

1 **Multiple equilibria in a cloud resolving model using** 2 **the weak temperature gradient approximation**

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3 **Abstract.** Multiple equilibria corresponding to either a state with per-
4 sistent, precipitating deep convection or a non-precipitating state are shown
5 to exist in a cloud resolving model employing the weak temperature gradi-
6 ent (WTG) approximation. WTG is important for the existence of both equi-
7 libria; numerical experiments show that a weak enforcement of WTG elim-
8 inates the dry non-precipitating state. In situations which support both equi-
9 libria, we find that the dry equilibrium has a small or negative gross moist
10 stability, while larger positive values of gross moist stability correspond to
11 the precipitating equilibrium. Our simulations also show that the gross moist
12 stability tends to become negative during times of rapid change in the at-
13 mospheric moisture, as occurs in the transient stages of convection.

1. Introduction

14 Recent work by *Sobel et al.* [2007, hereafter SBB07] has demonstrated the existence of
15 multiple equilibria in a single column model of the atmosphere using the weak temperature
16 gradient (WTG) approximation with parameterized deep convection. They showed that
17 in conditions with sufficient convective available potential energy (CAPE) for convection
18 to occur, an initially dry column would remain dry unless some mechanism (such as
19 horizontal moisture advection) sufficiently moistened the free troposphere. This work is
20 directly related to multiple equilibria of the Hadley circulation [*Bellon and Sobel, 2009*],
21 which suggests that understanding this phenomenon in limited-domain simulations may be
22 important to understanding the interaction between convection and large-scale dynamics
23 in the tropics.

24 In this work, we pursue the idea that multiple equilibrium states may exist in the
25 atmosphere by performing numerical experiments with a cloud resolving model (CRM)
26 which interacts with the implicit large-scale circulations in a parameterized way via the
27 WTG approximation. We demonstrate that multiple equilibrium states do exist in our
28 model, but only under strict conditions which depend on surface fluxes (controlled by
29 either imposed surface winds or sea surface temperatures (SSTs)), domain size, and the
30 time scale with which the local vertical profiles of potential temperature relax to that of
31 the large scale environment. The equilibrium state which is realized by the model depends
32 on the initial moisture in the modeled domain, which is consistent with the results from
33 SBB07.

34 In addition to determining the conditions for the existence of multiple equilibria within
35 our model, we investigate the gross moist stabilities of the transient and steady states.
36 We find that the gross moist stability is a particularly important parameter not only for
37 diagnosing the environment in either equilibrium, but in characterizing the development
38 of deep convection.

39 Since the WTG approximation and gross moist stability are both important concepts
40 in this paper, we review these in the following subsections (1.1 and 1.2). Our numerical
41 experiments are described in section 2; results are presented in section 3. The role of
42 gross moist stability in the context of multiple equilibria is discussed in section 4. Finally,
43 section 5 includes a general discussion and summary.

1.1. WTG in the model

44 We run a version of the cloud resolving model very similar to that described in *Raymond*
45 *and Zeng* [2005, hereafter RZ05]. The model implements the WTG approximation as
46 described in RZ05, which is based on the original ideas of *Sobel and Bretherton* [2000,
47 hereafter SB00]. The basic premise of WTG is that the potential temperature in the
48 tropics tends to be horizontally homogeneous, and any buoyancy anomalies produced by
49 surface heat fluxes, latent heat release or radiation redistribute quickly by means of gravity
50 waves [*Bretherton and Smolarkiewicz*, 1989; *Mapes and Houze*, 1995]. This redistribution
51 maintains the horizontal homogeneity in the temperature profiles. In the model, this
52 effect is achieved by the imposition of a hypothetical vertical velocity, called the WTG
53 velocity (w_{wtg}), which produces vertical advection of potential temperature sufficient to
54 counteract the effects of heating.

55 As we will demonstrate in section 3, the WTG approximation plays an important role
 56 in the existence of multiple equilibria in our model. Consequently, we will briefly highlight
 57 some of the details of implementation in our model. For a more thorough discussion, see
 58 RZ05. In the model, the total wind field, \mathbf{v}_T , is composed of a part that is computed
 59 explicitly by the model and a part that is generated to maintain the WTG approximation
 60 (\mathbf{v} and \mathbf{v}_{wtg} , respectively):

$$61 \quad \mathbf{v}_T = \mathbf{v} + \mathbf{v}_{wtg} \quad . \quad (1)$$

62 Each of the contributions satisfy mass continuity individually. The first contribution
 63 obeys cyclic boundary conditions on the lateral model boundaries, which implies that its
 64 contribution to the domain averaged vertical mass flux is zero. The second contribution
 65 has horizontal and vertical components, \mathbf{v}_{wtg}^h and w_{wtg} , which satisfy the anelastic mass
 66 continuity equation,

$$67 \quad \nabla \cdot (\bar{\rho} \mathbf{v}_{wtg}^h) + \frac{\partial(\bar{\rho} w_{wtg})}{\partial z} = 0 \quad , \quad (2)$$

68 where $\bar{\rho}(z)$ is the horizontally averaged density. w_{wtg} is taken to be horizontally uniform,
 69 and a horizontal average of \mathbf{v}_{wtg}^h is assumed to be zero at each level. We postulate that
 70 $|\mathbf{v}| \gg |\mathbf{v}_{wtg}|$, and thus the approximate treatment of \mathbf{v}_{wtg} above is justified.

71 The equation for potential temperature, θ , implemented in the model is

$$72 \quad \frac{\partial(\rho\theta)}{\partial t} + \nabla \cdot (\rho \mathbf{v} \theta + \mathbf{T}_\theta) \equiv \rho(S_\theta - E_\theta) \quad . \quad (3)$$

73 Here, \mathbf{T}_θ represents unresolved eddy and viscous transport, S_θ is the diabatic potential
 74 temperature source, and E_θ enforces the WTG approximation via a relaxation of the
 75 potential temperature to a reference profile $\theta_0(z)$:

$$76 \quad E_\theta = w_{wtg} \frac{\partial \bar{\theta}}{\partial z} = \sin(\pi z/h) \frac{\{\bar{\theta} - \theta_0(z)\}}{t_\theta} \quad . \quad (4)$$

77 The overbar indicates a horizontal average of potential temperature, h is the height of
 78 the tropopause, and t_θ is the time over which the vertical profile of potential temperature
 79 relaxes to the specified reference profile. Note the sinusoidal modulation of the potential
 80 temperature in the vertical. This is to emphasize the role of gravity waves in adjusting the
 81 buoyancy in the middle troposphere. Above the tropopause, the relaxation rate (t_θ^{-1}) is set
 82 to zero. Also, since WTG breaks down in the boundary layer [*Sobel and Bretherton, 2000*],
 83 the value at the top of the boundary layer (taken to be 1000 m) is linearly interpolated
 84 to zero at the surface.

