$$

369 where  $Q_R$  and  $SF$  are radiative and surface flux forcing, respectively, and  $\nabla \cdot (s_d \vec{v})$  is  
 370 the divergence of dry entropy. The authors show that convective development (precipi-  
 371 tation increases with time) occurs for GMS values lower than  $\Gamma_C$ —which turns out to be  
 372 approximately constant during TOGA COARE [*Inoue and Back*, 2015a]—while convective  
 373 decay (precipitation decreases with time) happens for GMS values higher than  $\Gamma_C$ . This  
 374 behavior can be seen for some of the convective events in figure 4a–b, which shows the  
 375 GMS (from figure 2g–h, thick lines), and the characteristic GMS ( $\Gamma_C$ , thin lines) for the  
 376 observations and the model; for comparison, the dashed vertical lines represent the NSA  
 377 precipitation maxima. The model follows the theory and observations from *Inoue and*  
 378 *Back* [2015a];  $\Gamma_C$  remains fairly constant, and  $\Gamma < \Gamma_C$  for developing modeled convection,  
 379 and  $\Gamma > \Gamma_C$  for decaying modeled convection. A particularly nice model example is seen  
 380 on days 50–60. The observed  $\Gamma_C$ , on the other hand, deviates somewhat from the theory;  
 381 at times the observed GMS is in phase with  $\Gamma_C$ , while at times it shows behavior opposite

382 to that of *Inoue and Back* [2015a]. This discrepancy might be influenced by noise in the  
 383 observational data, analysis of which is beyond the scope of this study.

384 In addition to the usefulness of the characteristic GMS time series in studying the  
 385 dynamics of convection, calculating a characteristic value for a particular data set can  
 386 also be useful. *Inoue and Back* [2015a] list several different methods for calculating the  
 387 characteristic GMS found in the literature; here we redefine them in terms of moist entropy  
 388 instead of moist static energy. According to *Inoue and Back* [2015a], the characteristic  
 389 GMS can be calculated as:

390 1. GMS at maximum anomalous precipitation:

$$391 \quad \Gamma'_{max} \equiv \Gamma|_{P_{max}}, \quad (20)$$

392 2. GMS computed from a scatter plot of anomalous  $\nabla \cdot \langle s\mathbf{v} \rangle$  versus  $\nabla \cdot \langle s_d\mathbf{v} \rangle$ :

$$393 \quad \tilde{\Gamma}' \equiv \frac{\overline{\langle \nabla \cdot (s\mathbf{v}) \rangle' \cdot \langle \nabla \cdot (s_d\mathbf{v}) \rangle'}}{(\overline{\langle \nabla \cdot (s_d\mathbf{v}) \rangle'})^2}, \quad (21)$$

394 3. GMS computed from a scatter plot of non-anomalous  $\nabla \cdot \langle s\mathbf{v} \rangle$  versus  $\nabla \cdot \langle s_d\mathbf{v} \rangle$ :

$$395 \quad \tilde{\Gamma} \equiv \frac{\overline{\langle \nabla \cdot (s\mathbf{v}) \rangle \cdot \langle \nabla \cdot (s_d\mathbf{v}) \rangle}}{(\overline{\langle \nabla \cdot (s_d\mathbf{v}) \rangle})^2}, \quad (22)$$

396 4. and, GMS in a quasi-steady state:

$$397 \quad \Gamma_0 \equiv \frac{\overline{\langle \nabla \cdot (s\mathbf{v}) \rangle}}{\overline{\langle \nabla \cdot (s_d\mathbf{v}) \rangle}}. \quad (23)$$

398 In this paper, we calculate the characteristic GMS,  $\Gamma_C$ , using each of these methods for  
 399 both the NSA and the SSA in modeled and observed data. These results are shown in  
 400 Table 1. Both the NSA and the SSA have characteristic GMS values close to 0.4, across all  
 401 of the definitions. The modeled values, on the other hand, show more variability across the  
 402 four definitions, and range from 0.1 to 0.2, with the SSA leaning toward higher values. We

403 address the non-trivial difference between the observed and modeled characteristic GMS  
 404 below, and in section 3.2.

405 The difference between the observed and modeled characteristic values can be analyzed  
 406 by decomposing the GMS into contributions from vertical,  $\Gamma_V$ , and horizontal,  $\Gamma_H$ , mo-  
 407 tions. To accommodate the coordinate systems of the observational and model data set,  
 408 we have:

$$409 \quad \Gamma_{obs-V} = \frac{\langle \omega \frac{\partial s}{\partial p} \rangle}{\langle \omega \frac{\partial s_d}{\partial p} \rangle}, \quad (24)$$

$$410 \quad \Gamma_{obs-H} = \frac{\langle \mathbf{v} \cdot \nabla s \rangle}{\langle \omega \frac{\partial s_d}{\partial p} \rangle}, \quad (25)$$

411  
 412 for the observations, and:

$$413 \quad \Gamma_{mod-V} = \frac{\langle w_{wtg} \frac{\partial \bar{s}}{\partial z} \rangle}{\langle w_{wtg} \frac{\partial s_d}{\partial z} \rangle}, \quad (26)$$

$$414 \quad \Gamma_{mod-H} = \frac{\langle (\bar{s} - s_{ent}) \frac{1}{\rho_0} \frac{\partial \rho_0 w_{wtg}}{\partial z} \rangle}{\langle w_{wtg} \frac{\partial s_d}{\partial z} \rangle}, \quad (27)$$

416 for the model (see term definitions in sections 2.1 and 2.2). Figure 4 (panels c–f) show  
 417 the horizontal and vertical components of the GMS numerator; they differ in magnitude,  
 418 both for the observations and the model. The NSA horizontal transport of moist entropy  
 419 (panel c) is about an order of magnitude smaller than the vertical transport (panel e)  
 420 during intense precipitation periods; the difference is smaller for the SSA (panels d and f).  
 421 This indicates the importance of the vertical transport in the development of convection  
 422 in DYNAMO, and it is in agreement with *Inoue and Back* [2015a]. The source of the  
 423 difference in the observed and the modeled GMS magnitudes seen in figure 4a,b can be  
 424 seen in figure 4e,f; the model consistently underestimates the vertical transport of moist  
 425 entropy compared to observed values.

426 The observed and modeled GMS denominator (figure 4g,h) are in excellent agreement  
427 for the NSA, while it is generally slightly underestimated for the SSA. The success of the  
428 model in reproducing the denominator—the divergence of the dry entropy,  $w_{wtg}\partial s_d/\partial z$ —is  
429 directly related to enforcement of the WTG approximation. In the WTG approximation  
430 the vertical gradient of the dry entropy,  $\partial s_d/\partial z$ , is constrained explicitly by the relaxation  
431 of the temperature to the reference environment. The WTG vertical velocity,  $w_{wtg}$ , on  
432 the other hand, is not explicitly constrained and is calculated according to equation 4  
433 or 9. Since the model captures the trends in the observed denominator, this indicates  
434 that the WTG approximation is adequately capturing the variations in the large scale  
435 vertical velocity. On the other hand, the weaker denominator for the SSA indicates that  
436 the modeled SSA vertical advection is too weak, and could possibly be adjusted by fitting  
437 the time scale  $\tau$  or length scale  $L$ . The difference between the NSA and the SSA WTG  
438 simulations might indicate differing dynamics in the two regions during DYNAMO, and  
439 may provide clues as to which are well represented by WTG.

### 440 3.1.3. Possible sources of stability changes.

441 So far our results suggest, in conjunction with previous work [*Raymond and Sessions,*  
442 2007; *Sessions et al., 2015; Raymond et al., 2015*], that increased atmospheric stability  
443 (quantified by lower instability index) increases the moisture content (quantified by higher  
444 saturation fraction), and consequently increases the precipitation rate. Since the WTG  
445 simulations are forced by atmospheric stability changes—changes in the reference potential  
446 temperature profiles—the question of the sources of those changes arises.

447 In a recent paper, *Raymond et al. [2015]* develop a hypothesis relating the interplay of  
448 dynamics and thermodynamics in tropical convection. Given an evolution of the poten-

449 tial vorticity field with necessary boundary conditions and a balance condition–nonlinear  
450 balance, in case of the tropics– one could deduce the evolution of atmospheric stability,  
451 via changes to the virtual temperature [which are closely related to changes in potential  
452 temperature, e.g., see *Raymond et al.*, 2014]. We do not know if this type of balanced  
453 dynamics is at work in the DYNAMO MJO, and although this data deserves a full anal-  
454 ysis in this context, we present preliminary results which relate some of the diagnostics  
455 used from *Raymond et al.* [2015] to the dynamics in the DYNAMO MJO. Specifically, we  
456 explore the possibility that there is a relationship between the vorticity and divergence in  
457 this case, as hypothesized for tropical convection [*Raymond et al.*, 2015].

458 Figure 5 shows budget derived [*Johnson and Ciesielski*, 2013; *Ciesielski et al.*, 2014a, b;  
459 *Johnson et al.*, 2015] time series of divergence and vorticity–smoothed with a five day  
460 moving average–for the NSA and the SSA. Here, we focus only on observations since the  
461 idealized setup of WTG simulations does not capture large scale vorticity and divergence.  
462 The variables were averaged vertically in layers–0-15 km, 0-3 km, 3-8 km, and 8-15 km–in  
463 order to analyze the vertical structure of vorticity and divergence, and to speculate on  
464 possible correlation with stability changes. Despite the fact that the tropospheric vertical  
465 average from 0 to 15 km (black solid line) shows no features which might correlate with  
466 stability changes, significant patterns emerge when we divide the troposphere into layers.