85 The equation for total water mixing ratio, r_t , is given by

$$86 \quad \frac{\partial(\rho r_t)}{\partial t} + \nabla \cdot (\rho \mathbf{v} r_t + \mathbf{T}_r) \equiv \rho(S_r - E_r) \quad , \quad (5)$$

87 where \mathbf{T}_r represents unresolved eddy and viscous transport, S_r is the source of r_t due to
 88 precipitation and evaporation, and E_r is an approximation which represents the entrain-
 89 ment of total advected water from the environment and the vertical transport of water
 90 by the large-scale vertical motion. Specifically,

$$91 \quad E_r = \frac{(\bar{r}_t - r_x)}{\bar{\rho}} \frac{\partial(\bar{\rho} w_{wtg})}{\partial z} + w_{wtg} \frac{\partial \bar{r}_t}{\partial z} \quad . \quad (6)$$

92 Here, r_x equals the reference profile $r_0(z)$ when $\partial(\bar{\rho} w_{wtg})/\partial z > 0$ and is equal to \bar{r}_t
 93 otherwise. This assignment assures that the outflowing air has a total water mixing ratio
 94 equal to the horizontal average in the domain (i.e., \bar{r}_t), while inflowing air has a mixing
 95 ratio representative of the environment surrounding the domain which is specified by the
 96 reference profile, $r_0(z)$. We note that equation 2 has been used to derive equation 6. For
 97 more details, see RZ05.

98 As described in RZ05, the WTG velocity substantially modifies the evolution of con-
99 vection. As equation 4 shows, the model does not strictly impose a background profile
100 of potential temperature, but rather allows the instantaneous local profile to relax to a
101 reference profile equal to the assumed large scale mean over a time scale t_θ . Physically,
102 we interpret t_θ to be the amount of time it takes a gravity wave to cross the modeled
103 domain. Though we may set this time scale according to realistic gravity wave speeds for
104 a given domain size (e.g., a gravity wave travelling at 50 ms^{-1} would take approximately
105 33 minutes to cross a 100 km domain), we vary this parameter to investigate the response
106 of the modeled convection. Furthermore, we may think of this relaxation time scale as
107 a measure of the strength with which the weak temperature gradient approximation is
108 enforced: $t_\theta = 0$ corresponds to strict enforcement of WTG ($\bar{\theta} \equiv \theta_0(z)$ in equation 4; this
109 was used in SB00 and SBB07) while $t_\theta = \infty$ turns off WTG mode in the simulations and
110 allows the model to evolve toward radiative convective equilibrium. The tropical environ-
111 ment falls somewhere in between these two extremes, depending on the time and space
112 scales of interest.

113 The model used in these simulations prescribes only the potential temperature relax-
114 ation time. This parameter controls the corresponding WTG vertical velocity which
115 vertically advects moisture. Horizontal advection of moisture from the surrounding envi-
116 ronment is determined by assuming that the advecting velocity is purely divergent, so that
117 it can be computed self-consistently from the WTG vertical velocity using mass continuity
118 (RZ05, and equation 2). This is equivalent to simulating a Lagrangian column moving at
119 the prescribed wind speeds. In SBB07, on the other hand, the time scale for horizontal
120 advection of moisture was specified a priori. This can be thought of as an assumption

121 that the rotational component of the advecting horizontal velocity is much larger than the
 122 divergent component, so that the latter can be neglected and the horizontal advection of
 123 moisture is decoupled from the WTG vertical velocity. SBB07 showed that the existence
 124 of multiple equilibria is sensitive to the moisture relaxation, and thus an important ques-
 125 tion arises: Does the existence of multiple equilibria depend on relaxation of the vertical
 126 profile of potential temperature or moisture? Because the horizontal advection time scale
 127 is not specified externally here, but rather determined interactively from the WTG verti-
 128 cal velocity, we cannot disentangle these effects and thus we do not address this question
 129 in the present study.

1.2. Normalized gross moist stability (NGMS)

130 The gross moist stability can be thought of as a measure of precipitation efficiency. It
 131 was first introduced in 1987 by *Neelin and Held* as a means to model tropical convergence
 132 based on the moist static energy budget (moist static energy being approximately con-
 133 served in moist processes). In this work, we choose to use the convention of *Raymond et*
 134 *al.* [2007], which defines the normalized gross moist stability (NGMS) in terms of moist
 135 entropy (which is also approximately conserved in moist processes). Specifically, we define
 136 Γ to be the NGMS:

$$137 \quad \Gamma = \frac{T_R[\nabla \cdot (s\mathbf{v})]}{-L[\nabla \cdot (r\mathbf{v})]} = \frac{T_R \frac{1}{g} \int \nabla \cdot (s\mathbf{v}) dp}{-L \frac{1}{g} \int \nabla \cdot (r\mathbf{v}) dp} \quad , \quad (7)$$

138 where s is the moist entropy, r is the total water mixing ratio, and \mathbf{v} is the horizontal
 139 wind. The square brackets indicate a vertical pressure integral over the troposphere, g
 140 is the gravitational acceleration, and ∇ is the horizontal divergence operator. T_R and
 141 L are a constant reference temperature and the latent heat of condensation, which are

142 included so that Γ is dimensionless. We thus define NGMS as the ratio of the vertically
 143 integrated lateral moist entropy export to moisture import. In a majority of cases, NGMS
 144 is a positive quantity, but can become negative if moist entropy and moisture are both
 145 imported into (or exported from) a convecting region.

146 Observations suggest that the sign of NGMS in a convecting region is related to the
 147 level of non-divergence in the horizontally averaged circulation [*López and Raymond, 2005*;
 148 *Back and Bretherton, 2006*]. In particular, when convergence is concentrated at low levels,
 149 as expected in the early stages of the development of deep convection, a vertical pressure
 150 integral of moist entropy or moist static energy divergence gives a net *import* of these
 151 quantities. Coupled with the moisture convergence, this results in a negative NGMS. In
 152 a region of subsidence, we expect moisture export, and either a modest import or export
 153 of moist entropy. The latter also results in a negative NGMS. As we will show in section
 154 4, both of these situations are realized in our model.

155 A thorough discussion on the role of NGMS in tropical dynamics can be found in
 156 *Raymond et al. [2009]*. As in *Raymond et al. [2007]*, the net precipitation in the steady
 157 state is related to the NGMS according to:

$$158 \quad L\Gamma(P - E) = T_R(F_s - R) \quad , \quad (8)$$

159 where P is the precipitation rate, E is the surface evaporation rate, F_s is the surface
 160 moist entropy flux due to surface heat and moisture fluxes, and R is the pressure integral
 161 of the entropy sink per unit mass due to radiation divided by the acceleration of gravity.
 162 Equation (8) suggests that the net precipitation ($P - E$) is inversely proportional to
 163 the NGMS for a given net entropy forcing. This is consistent with the results from
 164 *Raymond and Sessions [2007]*, which showed that thermodynamic profiles which are either

165 moister or more unstable at low levels result in smaller values of NGMS and increased
166 precipitation rates. The results described in section 4 are consistent with those findings,
167 and demonstrate that NGMS can be used as a diagnostic for characterizing the equilibrium
168 state of the system.

2. Experiments

169 For the implementation of the WTG approximation, we must specify the vertical pro-
170 files of potential temperature and mixing ratio representing the large-scale environmental
171 mean. For simplicity, we take this *reference profile* to be one of radiative convective equi-
172 librium (RCE), generated by averaging the last 3×10^6 s (≈ 35 days) in a 5×10^6 s (\approx
173 58 day) non-WTG simulation (i.e., $t_\theta = \infty$). RCE profiles are generated with imposed
174 horizontal surface wind speeds of 5 ms^{-1} over tropical oceans with SSTs of 303 K. The
175 horizontal wind is perpendicular to the two-dimensional plane of the domain (all simula-
176 tions are in 2D to decrease the computational demand). The simple radiative model used
177 in *Raymond and Zeng* [2000] calculates radiative cooling interactively in these simulations.

178 We have found that the RCE profiles are sensitive to the domain size and we thus
179 perform RCE simulations for each domain size and grid resolution used in the numerical
180 experiments. The vertical domain size used in the experiments is 20 km with a 250 m
181 vertical resolution. We used horizontal domain sizes of 50, 100 or 200 km. The 100 and
182 200 km domains had a horizontal resolution of 1 km (to reduce computational expense).
183 Most of the 50 km domains used a horizontal grid resolution of 500 m, though several
184 experiments used a 1 km grid to facilitate comparisons with the larger domains. Unless
185 otherwise noted, all references to the 50 km domain correspond to runs with a 500 m
186 resolution.

187 Figures 1A and 1B show a comparison between the horizontal and temporal averages
188 of potential temperature and mixing ratio of the RCE simulations. To emphasize the
189 differences, we plot the deviations of all profiles from the 50 km domain size with 500
190 m resolution. The differences between the logarithm of the mixing ratios is shown in
191 figure 1C to exaggerate the differences in the moisture content of the upper troposphere.
192 The solid lines correspond to 50 km domains with line thickness indicating grid resolution
193 (thin and thick lines corresponds to 500 m and 1 km resolution, respectively). Dashed and
194 dotted lines show deviations of the 100 km and 200 km domains from the 50 km domain.
195 Note that the larger domains are slightly warmer and drier in the free troposphere (from
196 about 2.5-12 km; figures 1A,B) than the 50 km domains, though the 50 km domain with
197 coarser resolution is considerably drier just above the boundary layer (1-2.5 km).

198 We do not fully understand the origin of these differences, though a comparison of the
199 fractional coverage of precipitating clouds for the different RCE simulations may provide
200 one possible explanation. Comparing the effects of grid resolution on the 50 km domains,
201 we find that the finer grid produces a considerably moister domain with precipitating
202 clouds covering a larger fraction of the space-time domain ($18.5 \pm 1.1\%$ versus $14.7 \pm 0.9\%$
203 of the domain has a precipitation rate of at least 1 mm day^{-1}). The larger domains have
204 a slightly lower fractional coverage ($13.1 \pm 0.7\%$ and $13.1 \pm 0.6\%$ with precipitation rate of
205 at least 1 mm day^{-1} for the 100 and 200 km domains, respectively), which suggests that
206 there is more room for descent to exist. This in turn allows the domain to warm and
207 dry slightly compared to the smaller domains. In section 3, we show that the differences
208 in the RCE reference profiles have essentially no effect on the WTG runs with 1 km

209 grid resolution, but that the moister profile associated with the finer resolution produces
 210 slightly higher precipitation rates (e.g., see figures 3 and 4).

211 Having specified the reference profile for the WTG simulations, we now describe the
 212 parameters in the modeled domain. We choose to fix the sea surface temperature at 303 K,
 213 and vary the strength of the surface horizontal wind. For each experiment, the wind speed
 214 is specified and approximately maintained throughout the duration of the simulation.
 215 Both SST and surface winds control surface fluxes, with larger values producing more
 216 convection and correspondingly greater precipitation rates (e.g., RZ05 and SBB07). In
 217 these experiments, the prescribed surface wind ranges from 0-20 ms^{-1} , with most runs set
 218 at 5, 7, 10 or 15 ms^{-1} .

219 As discussed in section 1.1, we also vary the time scale of relaxation of potential tem-
 220 perature toward the large-scale mean. This effectively controls the degree to which the
 221 model obeys the WTG approximation, and consequently controls the magnitude of the
 222 WTG vertical velocity. By mass continuity, this fixes the horizontal flow and controls the
 223 horizontal advection of moisture from the surrounding environment.

224 Each experiment is initiated with vertical profiles of potential temperature and total
 225 water mixing ratio which may differ from the RCE reference profile. The initial mixing
 226 ratio profile is taken to be

$$227 \quad r_{t(\text{init})}(z) = f r_{t0}(z) \quad , \quad (9)$$

228 where $r_{t0}(z)$ is the RCE profile which represents the surrounding environment, and f is
 229 a fractional multiplier ($0 \leq f \leq 1$). Initializing the domain with the moisture content
 230 of the surrounding environment corresponds to $f = 1$, while a domain that is initially
 231 completely dry has $f = 0$. Only $f = 0, 1$ were used by SBB07. In addition to these

232 extremes, we perform some experiments with the initial moisture of the modeled domain
 233 as some fraction of the RCE profile ($0 < f < 1$).

234 The initial potential temperature profile is given by

$$235 \quad \theta_{init} = \theta_0(1 + \delta\theta_{local}/\theta_0)(1 + \delta\theta_{random}/\theta_0) \quad , \quad (10)$$

236 where θ_0 is the RCE profile used to represent the large scale mean, $\delta\theta_{local}$ and $\delta\theta_{random}$
 237 represent localized and random perturbations respectively (see appendix for details on
 238 the perturbations). A limited number of simulations which vary $\delta\theta_{local}$ and $\delta\theta_{random}$ were
 239 performed to determine the sensitivity to initial conditions. Preliminary results suggest
 240 that the final state of the system is not very sensitive to these perturbations, though more
 241 investigation is necessary in order to determine this conclusively. We do note, however,
 242 that setting $\delta\theta_{random} = \delta\theta_{local} = 0$ and $f = 0$ will not generate convection no matter what
 243 the boundary conditions are.

244 One of the primary goals of this paper is to characterize the statistically steady states
 245 of the system, which we assume with some confidence will remain stable indefinitely. The
 246 steady state precipitation and NGMS are taken as averages over the last month of the
 247 simulation. In most cases, the simulation was run for 4 months, though the run time
 248 varied from 60 to 364 days¹. The shorter runs were justified from the observation that
 249 once the model began precipitating, it equilibrated rather quickly (over a period of hours),
 250 and remained in a statistically steady state. For the numerical experiments which were
 251 initially dry, it could take several days or months to develop precipitation (in one case,
 252 this transition occurred after 70 days, see figure 2). In those cases, longer run times may
 253 be necessary to observe a transition. For that reason, several experiments were run for a
 254 full year to verify whether the system would transition at a later time or if it was truly

255 in a stable equilibrium. Of these, most remained completely dry for the duration of the
256 model run. Two experiments showed moistening of the troposphere after 4 months which
257 was insufficient to overcome convective inhibition and trigger deep convection, even after
258 running the simulation for a full year.

259 Finally, we note that despite some model runs of up to one year, we cannot exclude
260 the possibility that convection could still develop after the model run time. Indeed, of
261 the initially dry experiments which eventually precipitated, we found that the amount of
262 time it took to initiate precipitation increased with decreasing surface wind speeds and
263 potential temperature relaxation time. This is shown in figure 2. On the left, the amount
264 of time it took for a precipitation rate of at least 1 mm day^{-1} to develop is plotted as a
265 function of the relaxation rate (t_θ^{-1}) for different surface wind speeds and domain sizes. In
266 all but one instance, the time to precipitate increased with decreasing t_θ , suggesting that a
267 further decrease in t_θ (which corresponds to a stricter enforcement of WTG) would require
268 a longer wait time to transition to the precipitating equilibrium for a given domain size
269 and wind speed. Considering that the real atmosphere will not maintain steady conditions
270 over a time scale of months, we do not give further consideration to this possibility. Also
271 note that surface fluxes play a role in the amount of time required for precipitation to
272 occur. As shown on the right panel of figure 2, more vigorous surface fluxes associated with
273 higher surface wind speeds initiate convection more quickly. In this case, if we consider
274 wind speeds close to or below the values used to compute the RCE reference profile, we
275 do not expect convection to initiate no matter how long we wait, since there will not be
276 sufficient CAPE. The choice of a 4 month simulation is somewhat arbitrary, and is chosen

277 in hopes that this gives initially dry atmospheres enough time to develop convection if
278 they are going to do so, without excessive computation time.