467 Both for the NSA and the SSA divergence (figure 5, panels a–b) shows strong positive  
468 values in the upper levels (8-15 km), signifying mass export, while weaker negative values  
469 of divergence occur at mid (3-8 km) and low (0-3 km) levels simultaneously, signifying  
470 mass import. Compared to the vorticity (panels c-d) the divergence is much stronger,

471 especially in the upper levels. Despite the fact that the divergence is much stronger than  
472 vorticity, divergence patterns show uniformity across the different convective events.

473 The vorticity, on the other hand, distinguishes differing dynamics between the first,  
474 second, and third convective events both in the NSA and the SSA. In the NSA, the first  
475 event (days 15–35), shows little vortical activity during the peak precipitation rate period  
476 from days 25–30. Vorticity shows patterns, however weak, during the developing (days 15–  
477 25) and decaying (days 30–35) phases of the first convective event. The vertical structure  
478 shows a dipole (negative vorticity at low levels, positive vorticity aloft) in the developing  
479 and the decaying phase; the weakness of the vorticity makes it hard to distinguish the  
480 vorticity significantly different than zero, so we cannot speculate on the dynamics of  
481 these vorticity features. For the second NSA event (days 50–60) the low and mid level  
482 vorticity show a prominent peak around the precipitation rate maximum, which could be  
483 associated with the atmospheric stability changes and convective development during that  
484 period. The second vorticity anomaly might be associated with a passing cyclone visible  
485 in infrared satellite imagery (not shown). The third precipitation event shows persistent  
486 surface vorticity with a developing mid level vortex which peaks around the precipitation  
487 maximum.

488 The SSA shows similar conditions with some major differences; apart from the fact  
489 that the SSA is on the southern hemisphere so that the vorticity bears, in general, the  
490 opposite sign compared to the NSA. The first precipitation event is accompanied by a  
491 persistent vorticity dipole from days 10–40. The constancy of this vorticity dipole might  
492 be associated with the moderate persistent precipitation rates from days 10–30, while the  
493 development of the mid level vorticity around day 33 might be associated with decay of



494 the convection. The second SSA precipitation event is associated with strong vorticity in  
495 low and mid levels, similar to the NSA. Lastly, the third precipitation event shows similar  
496 conditions as the NSA, with vorticity bearing the opposite sign.

497 In conclusion, balanced vorticity dynamics might be associated with atmospheric sta-  
498 bility changes, because of distinct patterns observed in the DYNAMO arrays. However,  
499 further work is necessary to confirm this hypothesis.

#### 500 **3.1.4. Individual stations in the DYNAMO array and WTG verification.**

501 To obtain time series and verify the performance of the WTG approximation in simulat-  
502 ing the convection during DYNAMO, we compared observed and modeled daily averaged  
503 time series of precipitation rate, saturation fraction, and instability index for WTG ap-  
504 plied to individual stations in DYNAMO. It is useful to compare these with the results  
505 from the NSA and the SSA so that we have a measure for how well WTG performs using  
506 observations over different spatial scales. This result might help interpret WTG results  
507 from data taken from isolated soundings from field campaigns that do not have an or-  
508 dered configuration of arrays like DYNAMO or TOGA COARE. A detailed account of  
509 the verification of WTG simulations on both station arrays (the NSA and the SSA), and  
510 individual stations, can be found in the Appendix.

511 In general, our comparisons of modeled and observed convective diagnostics for both  
512 regional arrays and single stations show that the WTG approximation qualitatively and  
513 quantitatively does a good job in reproducing the observed diagnostics with at least  
514 some of the differences accountable to the model parameters (e.g. microphysics and RCE  
515 profiles used in WTG reference profiles as explained above). There are some differences,  
516 however, that are not easily attributable to model parameters, which suggests that other

mechanisms which do not act through the thermodynamic environment are also important for modulating tropical convection. Although understanding what these are is important for improving convective parameterizations, a systematic analysis is beyond the scope of this study.

### 3.2. Diagnostic relationships

In this section we consider diagnostic relationships between precipitation rate, saturation fraction, instability index, and GMS. The relationship between these diagnostics has been studied in satellite observations as well as in models in past work. For example, *Bretherton et al.* [2004], and *Raymond et al.* [2007] have proposed a strong relationship between precipitation rate and saturation fraction. Also, numerical studies by *Raymond and Sessions* [2007], *Sessions et al.* [2015], and *Raymond et al.* [2015] showed a strong dependence of the saturation fraction on the stability of the column air. We compare diagnostic relationships derived from observations and WTG simulations to see how significant the relationships are in observed data, and test if the WTG approximation preserves the diagnostic relationships. We do so for the NSA, the SSA, and all the results from the single station simulations.

Figure 6 shows observed (blue and black) and modeled (red) relationships between precipitation rate and saturation fraction (panels a–c), precipitation rate and instability index (panels d–f), and saturation fraction and instability index (panels g–i), for the NSA (panels a, d, and g), the SSA (panels b, e, and h), and the single station simulations (panels c, f, and i). We divided the observed data into two regimes because of distinctive behavior in observed relationships; events with precipitation rates lower than  $P_B$ , and saturation fraction less than  $S_B$  (gray shaded area in panels a–c) are colored blue, while

539 the remaining data is colored black. We choose  $P_B = 5 \text{ mm day}^{-1}$ , and  $S_B = 0.73$   
 540 somewhat arbitrarily, but our choice does not qualitatively affect our results. We explain  
 541 the reasons for this separation in detail below.

542 For qualitative comparison with previous research, we overlay the precipitation rate–  
 543 saturation fraction functional relationships (figure 6a–c) from *Bretherton et al.* [2004,  
 544 dashed blue]:

$$545 \quad P(S) = \exp[b(S - S_D)], \quad (28)$$

546 where  $S$  is saturation fraction, and the numbers  $b = 15.6$ , and  $S_D = 0.603$ , are fitting  
 547 parameters, and from *Raymond et al.* [2007, solid blue];

$$548 \quad P(S) = R_{RCE} * \frac{S_C - S_{RCE}}{S_C - S}, \quad (29)$$

549 where the number  $S_C = 0.87$  is a fitting parameter, and  $R_{RCE} = 4 \text{ mm day}^{-1}$  and  
 550  $S_{RCE} = 0.81$  are steady state precipitation rate and saturation fraction for an RCE  
 551 simulation. For the saturation fraction–instability index relationship (figure 6g–i) we  
 552 overlay [*Raymond et al.*, 2015, solid blue]:

$$553 \quad S(II) = c * II + d, \quad (30)$$

554 where  $II$  is the instability index, and  $c = -0.00433$ , and  $d = 0.897$  are fitting parameters.  
 555 For quantitative comparison with previous work, we perform exponential and linear fits  
 556 for the precipitation rate–saturation fraction and saturation fraction–instability index  
 557 relationships, respectively, for the observations (dashed black curves) and model (dashed  
 558 red curves). Table 2 lists the fit parameters with 95% confidence intervals, together with  
 559 fit parameters for relationships from previous work for comparison.

**Precipitation rate–saturation fraction relationship.** The precipitation rate (figure 6a–c) increases with saturation fraction for both the average arrays and for all the single station simulations. The observed NSA and SSA precipitation rate–saturation fraction scatters follow both the fit line (black dashed line) and the satellite observational fit from *Bretherton et al.* [2005, equation 28, blue dashed line], while the model scatter–and its exponential fit (red dashed line)– is closer to the *Raymond et al.* [2007, equation 29] power law relationship; qualitatively it seems that the modeled relationship might follow an exponential form. Quantitative conclusions on whether this relationship follows exponential or power law forms is beyond the scope of this study; the number of DYNAMO observations might be too small to make a statistically significant conclusion. Furthermore, the SSA relationship for both model and observations seem to follow the NSA relationships but do not extend to higher precipitation rates. Indeed, fit parameters for the precipitation rate–saturation fraction relationship (table 2) are similar for the NSA and the SSA, within variability, for both the observations and the model. In contrast to the NSA and the SSA, the single station observations and simulations show more scatter. One possible reason for less scatter in the NSA and SSA relationships might be the smoothing effect of the objective analysis used to interpolate data sets [*Johnson and Ciesielski, 2013; Ciesielski et al., 2014a*] before they were averaged. Despite the scatter, however, the single station precipitation rate is a strong function of saturation fraction, similar to the NSA and the SSA, for both the observations and the model (table 2, and figure 6a–c). This is the most important point of this result.

Both the model and observed data seem to have better fits to the exponential forms [equation 28 *Bretherton et al., 2005*] than to the inverse relation with with the saturation

583 deficit [equation *Raymond et al.*, 2007]. We note that the *Raymond et al.* [2007] relation  
584 was obtained from idealized steady state WTG simulations; deviations from this may  
585 arise as a result of transient dynamics. Furthermore, the NSA and the SSA observations  
586 are averaged over a larger area, similar to satellite measurements. Other studies with  
587 satellite data [*Masunaga*, 2012] have shown that the precipitation rate–saturation frac-  
588 tion relationship is time dependent, and that it changes for different orders of convective  
589 organization.

590 **Precipitation rate–instability index relationship.** The precipitation rate increases  
591 with decreasing instability index (figure 6d–f) for the NSA, the SSA, and all the single  
592 stations. The NSA shows this prominently for both the observations and the model. How-  
593 ever, the NSA also exhibits events with low precipitation rate for low instability index,  
594 which the model does not capture. These outliers fall into the region of precipitation rates  
595 lower than  $5 \text{ mm day}^{-1}$  and saturation fractions lower than 0.73 (gray shaded region in  
596 figure 6a–c). This behavior suggests that the WTG approximation does not capture a  
597 mechanism which inhibits precipitation events for increased stability (i.e. decreased insta-  
598 bility index) of the outliers. We find similar behavior in the SSA though the relationship  
599 in the SSA is much less obvious. Compared to the NSA and the SSA, the single station  
600 simulations also show more variance in atmospheric stability for low rain rates. Never-  
601 theless, they still show a general increase in precipitation for more stable environments,  
602 consistent with the balanced dynamics hypothesis of *Raymond et al.* [2015].