3. Results

279 As in SBB07, we perform parallel experiments with the domain initialized either with
280 the moisture profile of the surrounding environment or completely dry. We then vary
281 the boundary conditions (surface wind speed, domain size and WTG relaxation time) in
282 each case to establish the range of conditions which support multiple equilibria. Upon
283 establishing this range, we vary the initial conditions to determine the sensitivity of the
284 realized equilibrium to the initial state of the domain. The results from SBB07 suggest
285 that the final equilibrium state depends on the initial moisture in the modeled domain.
286 We cannot say a priori whether or not the system always evolves to the precipitating
287 equilibrium state if *some* amount of moisture exists in the free troposphere, or if there is
288 a minimum amount required to overcome convective inhibition. We address this issue in
289 this section, as well as how this depends on the boundary conditions.

290 In addition to determining the range of boundary conditions which support multiple
291 equilibria, and the sensitivity of the final state to initial conditions, we consider the NGMS
292 as a diagnostic for characterizing the equilibrium state of a modeled domain. Furthermore,
293 we discuss the role of NGMS in developing (or decaying) convection as well as in the steady
294 state. These results are shown in section 4.

3.1. Dependence on surface wind speed

295 The first set of experiments aims to determine the range of wind speeds which support
296 multiple equilibria for a given domain size and potential temperature relaxation time

297 (with SST fixed). Figure 3 shows results for a 50 km domain with $t_\theta = 1.85$ hours.
298 Experiments initiated with RCE moisture profiles are indicated by circles (solid line),
299 while squares (dashed line) represent initially dry experiments. Thin lines correspond to
300 a grid resolution of 500 m, while thick lines give the 1 km grid results. This figure is
301 analogous to figure 2 in SBB07, except in SBB07, the wind speed was fixed and SSTs
302 were varied compared to the RCE value. As discussed in section 2, both have the effect
303 of modulating surface fluxes.

304 From figure 3 (top), we note that a single, precipitating equilibrium exists for wind
305 speeds of 12 ms^{-1} and greater, regardless of horizontal grid resolution. For the finer (500
306 m) grid, a single non-precipitating equilibrium exists below a wind speed of 5 ms^{-1} , while
307 horizontal wind speeds from 5 to 10 ms^{-1} can sustain either a dry or a precipitating
308 steady state, depending on the initial moisture profile. Specifically, in the range of wind
309 speeds that can sustain multiple equilibria, an initially dry domain will remain dry, while
310 an initially moist domain will sustain persistent convection. The only difference in the
311 simulations is the initial mixing ratio profile, which is consistent with results of SBB07.
312 Similar results also hold for the coarser (1 km) grid, though multiple equilibria persist
313 even for wind speeds as low as 3 ms^{-1} .

314 These results demonstrate that multiple equilibrium states exist under certain boundary
315 conditions in our CRM in the WTG approximation. As the wind speed is increased,
316 surface fluxes overcome convective inhibition, allowing shallow convection to penetrate
317 into the free troposphere thus effectively destroying the dry equilibrium state. The bottom
318 panel of figure 3 shows the steady state value of NGMS for the corresponding runs. The
319 significance of these values will be discussed further in section 4.2.

320 Experiments similar to those presented in figure 3 on 100 and 200 km domains show
321 similar behavior, with one important distinction: The larger the domain, the smaller
322 the range in which multiple equilibria exist. Figure 4 shows that with a thermodynamic
323 relaxation time of 1.85 hours, there is only a very narrow range of wind speeds which
324 permit multiple equilibrium on a 100 km domain, and this range decreases even more for
325 a computational domain of 200 km. Also note that, unlike the coarsely gridded 50 km
326 domain, the precipitating equilibrium is destroyed for wind speeds less than 5 ms^{-1} . We
327 discuss the dependence on domain size below.

3.2. Dependence on domain size and relaxation time

328 We did not systematically probe the dependence of precipitation rate on domain size for
329 different relaxation rates, but in comparable instances, we found very little difference in
330 the domain averaged precipitation rate as a function of wind speed in any of the domains
331 with 1 km horizontal grid resolution and wind speeds of 5 ms^{-1} and greater (as discussed
332 above, wind speed less than this sustained precipitation in the 50 km domain while other
333 domain sizes dried and remained dry). Furthermore, there is only a slight difference
334 between the results from a 1 km grid and the 500 m grid, with the finer grid producing
335 slightly higher precipitation rates (compare figures 3 and 4). As explained in section 2, we
336 suspect that this difference is attributed to the moister RCE reference profile generated
337 with a finer grid resolution. Accounting for this, we conclude that in the convecting
338 equilibrium state, there is little dependence of precipitation rate on domain size for a
339 given wind speed and potential temperature relaxation time.

340 Though the change in precipitation rate as a function of domain size in the precipitating
341 equilibrium is nearly negligible, the role of domain size in determining the existence of

multiple equilibria (as seen in figure 4) is quite significant. Smaller computational domains allow multiple equilibria over a larger range of wind speeds. This dependence on domain size is at least partially related to the distribution of convective inhibition in the modeled domain. Figure 5 shows a histogram of deep convective inhibition [DCIN, *Raymond et al.* 2003], where DCIN is defined as

$$DCIN = s_t - s_b \quad . \quad (11)$$

Here, s_t is the threshold entropy for convection and is defined as the vertical average of the saturated moist entropy over the height range 1750-2000 m; s_b is the boundary layer entropy defined as the average of moist entropy over 0-1000 m. The curves on the right correspond to simulations which were initiated with dry mixing ratio profiles ($f = 0$ from equation 9), while curves on the left correspond to simulations initiated with the RCE mixing ratio profiles ($f = 1$). For this set of boundary conditions, all simulations which were initially dry remained dry, and the values of DCIN were taken from the last month and over the entire domain of each simulation (the number of occurrences was normalized to account for a larger number of grid points on larger domain sizes). Even in runs which remained dry, the 200 km domain experienced a higher frequency of lower values of DCIN, which increases the possibility that convection will spontaneously initiate. Indeed, this does occur when the surface wind speed is increased from 5 to 7 ms^{-1} (see, e.g. figure 4). Though we believe that this contributes to the dependence of multiple equilibria on domain size, the similarity between the DCIN distributions in the 50 km and 100 km domains suggests that this is not the whole story.

The results from SBB07 showed that the existence of multiple equilibrium states depends on the time scale of horizontal moisture advection. As we explained earlier, our

365 model ties moisture advection to the time scale of the potential temperature relaxation in
366 the WTG implementation via mass continuity. To explore the implications of this relation,
367 we performed several experiments to determine the sensitivity of multiple equilibria with
368 respect to the WTG potential temperature relaxation time, t_θ . Decreasing the value of t_θ
369 is tantamount to decreasing the amount of time it takes to remove buoyancy anomalies in
370 the vertical profile of potential temperature. A relaxation time of zero corresponds to an
371 instantaneous relaxation to the specified environmental profile, and thus strictly enforces
372 the WTG approximation, and $t_\theta = \infty$ turns off WTG entirely.

373 Figure 6 illustrates the dependence of the final precipitation state on the potential tem-
374 perature relaxation time for the different domain sizes. All of the experiments shown
375 were initiated with zero moisture and have an imposed horizontal wind speed of 10 ms^{-1} .
376 The larger values of t_θ result in a precipitating state, and all domain sizes can main-
377 tain a dry equilibrium for sufficiently small t_θ . This means that the more strongly the
378 WTG approximation is enforced, the more likely the system is to sustain multiple equi-
379 libria. Furthermore, the relaxation time scale at which the dry equilibrium is destroyed
380 decreases with increasing domain size. This implies that the WTG approximation must
381 be more vigorously enforced the larger the domain in order to maintain multiple equi-
382 libria. To rationalize this tendency, consider that smaller relaxation times prevent the
383 development of domain-averaged temperature profiles which differ significantly from that
384 of the reference profile (see figure 8 and the discussion below). Perhaps such temperature
385 profile anomalies are more conducive to the development of deep convection. In the larger
386 horizontal domains, these anomalies may contribute to more developed circulations which
387 ultimately destroy the dry equilibrium state. Presumably in the presence of radiative

388 cooling, if there is no convection and t_θ is finite, the troposphere cools which increases
389 CAPE and decreases convective inhibition, resulting in a more favorable environment for
390 convection.

391 Figure 7 encapsulates the dependence of multiple equilibria on domain size and poten-
392 tial temperature relaxation time for given surface flux conditions. All symbols represent
393 experiments which were initiated with completely dry profiles. Solid circles represent
394 simulations which eventually precipitated while open squares indicate that the domain
395 remained dry. This shows that precipitation is more likely to develop with longer re-
396 laxation times or larger domains. Furthermore, this type of figure shows the range of
397 parameters under which our model supports multiple equilibria since we can conclude
398 that the dry equilibrium is destroyed under the conditions which developed precipitation.

399 We now discuss the interpretation of t_θ as a measure of the WTG approximation, and
400 the relationship between t_θ and precipitation rate. An inspection of the vertical profile of
401 potential temperature in the steady state suggests a nearly constant deviation from the
402 reference profile in the free troposphere (not shown). Taking the free troposphere to be
403 between 4 km and 12 km, we define $\delta\theta$ to be the vertical average over this height range,
404 the horizontal average over the model domain, and the time average over the last month
405 of the simulation. Figure 8 shows a plot of $\delta\theta$ as a function of t_θ for simulations with
406 all ranges of surface wind speeds, all domain sizes, and arbitrary initial moisture. The
407 black filled triangles represent the simulations which converged to the dry equilibrium,
408 independent of model parameters, while all other symbols represent simulations which
409 eventually sustained precipitation and are coded to indicate wind speed. There are several
410 observations we can make from this figure. First note that strong convection (i.e., when

411 convective heating is greater than radiative cooling) corresponds to $\delta\theta > 0$, with stronger
412 heating resulting in larger temperature deviations (indicated by steeper slopes in figure
413 8). Weak or non-existent convection results in $\delta\theta < 0$. The WTG formulation of the
414 temperature basically guarantees this behavior; Since we parameterize the large-scale
415 vertical advection as a relaxation (measured by t_θ), it responds to the tendencies given by
416 radiation and convection, which implies that those control the sign of $\delta\theta$. So here we see
417 how convection controls the temperature anomalies. However, the temperature anomalies
418 are also known to influence convection (see e.g., *Raymond and Sessions* [2007]), which
419 implies a coupling between the two.

420 To see how the coupling between convection and temperature anomalies influence pre-
421 cipitation rate as we vary t_θ , we plot precipitation rate versus $\delta\theta$ for all experiments
422 (shown in figure 9). Again we see that negative temperature deviations correspond ei-
423 ther to the dry equilibrium (precipitation rate is zero) or to cases where convection is
424 weak (as when the surface wind speed is equal to the ambient RCE value, in this case 5
425 ms^{-1}). Stronger convection associated with positive $\delta\theta$ have non-zero precipitation rates.
426 Note that for a given surface wind speed, there is an increase in precipitation rate as the
427 WTG approximation is relaxed (t_θ and $\delta\theta$ increase proportionally from zero), followed by
428 a subsequent decrease. The rise corresponds to a switch from the dry equilibrium to the
429 precipitating one, while the fall corresponds to the slower shift from a WTG-dominated
430 simulation to one approaching RCE (as $t_\theta \rightarrow \infty$). This behavior emphasizes the coupling
431 between the atmospheric stability and precipitation.

3.3. Dependence on initial conditions

432 So far we have confirmed the existence of multiple equilibria in our CRM using the WTG
433 approximation, and determined the range of parameters which may sustain both states.
434 Given a specific set of boundary conditions that may sustain both equilibria, we find that
435 the state which is ultimately realized by the model depends on the initial moisture profile
436 of the modeled domain. This is identical to the conclusions in SBB07. SBB07 suggested
437 that the persistence of the dry equilibrium in the presence of positive CAPE requires that
438 the free troposphere be able to remain completely dry, even over a moist boundary layer.
439 Though not shown, all experiments which remained dry observed this condition.

440 To further test the sensitivity of the final equilibrium state to the initial moisture, we
441 performed a series of experiments which were initiated with a mixing ratio profile equal
442 to a non-zero fraction of the reference profile. We find that there is a minimum amount
443 of moisture necessary to move the system from the non-precipitating to the precipitating
444 equilibrium state. An example of this is shown in figure 10. Here, each bullet represents
445 a simulation performed on a 50 km domain with $t_\theta = 17$ min. Line styles and symbols
446 indicate different wind speeds. The initial moisture is taken to be a fraction of the RCE
447 mixing ratio profile (with the fraction given by f in equation (9)). These results show that
448 even though some simulations are initiated with non-zero moisture in the troposphere, the
449 amount is insufficient to initiate deep convection and transition to the precipitating equi-
450 librium. The larger the surface fluxes, the more likely a small amount of moisture will
451 result in persistent deep convection. However, domains with low surface winds may toler-
452 ate a relatively large fraction of moisture without initiating deep convection. For example,
453 one experiment with a wind speed of 7 ms^{-1} remained dry even with an initial moisture

454 profile equal to 80% of the moisture of the surrounding environment. In this particular
455 case, the initial moisture in the free troposphere rapidly vanishes due to subsidence and
456 lateral export, which can be deduced from figures 11 and 12.

457 Figure 11 shows the evolution of relative humidity for the 7 ms^{-1} experiment on a 50
458 km domain initialized with 80% of the reference moisture profile (white is zero relative
459 humidity, while black indicates 100%; in this experiment, $t_\theta = 17 \text{ min.}$). Superimposed is
460 a decomposition of the contributions of the total water mixing ratio tendencies into the
461 free troposphere by eddy motions and parameterized lateral fluxes (the latter is due to the
462 WTG motions given by equations (4-6)). We have also plotted the moisture tendency into
463 the boundary layer from surface fluxes. Explicit and unresolved eddy contributions are
464 taken as vertical fluxes into the free troposphere ($\overline{\rho w' r_t'}$ and $-k \partial r_t / \partial z$, respectively; The
465 overbar represents a horizontal average, primes indicate perturbations from the mean, w
466 is the vertical velocity, and k is the eddy mixing coefficient given in RZ05), where we
467 approximate the lower free tropospheric boundary to be at 2 km for this purpose. The
468 contribution by lateral fluxes implied by WTG is vertically integrated over the entire
469 depth, and convergence into the boundary layer is assumed to quickly enter the free tro-
470 posphere. This assumption is justified by the boundary layer quasi-equilibrium hypothesis
471 [*Emanuel, 1995; Raymond, 1995*]. In the first 5 days of the experiment, unresolved eddies
472 transport moisture into the free troposphere while the resolved ones have a small drying
473 tendency. Moisture is also removed by the WTG horizontal flux divergence. In the steady
474 state, the horizontal fluxes remove moisture from the boundary layer (note the balance
475 between horizontal and surface fluxes). The evolution of relative humidity in figure 11
476 shows that nearly all of the initial moisture in the free troposphere is gone by day 15.