603 As discussed in section 3.1.3, the balanced dynamics hypothesis explains how precipi-  
604 tation rate increases for lower instability index. A more stable atmosphere (quantified by  
605 a lower instability index) produces more bottom heavy mass flux profiles which entrain

606 more moist air from the low levels [*Raymond and Sessions, 2007; Raymond et al., 2011;*  
607 *Gjorgjievska and Raymond, 2014; Raymond et al., 2014; Sessions et al., 2015; Raymond*  
608 *et al., 2015*]. This increases the saturation fraction, and consequently the precipitation  
609 rate (figure 6a–c). Data for low precipitation rates and low saturation fractions do not  
610 seem to follow this general trend (blue dots, figure 6d–f), perhaps because of dry air intru-  
611 sion that suppresses saturation fraction and therefore precipitation rates in more stable  
612 environments (this, however, has not been tested).

613 **Saturation fraction–instability index relationship.** The saturation fraction, in  
614 general, increases with lower instability index (figure 6g) in the NSA, both for the model  
615 and the observations. The outliers from the observed precipitation rate–instability in-  
616 dex relationship were excluded before fitting a linear relationship on the data (table 2)  
617 for comparison with the modeled relationship. The WTG approximation captures the  
618 observed strong dependence of saturation fraction on instability index, for high satura-  
619 tion fraction events. For comparison, we overlaid the relationship from *Raymond et al.*  
620 [2015, solid blue], so far the only such relationship in the literature, which was derived  
621 for tropical cyclogenesis data for regions between 10 and 25 degrees in the Pacific and  
622 North Atlantic (THORPEX Pacific Asian Regional Campaign, T-PARC, Tropical Cy-  
623 clone Structure-08, TCS-08, and the Pre-Depression Investigation of Cloud-Systems in  
624 the Tropics, PREDICT), there is no a priori reason for it to hold at the equator, or to  
625 hold for the DYNAMO convection which might differ in controlling mechanisms. How-  
626 ever, this comparison shows that the saturation fraction–instability index relationship  
627 derived from DYNAMO data is a more sensitive function of the instability index than the  
628 cyclogenesis results from T-PARC, TCS-08 and PREDICT. It would be interesting to de-

629 termine if this was a consequence of latitude or different dynamics related to cyclogenesis  
630 compared to the MJO.

631 The SSA observations (figure 6h) differ from the NSA observations. There is more  
632 scatter in the SSA, and the outliers with precipitation rates lower than  $P_B$  and saturation  
633 fractions lower than  $S_B$ , do not separate neatly; there are precipitation events greater than  
634  $P_B$  for saturation fractions lower than  $S_B$ . This scatter produces a more uncertain slope  
635 for the SSA observational fit (table 2, coefficient  $c$  for the SSA). The model fit, on the other  
636 hand, shows behavior similar to the NSA fit, with a slightly lower slope. The differences  
637 between the NSA and the SSA in all of the relationships so far, suggest a distinct feature;  
638 the WTG approximation seems to model the NSA region better than the SSA. The NSA  
639 experiences in stronger precipitation rates which the WTG approximation captures, which  
640 might indicate that the WTG approximation works better in organized convection if we  
641 take the intensity of precipitation to be a measure of convective organization.

642 As expected, the single station observations (figure 6i) show more scatter than the  
643 NSA and the SSA. Here, similar to the SSA, the outliers with precipitation rates lower  
644 than  $P_B$  and saturation fractions lower than  $S_B$ , exist for a wide range of the instability  
645 index. The scatter affects the linear fit for the observations in a similar manner as in the  
646 SSA; the correlation is weaker, and the slope is even lower than in the SSA. The scatter  
647 in the single station simulations might also reflect differences in local conditions for the  
648 individual stations.

649 **Gross moist stability relationships.** Given the existence of a characteristic GMS  
650 value related to peak precipitation rates [*Inoue and Back, 2015a*], we consider the rela-  
651 tionship between the GMS and the precipitation rate, the saturation fraction, and the

652 instability index. This can give us clues as to how the GMS is related to the convective  
653 development and decay in DYNAMO.

654 Figure 7 shows precipitation rate–GMS (panels a–b), saturation fraction–GMS (panels  
655 c–d), and instability index–GMS (panels e–f) relationships, for the NSA (panels a, c,  
656 and e), and the SSA (panels b, d, and f). We did not calculate the single station GMS  
657 relationships because it is impossible to calculate divergences for single point soundings.  
658 The scatter plot of precipitation rate and GMS shows the non-negative characteristic GMS  
659 value—here defined as the value of GMS at maximum precipitation rate—in observations  
660 and simulations, both in the NSA, and the SSA. We calculate the characteristic value as  
661 the average GMS for precipitation rates greater than 10 mm/day, and mark it in the figure  
662 as a vertical dashed line (black for observations, red for model). This derived characteristic  
663 GMS value is also shown on remaining scatter plots in figure 7 for comparison, for the  
664 NSA and the SSA, respectively. For comparison with figure 6, events with precipitation  
665 rates less than  $P_B$ , and saturation fractions less than  $S_B$ , are shown in blue.

666 Although the precipitation–GMS relationship (panels a–b) is similar for the observations  
667 and the model; the characteristic GMS is different (0.41 for the observations, and 0.1–0.2  
668 for the model). One possibility for this discrepancy is a consequence of the difference  
669 between the observed and the modeled (RCE) mean potential temperature and mixing  
670 ratio used in the WTG simulations. However, preliminary sensitivity tests (not shown)  
671 reveal that using RCE profiles closer to the observed mean profiles do not significantly  
672 affect the modeled characteristic GMS. An alternative reason for the discrepancy might  
673 be a result of either the surface fluxes or the radiation, according to one definition of  
674 the characteristic GMS (equation 19). Sensitivity simulations (not shown) indicate that



675 radiation effects—adjusted by changing the strength of the imposed RCE radiative cooling  
676 profile—does not affect the value of the characteristic GMS; the divergence of dry entropy  
677 adjusts to preserve the characteristic GMS value of 0.1-0.2. Given that neither the refer-  
678 ence thermodynamic environment or radiation seem to affect the characteristic value of  
679 the GMS, it would be interesting to further investigate what sets this characteristic value.

680 The saturation fraction–GMS (figure 7c–d) and instability index–GMS (figure 7e–f)  
681 relationships show behavior similar to the precipitation rate–GMS relationship. The peak  
682 saturation fraction and smallest instability index are aligned with the characteristic GMS  
683 value from peak precipitation rates (figure 7a–b) both for the observations and the model,  
684 though the result is much stronger in the modeled convection. The instability index–  
685 GMS relationship (figure 7e–f) is somewhat obscured by low precipitation rate and low  
686 saturation fraction events (shaded blue).

687 In conclusion, the characteristic GMS permeates the dynamics of convective devel-  
688 opment in the DYNAMO field campaign. We consider the instability index to be the  
689 independent variable in the WTG simulations: it is determined by the reference potential  
690 temperature profile, which in turn sets the convective profile which modulates the lateral  
691 import/export of moisture and moist entropy, thus controlling the saturation fraction, the  
692 GMS, and the precipitation rate. There is a strong relationship between the characteristic  
693 GMS and instability index, though further work is necessary to explain the link between  
694 the two.

#### 4. Conclusions

695 In order to better understand how local convection interacts with large scale condi-  
696 tions, we compared diagnostic relationships in observations and weak temperature gra-

697 dent (WTG) simulations of Dynamics of the Madden-Julian Oscillation (DYNAMO)  
698 convection. First, we imposed observed variations of thermodynamic variables—potential  
699 temperature, water vapor mixing ratio, wind speed, and sea surface temperatures (SST)—  
700 in WTG simulations of the north sounding array (NSA), the south sounding array (SSA),  
701 and six individual stations from the DYNAMO campaign. By comparing precipitation  
702 rate, saturation fraction (which quantifies moisture content) and instability index (which  
703 quantifies atmospheric stability), we found that the model reproduces the observations  
704 reasonably well, both quantitatively and qualitatively, with some systematic differences.  
705 These differences are possibly attributable either to model parameters used to construct  
706 the WTG simulations, or to dynamics that are not captured in the WTG approximation.

707 Second, we compared diagnostic relationships between the precipitation rate, the satu-  
708 ration fraction, the instability index, and the gross moist stability (GMS, which quantifies  
709 the interaction between local convection and large scale conditions), in observations and  
710 simulations. We found that:

711 1. The observed precipitation rate-saturation fraction relationship agrees with previous  
712 work [*Bretherton et al.*, 2004; *Raymond et al.*, 2004]: precipitation rate is a strong function  
713 of saturation fraction. The model shows similar behavior.

714 2. The observed and the modeled precipitation rate–instability index relationship agree,  
715 and suggest that the precipitation rate is a sensitive function of atmospheric stability; a  
716 more stable environment (quantified by low instability index) precipitates more efficiently.  
717 Outliers to this relationship suggest that the WTG approximation does not capture dy-  
718 namics which suppress precipitation in more stable environments which otherwise support  
719 strong precipitation. Alternatively, the relationship between precipitation rate and atmo-

720 spheric stability occurs in more organized convection where the WTG approximation  
721 seems to perform well.