477 Figure 12 shows the corresponding profile of vertical mass flux due to the WTG vertical
478 velocity for the first 10 days. Each profile represents an average of the entire day. Initially,
479 there is subsidence throughout the entire troposphere, with a maximum downdraft at 12
480 km. Subsequent days acquire a small updraft in the boundary layer which lies just below
481 a sharp downdraft at about 2 km. The downdraft is a result of the strong radiative
482 emission associated with the large moisture gradient that occurs when a dry troposphere
483 overlies a moist boundary layer. The updraft is associated with shallow convective heating.
484 Together, this behavior suggests a thin horizontal outflow at the level of zero mass flux
485 (just above 1 km). Physically, this corresponds to shallow non-precipitating clouds mixing
486 boundary layer air with air aloft and detraining air near cloud base, similar to the process
487 described by *Raymond and Blyth* [1986]. In the model it is effected both by the subgrid
488 mixing process and by explicit convection (see relative contributions of eddy motions
489 to the free troposphere mixing ratio in figure 11). The mixing process in combination
490 with horizontal flux through the lateral boundaries removes the moisture from the free
491 troposphere. By day 9, the system is completely subsiding. Once the moisture in the free
492 troposphere has been evacuated, the boundary layer also becomes unsaturated, though
493 it remains moist in the steady state. The mass flux profile changes very little from the
494 last day plotted in this figure. From this we conclude that the moisture convergence in
495 the boundary layer in the early days of the simulation is too weak to overcome convective
496 inhibition and the troposphere dries and remains dry.

497 The ability of our model to sustain multiple equilibria depends on the ability of shallow
498 convection to moisten the free troposphere. The relative contributions of moisture to the
499 free troposphere shown in figures 11 and 14 suggest that this moistening is primarily the

500 result of subgrid eddies. Therefore, the choice of subgrid parameterization may affect the
501 range of parameters which permit multiple equilibria. We also note that restricting the
502 motions to two dimensions may affect entrainment and detrainment which could further
503 modify the parameter space permitting multiple equilibria [*Petch et al.*, 2008].

504 Within the parameter space explored in this study, we conclude that the initial mois-
505 ture determines which equilibrium state is realized, while the boundary conditions (sur-
506 face wind speeds, SST, domain size and relaxation time scale in WTG mode) determine
507 whether there are multiple equilibria.

4. NGMS as a diagnostic

508 *Raymond et al.* [2009] discussed the role of NGMS in transient flows as well as in a
509 multiple equilibrium situation. In this section, we study the evolution of the NGMS in the
510 various model integrations in order to gain further insight into the nature of the equilibria
511 and the transitions between them. Of particular interest is the possibility of a negative
512 NGMS, which may occur during periods of rapid change in the environmental state or in
513 a dry equilibrium steady state.

4.1. NGMS in transient flows

514 The nature of the experiments reported here provides an opportunity to consider situa-
515 tions which make a transition from a dry to precipitating state or vice versa. The former
516 case is encountered in an initially dry atmosphere in which the troposphere proceeds to
517 moisten and the system initiates and sustains deep convection. During such an evolu-
518 tion, the initial convective vertical mass flux profile typically has a maximum at lower
519 levels. By mass continuity, this suggests that the mass convergence is concentrated near

520 the surface, where the moist entropy is greater than at middle levels [*Raymond et al.*,
521 2009]. Alternatively, the radiative cooling profile can cause a net inflow at high levels
522 where moist entropy is large compared to mid-levels (this happens in our simulations, see
523 e.g., figure 15). A vertical pressure integral under either of these conditions results in a
524 net import of moist entropy. Furthermore, moisture is likely to be imported under such
525 conditions, which implies a period where the system has negative NGMS [*Sobel*, 2007].
526 An example of the transition from a dry state to a precipitating one is shown in figures 13
527 and 14. Here, the saturation fraction increases gradually (figure 13) as moisture begins
528 to penetrate into the lower free troposphere (figure 14). As seen in figure 14, unresolved
529 eddies associated with shallow convection are largely responsible for the initial moisten-
530 ing. During this period, the resolved eddies which represent the mesoscale response to
531 the convection, actually act to dry the free troposphere. The net result, however, is mois-
532 ture penetrating into the free troposphere which is accompanied by an increase in the
533 saturation fraction. After about 24 days of the simulation, a sharp increase in the satu-
534 ration fraction triggers the onset of precipitation. At this time, a change in the implicit
535 large-scale divergence profile associated with the convective heating causes the descent in
536 the lower free troposphere to weaken, which in turn causes the net moisture convergence
537 ($-\nabla \cdot r\mathbf{v}$) to go from negative to positive. Consequently, the NGMS (see eq. (7)) goes
538 from large and positive to large and negative. For a period of about 2.5 days, the domain
539 is importing both moisture and moist entropy. When the system begins to export moist
540 entropy ($\nabla \cdot s\mathbf{v}$ goes from negative to positive), the saturation fraction and precipitation
541 rate level off. In this case, the steady state is exporting moist entropy, importing moisture,
542 resulting in a positive NGMS in the precipitating equilibrium.

543 Figure 15 shows the evolution of the vertical mass flux (left) and moist entropy (right) for
544 days 22-27 of the simulation, during which the domain makes the transition from the dry to
545 the precipitating state. This progression begins with a descent throughout the troposphere
546 with a maximum near 2 km and small updraft near the surface, indicating detrainment
547 just above 1 km in height. At this time, moisture is being exported from the domain,
548 and there is a net import of moist entropy due to the circulation near the surface. During
549 the period with negative NGMS (days 24-25), there is very little average vertical motion
550 in the free troposphere (which indicates the existence of some convective updrafts, whose
551 condensation heating balances radiative cooling), and the low level downdraft decays
552 which weakens the divergence near the surface, allowing moisture from the boundary layer
553 to penetrate into the free troposphere. On day 25, levels of convergence (surface, 9 km
554 altitude) import higher values of moist entropy than exported at the levels of divergence
555 (4 km), thus maintaining the import of moist entropy. By day 26, there is upward vertical
556 motion from the surface to the tropopause, which progresses to strong convergence near
557 2.5 km. The final stages produce a net export of moist entropy (and $\text{NGMS} > 0$).

558 In addition to convection developing in a quiescent region, negative NGMS can also
559 occur in decaying convection as might be expected with entrainment of dry air as in *Sobel*
560 *and Bellon* [2008]. In our simulations, this could happen for instance if a system is initiated
561 with a moisture profile representative of the surrounding environment, but with surface
562 wind speeds less than the value used for calculating RCE, or if the initiated moisture is
563 insufficient to sustain convection (as in the case shown in figures 11 and 12). Figure 16
564 is an example of the first possibility. In this case, we see moisture is imported into the
565 tropospheric column; rainfall rate drops along with the saturation fraction, and NGMS

566 is negative until the domain begins to export moisture. We note that the precipitation
567 rate is calculated as a time derivative of the cumulative precipitation in the domain. The
568 initial precipitation rate is artificially high since it is near an end point in the centered
569 difference scheme used in the derivative calculation. This does not affect the calculation of
570 NGMS. At day 11, moist entropy is exported for a period of about 3 days, again resulting
571 in a negative NGMS. Note that the period of negative NGMS is associated with a strong
572 time derivative of the saturation fraction. The decrease in saturation fraction is a result
573 of the export of moist entropy. In the steady state, surface fluxes and radiation balance
574 the import/export of moist entropy which results in a steady saturation fraction and
575 a small NGMS (which is positive in this case). The evolution of mass flux and domain
576 averaged moist entropy for the first 16 days of this simulation is shown in figure 17. Times
577 where moisture and moist entropy are imported/exported are emphasized along with the
578 corresponding sign of NGMS. An interesting observation from this time sequence is that
579 the atmosphere near the surface is close to saturation for times when the lateral export
580 of moist entropy is increasing; as this quantity decreases, the atmosphere just above the
581 surface becomes unsaturated. This implies that the surface fluxes are responsible for the
582 negative NGMS during days 11-17.