722 3. The observed and the modeled saturation fraction–instability index relationships  
723 also agree; though, more so for the NSA than the SSA and the individual stations in  
724 DYNAMO, which suggests different dynamics or perhaps different degrees of organization  
725 in the NSA compared to the SSA.

726 4. The relationships between the GMS and the precipitation rate, the saturation frac-  
727 tion, and the instability index, show that the characteristic GMS value [*Inoue and Back*,  
728 2015a] differs between the observations (0.41) and the model (0.1-0.2). However, the char-  
729 acteristic GMS value is ubiquitous to all of the diagnostics; maximum precipitation rates,  
730 saturation fractions, and minimum instability index values all occur at the characteristic  
731 GMS, both for the observations and the model.

732 Despite the differences in magnitudes, the modeled gross moist stability (GMS) captures  
733 the general characteristics of the observed GMS; in both the observations and simulations,  
734 the GMS goes from small and/or negative values in the developing stages of the convec-  
735 tion, to larger positive values in the decaying phases of the strong convective events. In  
736 agreement with the theory of *Inoue and Back* [2015a], GMS smaller than the character-  
737 istic occurs during developing convection, GMS larger than the characteristic indicates  
738 decaying convection (at least in strong MJO events).

739 Combining the theory of *Inoue and Back* [2015a], and the effects of atmospheric stability  
740 on atmospheric moisture content and precipitation rates—described in section 3, and in  
741 *Raymond and Sessions* [2007], *Sessions et al.* [2015], and *Raymond et al.* [2015]—gives

742 us a possible picture of how local convection interacts with large scale changes in the  
743 DYNAMO campaign:

744 1. The stability of the environmental air sets the type of the convection, via the ver-  
745 tical structure of the mass flux profile; a more stable environment (quantified by lower  
746 instability index) produces more bottom heavy mass flux profiles [*Raymond and Sessions,*  
747 2007; *Raymond et al.*, 2011, 2014; *Gjorgjievska and Raymond*, 2014; *Sessions et al.*, 2015;  
748 *Raymond et al.*, 2015].

749 2. The mass flux profile then sets the amount of moisture available for precipitation;  
750 a more bottom heavy mass flux profile vertically advects moister lower-tropospheric air  
751 while also importing more moist lower tropospheric air via mass convergence at low levels  
752 [*Raymond and Sessions*, 2007].

753 3. Finally, the amount of moisture (quantified by saturation fraction) sets the precipi-  
754 tation rate since precipitation rate is a sensitive function of saturation fraction [*Bretherton*  
755 *et al.*, 2004; *Raymond et al.*, 2004].

756 This dynamical relationship has been demonstrated in observations of tropical cyclo-  
757 genesis [*Gjorgjievska and Raymond*, 2014], but also seems to hold for the data presented  
758 here. This suggests, at least in strong (presumably more organized) convection, balanced  
759 dynamics also plays a role in the DYNAMO convection. A loose end in this study is the  
760 initial source of stability changes which eventually results in stronger precipitation rates.  
761 *Raymond et al.* [2015] indicates a potential vorticity anomaly is responsible for inducing  
762 the stability changes—further work, however, is necessary to study this possibility.

## Appendix A: Verification of Weak Temperature Gradient Simulations

763 We verify the WTG simulations using reference profiles from individual stations, in ad-  
764 dition to reference profiles derived from array averages (as a reminder, we use the spectral  
765 WTG approximation in these simulations). The combination of high resolution radiosonde  
766 data and large scale satellite derived precipitation rate, necessary for validating the WTG  
767 approximation on single stations, was not available until the DYNAMO campaign. Veri-  
768 fying the WTG approximation with precipitation gauge data is hindered by coarseness of  
769 precipitation gauge coverage. That is why we use Tropical Rainfall Measurement Mission  
770 (TRMM) data in tandem with DYNAMO sounding measurements for verifying both the  
771 sounding array, and the single station simulations. The single station verification sets  
772 precedent for future studies for using the WTG simulations in cases where large arrays  
773 are not present, and only individual station soundings are available.

774 Figures 8, 9, and 10 show daily averaged observed and modeled precipitation rate,  
775 saturation fraction, and instability index, respectively, for NSA and SSA (panels a–b),  
776 and individual stations (panels c–h). Correlation coefficients for each observation–model  
777 pair are also given. Table 1 lists the mean and standard deviation for all observed and  
778 modeled diagnostics with corresponding lag 0 correlation coefficients. Figure 8 also shows,  
779 in the panel titles, the mean imposed wind speed with respective standard deviations, for  
780 comparison.

781 The NSA and the SSA precipitation rate (figure 8a–b), saturation fraction (figure 9a–  
782 b), and instability index (figure 10a–b) are well reproduced, with SSA precipitation rate  
783 and saturation fraction having a weaker correlation with observations (table 3). Modeled  
784 diagnostics for Male, Colombo, Gan, and Diego Garcia (figures 8, 9, and 10, panels c, d, e,

785 and g) have a stronger correlation with observations than those for R/V Revelle and Mirai  
786 (figures 8, 9, and 10, panels f and h; table 3). Looking at differences between observed and  
787 modeled diagnostics, the model sometimes underestimates stronger precipitation rates for  
788 Gan and Diego Garcia (figures 8, 9, and 10, panels e and g), which suggests that the  
789 WTG relaxation length scale ( $L$ ) might depend on geographical location. The modeled  
790 precipitation rate at Male (figure 8c) is stronger than the observed, and R/V Revelle and  
791 Mirai precipitation rates (figure 8f,h) between the observations and the model are poorly  
792 correlated, possibly because the model imposed winds, which are derived from observa-  
793 tions, are stronger than those at the other stations; surface fluxes might be overwhelming  
794 or amplifying WTG effects. Modeled saturation fraction and instability index are consis-  
795 tently larger and smaller, respectively, than observed; section 3.1.1 lists possible reasons  
796 for this discrepancy.

797 The poor correlation between observed and modeled diagnostics for stations R/V Rev-  
798 elle and Mirai (figures 8, 9, and 10) might come from processes that cannot be captured  
799 by the WTG modeling paradigm. Qualitatively and quantitatively, the Mirai modeled  
800 precipitation rate (figures 8h) follows observations well, except for a dry spell from day  
801 15 to day 25. A possible reason for the model missing the dry spell might be because the  
802 model did not capture a process that lead to a non-precipitating state from days 15 to  
803 25. Modeled deep convective inhibition (DCIN, not shown)—defined as the average moist  
804 entropy from 0 to 1.75 km subtracted from the average saturated moist entropy from 1.75  
805 to 2 km—had low values while the observed DCIN was substantial, which supports the  
806 hypothesis that the model did not capture the process leading to a dry state. R/V Revelle  
807 (figures 8f) exhibits a similar behavior: after the first couple of days the model misses

808 the dry spell, and goes into a state that does not resemble the observed. Further study  
809 is necessary to confirm the source of this behavior; these outliers, however, do not affect  
810 the conclusions of this paper.