583 So we see in both developing and decaying stages of deep convection, NGMS may
584 become negative. It seems that NGMS tends to become negative during times of rapid
585 change in the modelled atmospheric state.

586 The sign of NGMS in the transient stages of convection is particularly salient with
587 respect to observations reported by *López and Raymond* [2005] and *Back and Bretherton*
588 [2006]. *Back and Bretherton* considered the moist static energy budget (an alternative to

589 moist entropy), and found that the sign of moist static energy export was determined by
590 the strength of the contribution due to horizontal motions in comparison to the vertical
591 contribution. While horizontal motions import moist static energy to the west and east
592 Pacific, vertical motions imported moist static energy in the west Pacific and exported it
593 from the east Pacific. The sign of the total import in the east Pacific changed depending
594 on the size of the import by horizontal motions. Both *Back and Bretherton* [2006] and
595 *López and Raymond* [2005] conclude that import or export of thermodynamic quantities
596 by vertical motions (which contribute to the sign of NGMS) are determined by the shape
597 of the vertical motion profile. Our results support these conclusions.

4.2. NGMS in steady state

598 Now we consider the steady state NGMS as a diagnostic for characterizing the equilib-
599 rium state. For example, figure 3 compares the steady state values of NGMS for the two
600 equilibrium states for the 50 km domains (500 m and 1 km grid resolutions). In the case
601 where a single equilibrium exists (either dry or precipitating), the steady state NGMS is
602 the same for both sets of experiments, independent of the initial moisture profile. In the
603 range of wind speeds where both equilibria exist, the value of NGMS for the precipitating
604 state is considerably larger than that of the dry state, and we can see that NGMS provides
605 a valuable characterization of the environmental flows. Note that the value of NGMS for
606 a surface wind speed of 5 ms^{-1} is either very large or very small for the precipitating
607 equilibrium, depending on grid resolution (figure 3). We know that RCE is a possible
608 solution for these boundary conditions, so we might expect the WTG model to reach that
609 solution. In that case, NGMS would be undefined since there is already a balance in mois-
610 ture and moist entropy, and a time average of the entrainment or detrainment of these

611 quantities should be close to zero and their ratio undefined. However, that need not occur
612 exactly, so instead we see large fluctuations in NGMS at this wind speed. For this reason,
613 we exclude calculations of NGMS in WTG simulations with surface winds of 5 ms^{-1} (or,
614 more generally, under RCE boundary conditions). For wind speeds greater than or less
615 than 5 ms^{-1} , either convective updrafts or subsidence dominates, which implies non-zero
616 averages of moisture and moist entropy convergence. Thus, for non-RCE boundary con-
617 ditions, NGMS computed from time-averaged quantities is useful (and we emphasize that
618 the time average is performed before the ratio of entropy divergence to moisture conver-
619 gence is taken). There was one exception to this in our simulations. The 50 km domain
620 with 1 km grid resolution sustained convection with wind speeds less than 5 ms^{-1} . We
621 are not sure why this happens, but it also causes large fluctuations in NGMS due to the
622 fluctuations between moisture import and export. Consequently, we also exclude these
623 values of NGMS.

624 To further demonstrate the value of NGMS as a diagnostic, figure 18 shows a scatter
625 plot of precipitation rate as a function of NGMS for all experiments with surface winds
626 greater than 5 ms^{-1} . Those with surface winds of 5 ms^{-1} are excluded since there is
627 essentially no correlation between the NGMS and precipitation for the reason discussed
628 above. The experiments with wind speeds less than 5 ms^{-1} are dry in the steady state,
629 with NGMS values less than 0.25. The symbols indicate the prescribed surface wind
630 speed in the particular experiment. Each symbol represents the steady state rain rate
631 and NGMS, independent of the initial moisture and domain size. There are three striking
632 observations from figure 18. First, *all* simulations which resulted in a dry equilibrium
633 have an NGMS less than 0.25, while those in the convecting equilibrium state have an

634 NGMS greater than 0.35. A vertical line corresponding to $\Gamma = 0.3$ clearly divides the
635 precipitating and dry equilibria.

636 The sharp transition in NGMS for the precipitating versus non-precipitating equilibrium
637 states could be related to the *Peters and Neelin* [2007; also *Neelin et al.*, 2009] hypothesis
638 that the tropical atmosphere represents an observable example of self-organized criticality.
639 Though further work is necessary to verify this in the context of our CRM, the current
640 work suggests that NGMS could be an important parameter for understanding this phe-
641 nomenon. Furthermore, the WTG approximation could prove to be important for probing
642 this possibility numerically.

643 *Raymond and Sessions* [2007] showed that increasing the atmospheric stability or mois-
644 ture of the reference profile decreases NGMS and increases the precipitation rate for a
645 given set of boundary conditions. Results from that work showed that the highest pre-
646 cipitation rates corresponded to values of NGMS which would be characterized by the
647 dry equilibrium in this work (i.e., $\text{NGMS} < 0.2$). This suggests that the reference profiles
648 also influence the range of boundary conditions which support multiple equilibria. More
649 specifically, this implies that the value of NGMS which separates the two equilibria de-
650 pends on the surrounding environment. Therefore, the dotted line in figure 18 is valid
651 only for a RCE reference profile. We expect that the location of this line will change for
652 different reference profiles.

653 The second observation to make from figure 18 is that, for simulations which sustain
654 convection and precipitation, the precipitation rate increases as NGMS decreases for a
655 given wind speed. The multiple values of rain rate for a given wind speed are a result of
656 the different values of the potential temperature relaxation time scale, t_θ (see discussion in

657 section 3). Furthermore, the precipitation for a given value of NGMS increases with wind
658 speed. The qualitative relationship between precipitation, NGMS and surface fluxes as
659 shown in figure 18 agrees with the steady state condition given by equation (8). Also note
660 that while there is a relationship between NGMS and precipitation within an individual
661 set of simulations, across all sets there is none, except for the split between the two types
662 of equilibrium.

663 The final observation regarding data in figure 18 is that for simulations which resulted in
664 a dry equilibrium, NGMS can be either weakly positive or negative. In all cases, subsidence
665 results in moisture export, and therefore the sign of NGMS in the dry equilibrium is de-
666 termined by whether the moist entropy is imported or exported. Near-surface circulations
667 determine this. Negative values of NGMS have been previously reported in RCE simula-
668 tions by *Bretherton et al.* [2005]. In that work, randomly seeded convection self-organized
669 into a single intensely convecting region surrounded by a dry subsiding atmosphere. The
670 most intensely convecting regions exhibited a positive gross moist stability, while negative
671 values existed elsewhere. Comparing those results to the steady-state NGMS values in
672 our simulations suggests that there is indeed an analogy between the dry and convecting
673 regions in *Bretherton et al.*'s [2005] large RCE domain and our domain-wide dry or moist
674 WTG equilibria (this was also pointed out in SBB07).