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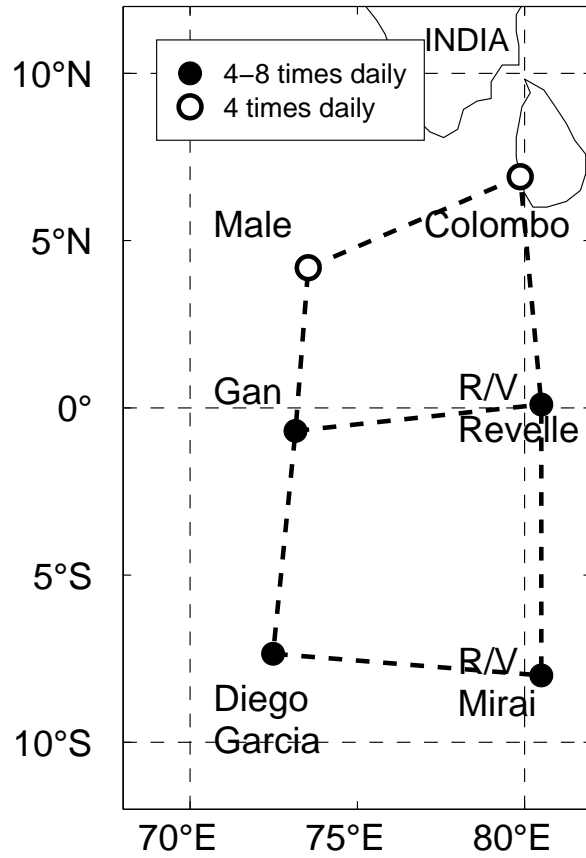
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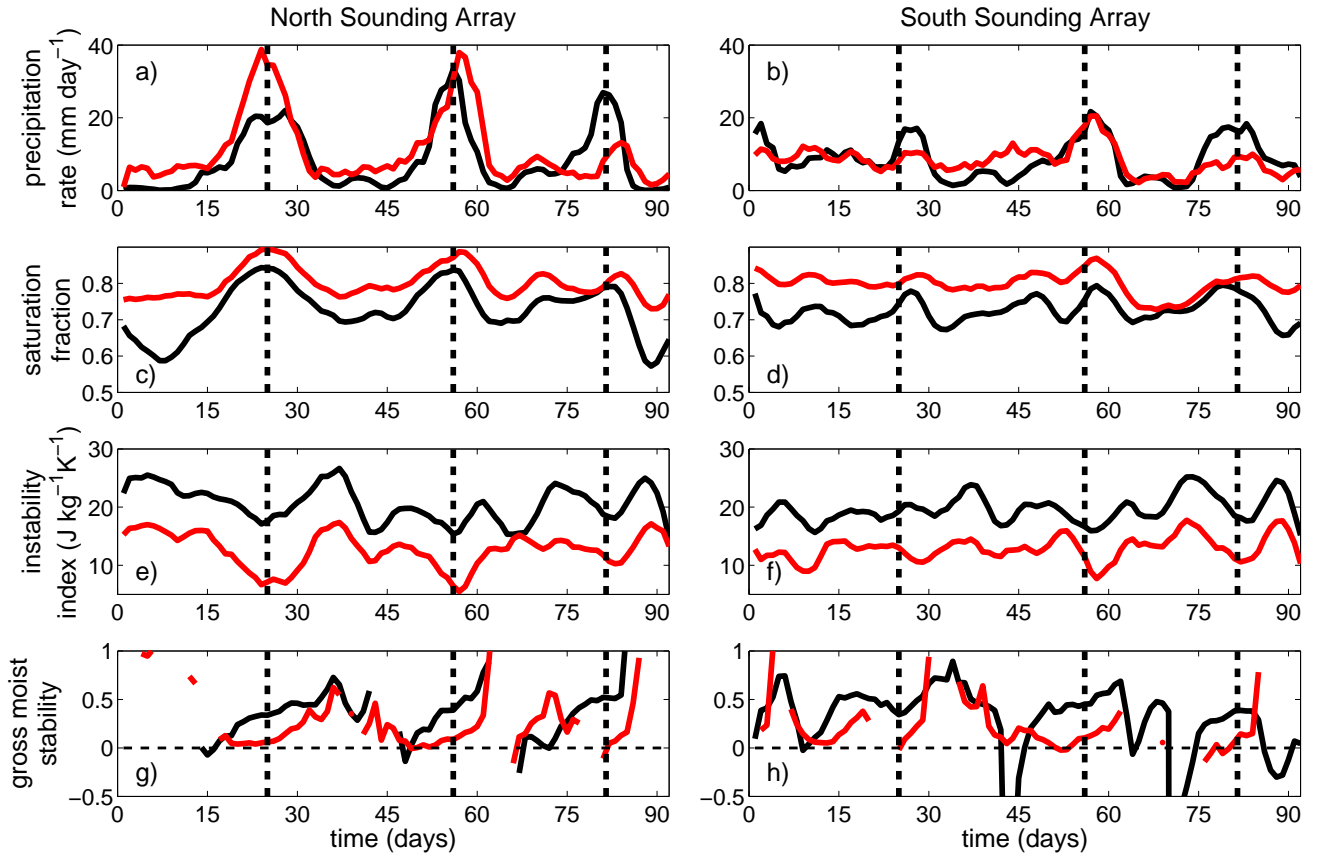
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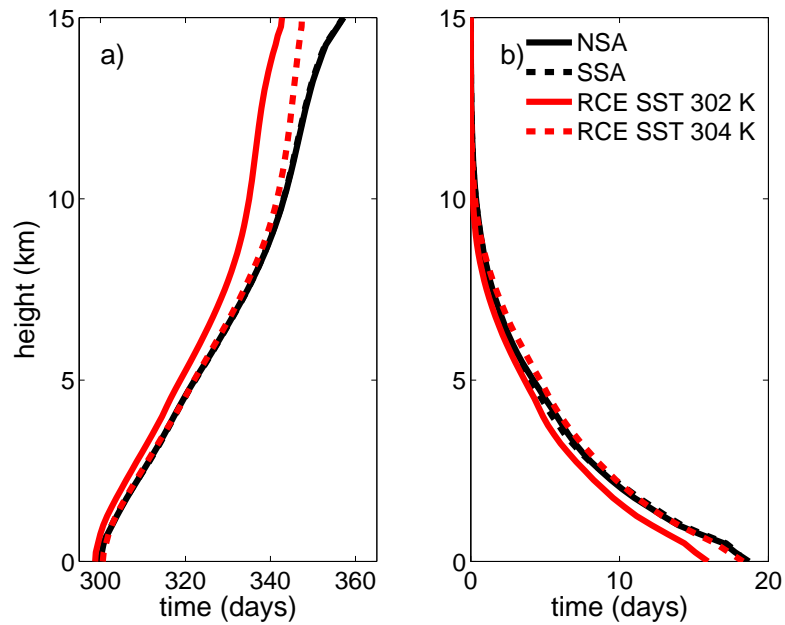


**Figure 1.** Station positions in the Dynamics of the Madden-Julian Oscillation (DYNAMO) campaign array.

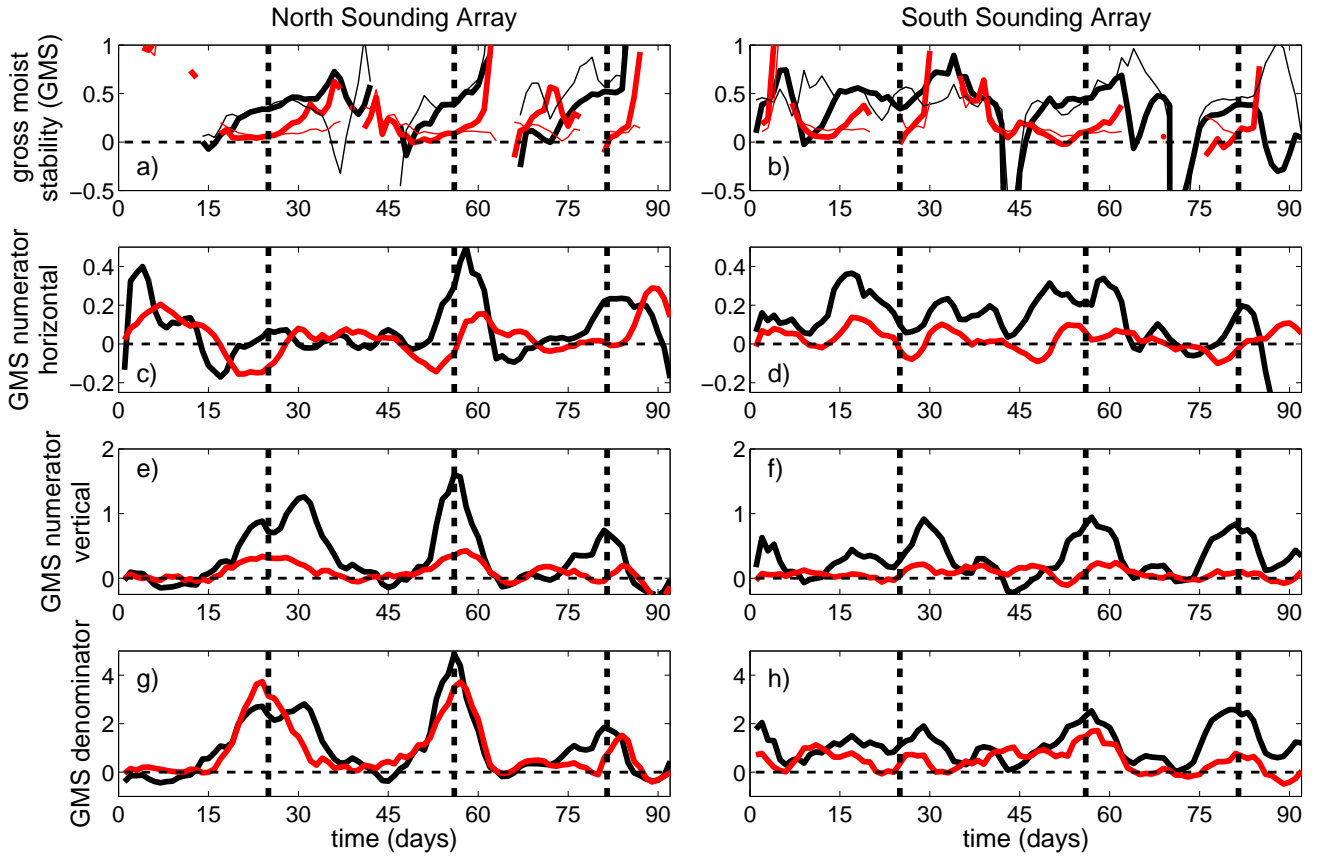


**Figure 2.** Smoothed (five day moving average) time series of precipitation rate (a–b), saturation fraction (c–d), instability index (e–f), and gross moist stability (GMS, g–h), for observations (black), and weak temperature gradient simulations (red). The North Sounding Array (NSA) are on the left (panels a, c, e, and g), and the South Sounding Array (SSA) are on the right (panels b, d, f, and h). Vertical dashed lines are rough guidelines aligned with observed precipitation maxima in the NSA. For comparison, NSA guidelines are plotted on the SSA panels (b,d,f, and h).

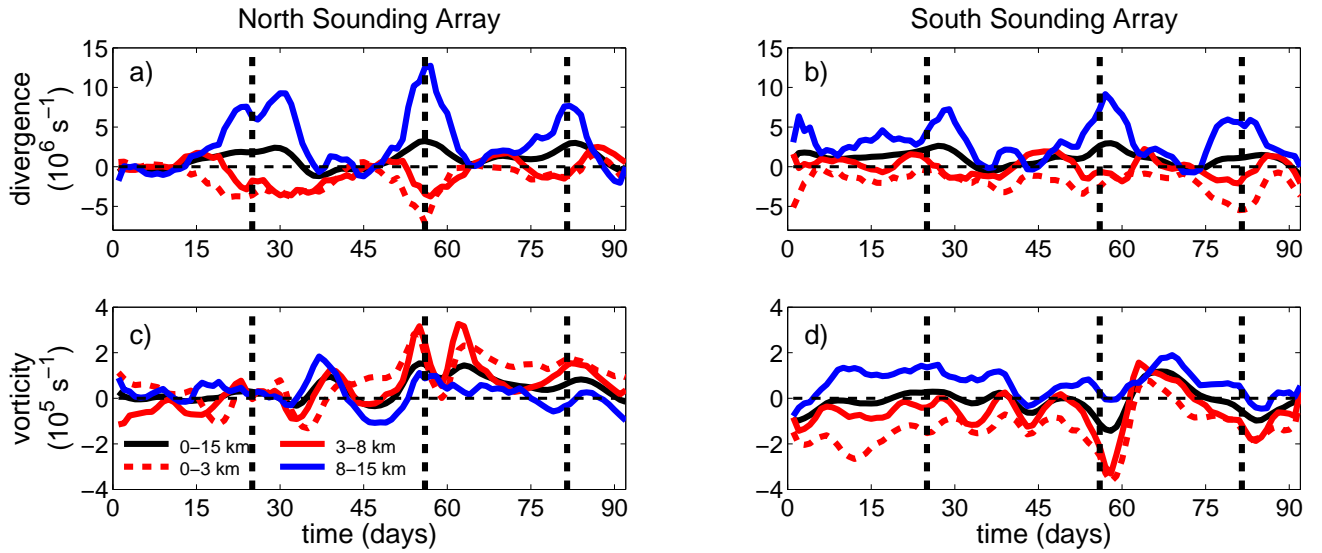




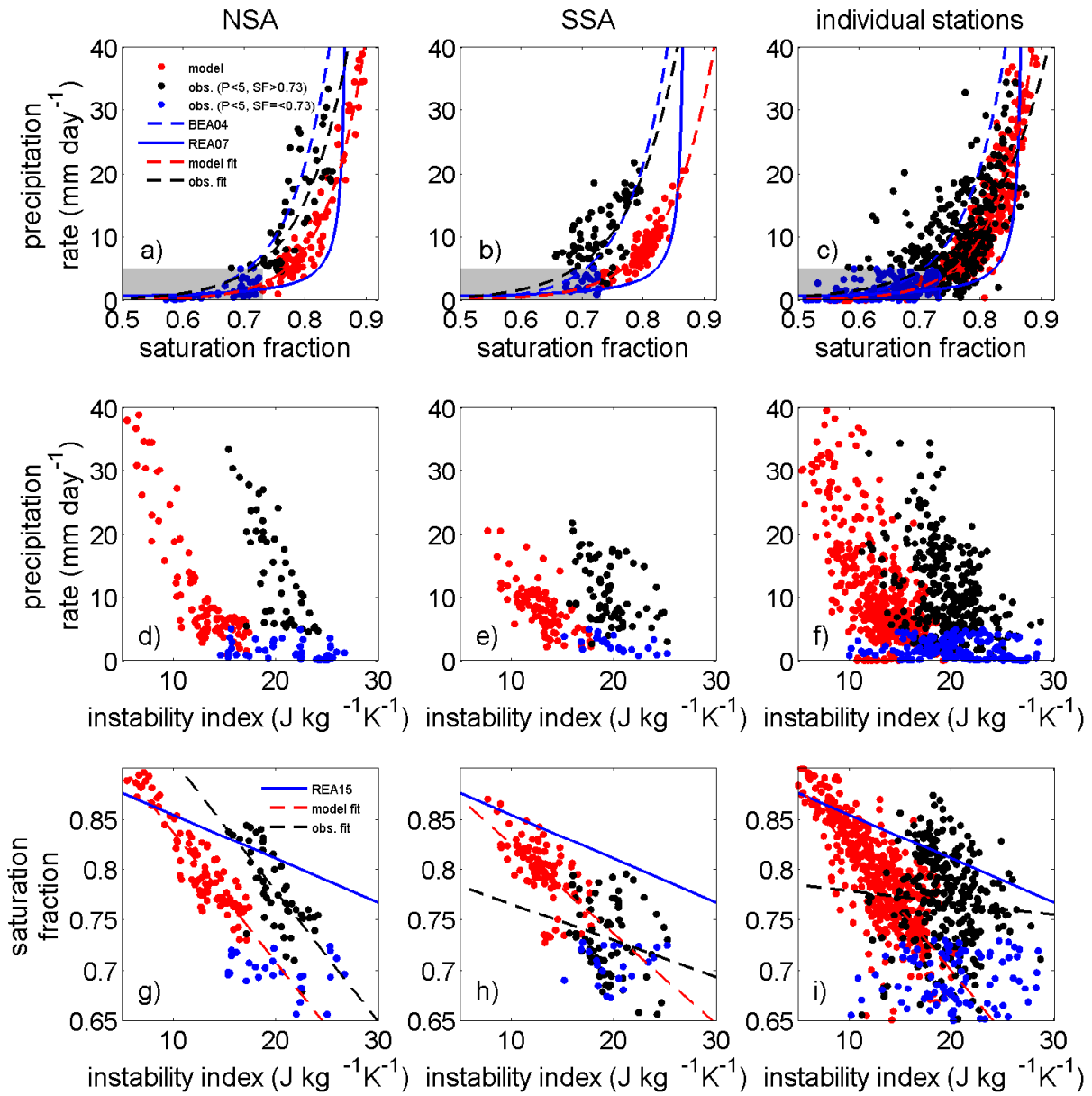
**Figure 3.** a) Potential temperature, and b) water vapor mixing ratio, for the model radiative-convective equilibrium (RCE, red), and observations: North Sounding Array (NSA, solid black) and South Sounding Array (SSA, dashed black). Note that the NSA and the SSA curves overlap. The RCE mean profiles corresponding to mean observed SSTs are cooler and dryer than the observed mean.



**Figure 4.** Smoothed (five day moving average) time series of the gross moist stability (GMS, panels a–b) and characteristic GMS(defined in equation 19, panels a–b), horizontal (c–d) and vertical (e–f) components of the GMS numerator, and the GMS denominator (g–h), for observations (black), and spectral weak temperature gradient simulations (red); for the North Sounding Array (NSA, panels a, c, e, and g), and the South Sounding Array (SSA, panels b, d, f, and h). Vertical dashed lines are rough guidelines aligned with observed precipitation maxima in the NSA. For comparison, NSA guidelines are plotted on the SSA panels (b, d, f, and h).



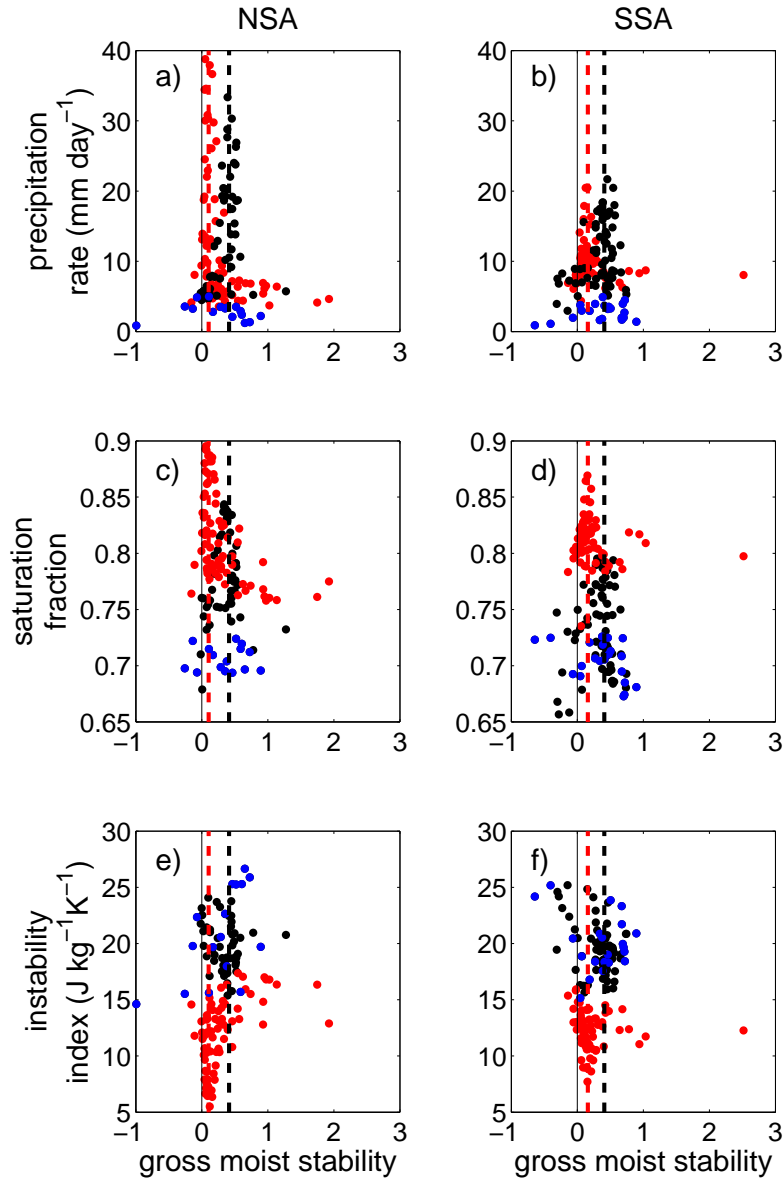
**Figure 5.** Observed time series of vorticity (a–b) and divergence (c–d) for the North Sounding Array (NSA, panels a and c), and the South Sounding Array (SSA, panels b and d). Time series were smoothed with with a five day moving average, and averaged vertically in 0–15 km (solid black), 0–3 (dashed red), 3–8 km (solid red), and 8–15 km (solid blue) layers. Vertical dashed lines are rough guidelines aligned with observed precipitation maxima in the NSA. For comparison, NSA guidelines are plotted on the SSA panels (b, and d).



**Figure 6.** Relationships between precipitation rate and saturation fraction (top panels, a–c), precipitation rate and instability index (middle panels, d–f), and saturation fraction and instability index (bottom panels, g–i), for the North Sounding array (NSA, left panels, a, d, and g), South Sounding Array (SSA, middle panels, b, e, and f), and single station simulations (SS composite, right panels, c, f, and i).

**Figure 6.** (continued)

Observations are shaded blue for precipitation rates and saturation fractions smaller than 5 mm day<sup>-1</sup> and 0.73, respectively, and black for the rest of the observed data, while the model is shaded in red. Precipitation rate–saturation fraction relationships from previous work are overlaid for comparison in the top row—dashed blue for *Bretherton et al.* [2004], and solid blue for *Raymond et al.* [2007]—while exponential fits for the DYNAMO observations and the model are the dashed black and red lines, respectively. The saturation fraction–instability index relationship from *Raymond et al.* [2015] has been overlaid for comparison in the bottom row—while linear relationship fits for the DYNAMO observations and the model are the dashed black and red lines, respectively. See table 2 for details about the fits.



**Figure 7.** Relationships between the precipitation rate and the gross moist stability (GMS, top panels, a–b), saturation fraction and GMS (middle panels, c–d), and instability index and GMS (bottom panels, e–f), for the North Sounding array (NSA, left panels, a, c, and e), South Sounding Array (SSA, middle panels, b, d, and f). Observations are shaded blue for precipitation rates and saturation fractions smaller than  $5 \text{ mm day}^{-1}$  and 0.73, respectively, black for the rest of the observed data, while the model is shaded red. Vertical dashed lines represent the characteristic GMS value defined as the GMS at peak precipitation rates; black for observations (NSA 0.41, SSA 0.41), and red for the model (NSA 0.10, SSA 0.16).

**Table 1.** Characteristic gross moist stability\* (GMS).

<b>Characteristic GMS (<math>\Gamma_C</math>):</b>	$\Gamma'_{\max}$	$\tilde{\Gamma}'$	$\tilde{\Gamma}$	$\Gamma_0$
<b>NSA Observations</b>	0.412	0.428	0.395	0.372
<b>NSA SWTG</b>	0.103	0.065	0.098	0.137
<b>SSA Observations</b>	0.409	0.455	0.398	0.378
<b>SSA SWTG</b>	0.160	0.054	0.138	0.169

\* Characteristic GMS calculated by all four methods listed by *Inoue and Back* [2015a, see text for definitions]; values for the North (NSA) and South (SSA) Sounding Arrays, for the observations, and spectral weak temperature gradient (SWTG) simulations.

**Table 2.** Diagnostic relationship fits\*.

	Precipitation rate– saturation fraction $P(S) = \exp(b * (S - S_D))$		Saturation fraction– instability index $S(II) = c * II + d$	
	$b$	$S_D$	$c$ (kg K J <sup>-1</sup> )	$d$
<b>Bretherton et al. (2004)</b>	15.6	0.603	/	/
<b>Raymond et al. (2015)</b>	/	/	-0.0433	0.897
<b>NSA</b>	$13.39 \pm 1.96$	$0.59 \pm 0.03$	$-0.013 \pm 0.004$	$1.04 \pm 0.07$
	$16.36 \pm 1.01$	$0.67 \pm 0.01$	$-0.013 \pm 0.01$	$0.97 \pm 0.02$
<b>SSA</b>	$12.47 \pm 2.69$	$0.56 \pm 0.04$	$-0.0036 \pm 0.0038$	$0.80 \pm 0.07$
	$14.10 \pm 1.31$	$0.65 \pm 0.02$	$-0.0089 \pm 0.0023$	$0.91 \pm 0.03$
<b>Single station simulations</b>	$10.4 \pm 1.04$	$0.55 \pm 0.02$	$-0.00118 \pm 0.0021$	$0.79 \pm 0.04$
	$16.11 \pm 0.62$	$0.66 \pm 0.01$	$-0.012 \pm 0.001$	$0.94 \pm 0.02$

\* The precipitation rate–saturation fraction, and saturation fraction–instability index relationship fits, for the North Sounding Array (NSA), the South Sounding Array (SSA), and individual station simulations, compared to the relationships from previous work [*Bretherton et al.*, 2004; *Raymond et al.*, 2015].

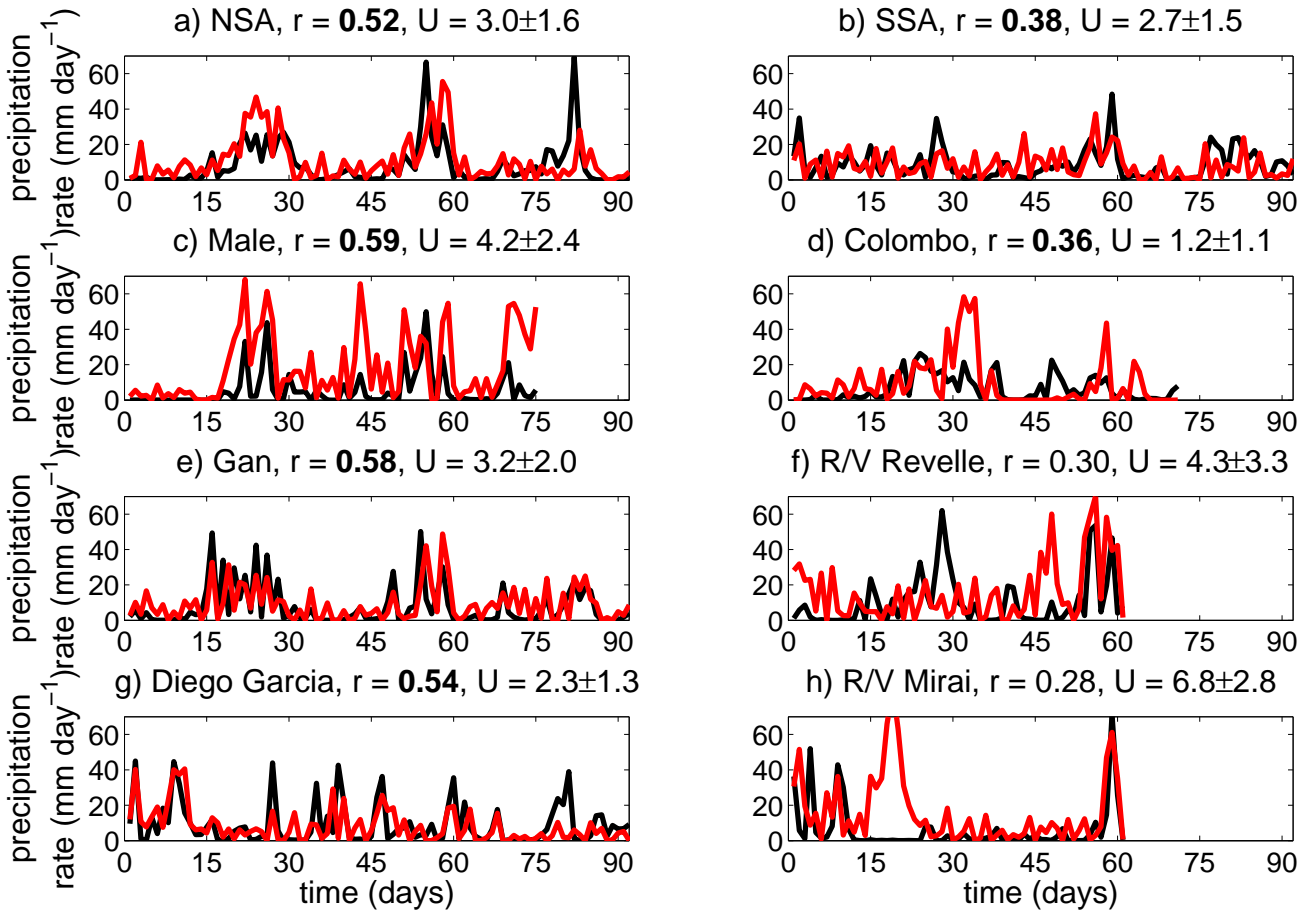


**Table 3.** Modeled and observed statistics\*.

	N (days)	Precipitation rate (mm day <sup>-1</sup> )	r	Saturation fraction (100%)	r	Instability Index (J kg <sup>-1</sup> K <sup>-1</sup> )	r
NSA	92	8.4 ± 12.2	<b>0.52</b>	0.73 ± 0.08	<b>0.80</b>	20.6 ± 4.0	<b>0.62</b>
		<i>11.1 ± 12.4</i>		<i>0.80 ± 0.05</i>		<i>12.7 ± 3.9</i>	
SSA	92	8.7 ± 8.5	<b>0.38</b>	0.72 ± 0.05	<b>0.51</b>	19.7 ± 3.4	<b>0.65</b>
		<i>8.4 ± 7.3</i>		<i>0.80 ± 0.04</i>		<i>12.9 ± 3.3</i>	
Male	75	5.6 ± 9.9	<b>0.59</b>	0.70 ± 0.11	<b>0.86</b>	21.5 ± 6.2	<b>0.60</b>
		<i>18.6 ± 19.1</i>		<i>0.81 ± 0.07</i>		<i>12.2 ± 5.0</i>	
Colombo	71	6.6 ± 7.3	<b>0.36</b>	0.72 ± 0.12	<b>0.66</b>	20.0 ± 5.4	<b>0.46</b>
		<i>9.2 ± 13.9</i>		<i>0.74 ± 0.11</i>		<i>12.6 ± 4.4</i>	
Gan	92	7.5 ± 11.2	<b>0.58</b>	0.75 ± 0.08	<b>0.72</b>	19.7 ± 4.7	<b>0.57</b>
		<i>9.1 ± 9.8</i>		<i>0.79 ± 0.05</i>		<i>13.1 ± 4.2</i>	
R/V Revelle	61	10.5 ± 14.5	0.30	0.77 ± 0.07	0.07	19.9 ± 3.8	0.01
		<i>15.9 ± 16.9</i>		<i>0.81 ± 0.05</i>		<i>16.6 ± 4.0</i>	
Diego Garcia	92	9.1 ± 11.8	<b>0.54</b>	0.73 ± 0.09	<b>0.65</b>	20.8 ± 5.4	<b>0.78</b>
		<i>8.1 ± 9.5</i>		<i>0.78 ± 0.05</i>		<i>13.0 ± 4.9</i>	
R/V Mirai	61	6.2 ± 13.4	0.28	0.65 ± 0.12	-0.03	15.5 ± 4.9	0.02
		<i>15.6 ± 18.9</i>		<i>0.80 ± 0.07</i>		<i>13.0 ± 5.1</i>	

\* Mean and standard deviation (observations roman, modeled italic), and lag zero correlation

coefficients,  $r$ , between observed and modeled precipitation rate, saturation fraction, and instability index, for the DYNAMO North (NSA) and South (SSA) Sounding Arrays, and individual stations.  $N$  is the number of daily averages in time series (sample size). Correlation coefficients significant at the 98% level are in bold.



**Figure 8.** Observed (black) and modeled (red) daily precipitation rate time series for the north sounding array (NSA, panel a), south sounding array (SSA, panel b), and individual stations (panels c–h) from the Dynamics of the Madden-Julian Oscillation (DYNAMO) campaign. Correlation coefficients ( $r$ ) between the observations and the model significant to 98% are in bold. The mean and the standard deviation of surface wind speeds is also given, for comparison.

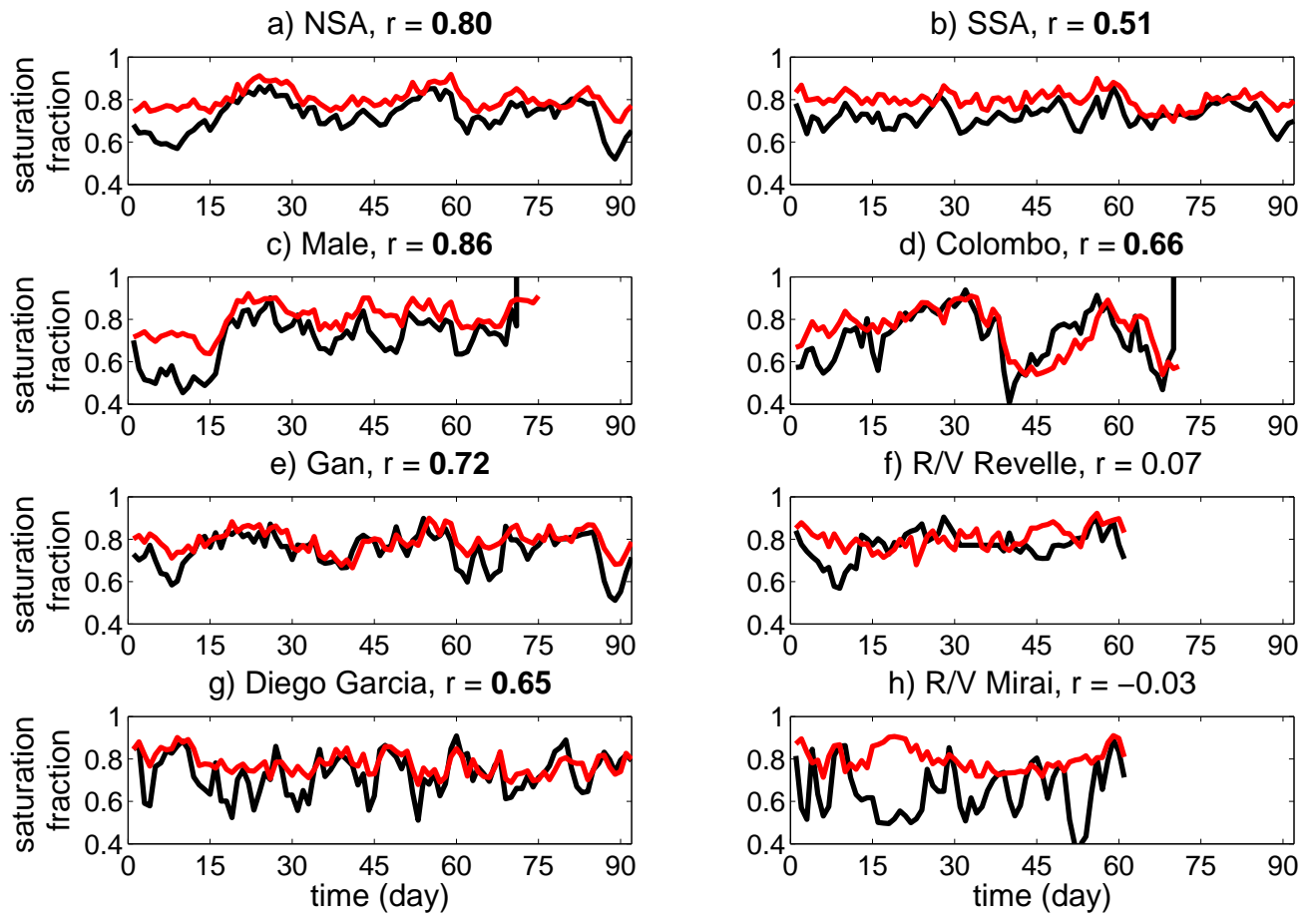


Figure 9. Same as figure 8, but for the saturation fraction.

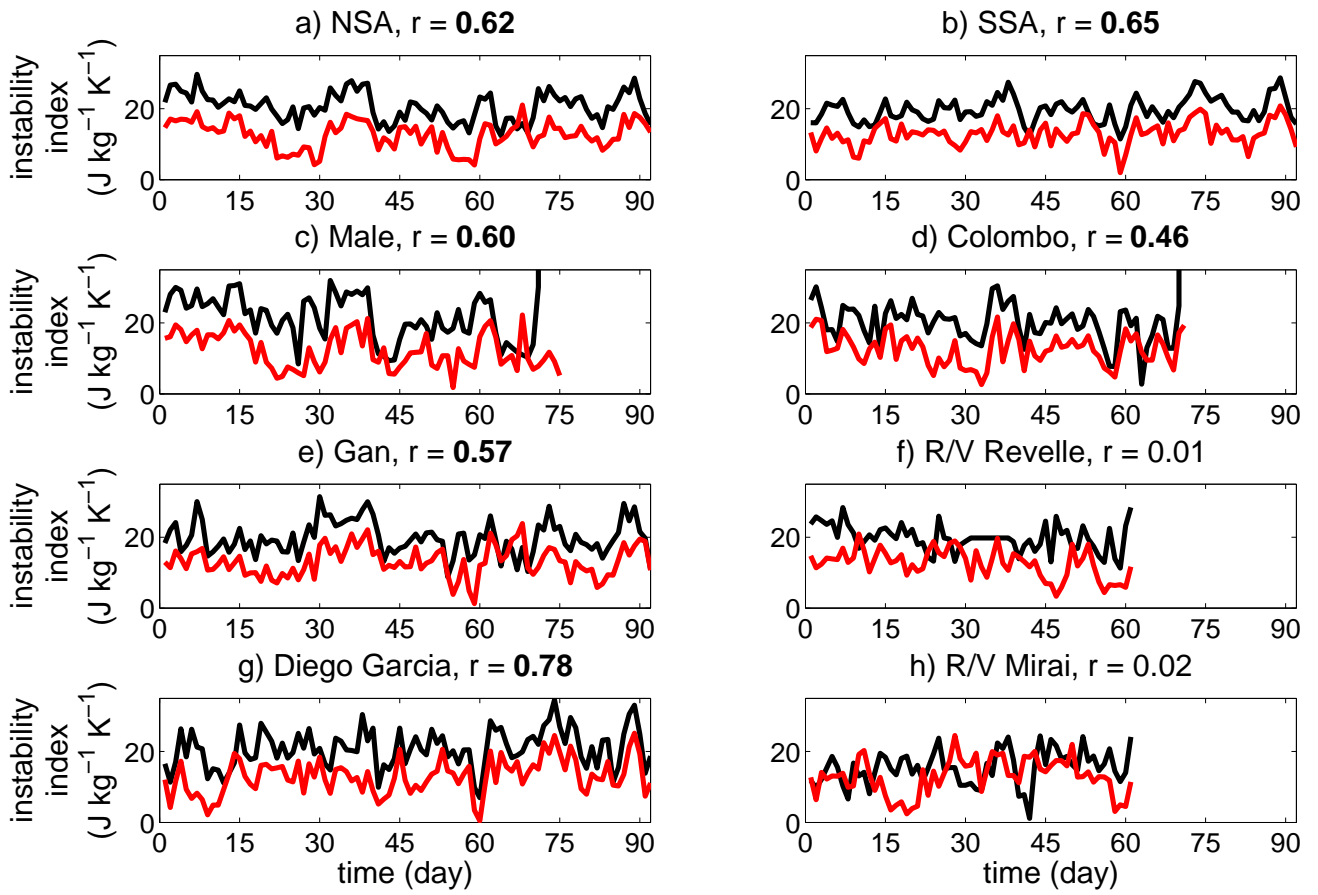


Figure 10. Same as figure 8, but for the instability index.