675 *Raymond et al.* [2009] also addressed the role of NGMS in the steady state in multiple
676 equilibrium. The authors discussed the idea of having two stable equilibria—one dry and
677 one with persistent deep convection and precipitation—separated by an unstable equilib-
678 rium. Figure 19 is the conceptual picture from *Raymond et al.* [2009], reprinted here for
679 convenience. The existence and location of the unstable equilibrium relative to the stable

680 equilibria is dependent on the boundary conditions. Initial conditions and the amplitude
681 of external forcing, if any, determine the state of the system relative to the unstable
682 equilibrium, and hence which equilibrium is ultimately realized in the steady state.

5. Summary and Discussion

683 We have established the existence of a dry and a precipitating steady state in a cloud
684 resolving model employing the WTG approximation. In this study, we have identified
685 several parameters which control the existence of two equilibrium states: 1) surface fluxes
686 (modulated by SST or surface winds), 2) domain size, and 3) time scale for relaxation
687 of the potential temperature to the environmental mean. Our results suggest that cer-
688 tain combinations of these are necessary for supporting multiple equilibria, and provide
689 some guidance for atmospheric conditions that may also be good candidates for multiple
690 equilibrium states.

691 Given an initially dry environment, the ability of the domain to initiate and sustain deep
692 convection hinges on its capacity to moisten the free troposphere in the face of radiatively
693 induced subsidence and maintain a sufficient saturation fraction. As shown in numerous
694 studies [*Bretherton et al.*, 2004; *Raymond et al.*, 2007], the precipitation is a sensitive
695 function of moisture in the environment, so in order for deep convection to develop,
696 there must be an external source of moisture and a mechanism which serves to transport
697 that moisture into the troposphere. Moisture from the boundary layer is accessible as
698 long as convection can overcome convective inhibition. The most effective channel for
699 this is increasing surface fluxes. Alternatively, some conditions may be conducive to the
700 initiation and development of deep convection, such as regions of low convective inhibition
701 (as occurs with larger domain sizes). Thus, increasing the domain size will increase the

702 likelihood that convection will occur spontaneously and allow the troposphere to tap into
703 the moisture of the boundary layer. In addition to boundary layer moisture, atmospheric
704 circulations can horizontally advect moisture from the surrounding environment, providing
705 an alternate mechanism for the system to transition from the dry state to the precipitating
706 one. This mechanism is likely influenced by the potential temperature relaxation time,
707 t_θ , since this controls the parameterized large-scale vertical motion. By mass continuity,
708 this also affects horizontal flows which may advect moisture. Our results indicate that
709 larger values of t_θ permit the free tropospheric moistening necessary to initiate deep
710 convection. These results are qualitatively consistent with the dependence of multiple
711 equilibria on the moisture relaxation time used in SBB07. In SBB07, longer relaxation
712 times for moisture advection killed the dry equilibrium state. If any of these mechanisms
713 successfully bring the system to the convecting equilibrium from a completely dry state,
714 we assume the conditions are insufficient to sustain a dry equilibrium. Failure, on the
715 other hand, implies the existence of multiple equilibria under the specific set of boundary
716 conditions.

717 It is important to also point out that the dependence on the time scale for relaxation
718 of the local potential temperature profiles to the large scale mean suggests that nonlocal
719 adjustment, parameterized here by the weak temperature gradient approximation, is an
720 important ingredient in producing multiple equilibria. Our results clearly show that if the
721 WTG approximation is only weakly obeyed (as measured by the magnitude of t_θ), the dry
722 equilibrium is destroyed. The multiple equilibria are local phenomena which occur over
723 areas small compared to some nominal larger domain (outside the computational domain,
724 and thus implicit in our calculations), with the mass circulation implied by the WTG

725 vertical velocity closed everywhere in the domain. A dry equilibrium cannot occur in the
726 spatial average over a domain with a truly closed mass budget (represented by RCE) since
727 in that case deep convection must occur in order to provide heating to balance radiative
728 cooling.

729 Upon establishing a set of boundary conditions capable of supporting multiple equilibria,
730 the actual state realized by the model will depend on the initial moisture in the free
731 troposphere. In this case, there is a threshold fraction of the mixing ratio profile below
732 which the initial moisture is advected out and the system sits at the dry equilibrium.
733 Moisture profiles which exceed the threshold fraction can sustain a convecting equilibrium.
734 The threshold value is a function of boundary conditions, being small for conditions which
735 support heavy precipitation (as with large surface winds) and large in conditions which
736 result in smaller precipitation rates.

737 Finally, we consider NGMS as a diagnostic of environmental characteristics. In the
738 steady state, this set of simulations found that values of NGMS less than 0.25 correspond
739 to the dry equilibrium, while values greater than 0.35 all correspond to a precipitating
740 equilibrium. The dry equilibrium physically represents a region of subsidence where mois-
741 ture is being exported from the domain and there is relatively little exchange of moist
742 entropy between the modeled domain and the surrounding environment, giving rise to
743 the smaller magnitudes of NGMS. Under these conditions, the vertical integral of moist
744 entropy can give a net import or export, depending on circulations in the boundary layer.
745 Completely divergent mass flux profiles imply net entropy export which, when coupled to
746 moisture export, implies a negative value of NGMS in the steady state. Any convergence

747 in the boundary layer is sufficient to import moist entropy and result in a small, positive
748 NGMS.

749 The NGMS also provides insight with respect to transient stages of developing or de-
750 caying deep convection. When convection is developing in an initially dry environment,
751 the domain goes from having a net export of moisture to a net import. Furthermore, the
752 moist entropy convergence of the dry state transitions to a net divergence in the mature
753 stages of convection. As these sign changes need not occur simultaneously, there may be
754 a period of time when both moisture and moist entropy are being imported, resulting in a
755 negative value of NGMS. A similar argument holds also for decaying convection, in which
756 case the period of negative NGMS corresponds to both quantities being exported. Our
757 simulations showed that periods of negative NGMS correspond to times of rapid change
758 in the saturation fraction.

Appendix

1. Initialization of thermodynamic profiles

759 As mentioned in section 2, the initial potential temperature profile is given by

$$760 \quad \theta_{init} = \theta_0(1 + \delta\theta_{local}/\theta_0)(1 + \delta\theta_{random}/\theta_0) \quad , \quad (1)$$

761 where θ_0 is the height dependent RCE profile used to represent the large scale mean,

762 $\delta\theta_{local}$ and $\delta\theta_{random}$ represent localized and random perturbations respectively:

$$763 \quad \begin{aligned} \delta\theta_{local} &= \delta\theta_{local}^{max} \exp[-(x_s^2 + z_s^2)] \\ \delta\theta_{random} &= \delta\theta_{random}^{max} z_s \exp[1 - z_s] \end{aligned} \quad . \quad (2)$$

764 Here, x_s and z_s are scaled Cartesian coordinates (i.e., $z_s = z/z_{scale}$, where z_{scale} defines the

765 width of the localized Gaussian perturbation; an analogous definition holds for x_s). For

766 most simulations, $x_{scale} = 3$ km, $z_{scale} = 1$ km. The maximum values of the perturbations

767 are given by $\delta\theta_{local}^{max} = f_{local}\theta_0$ and $\delta\theta_{random}^{max} = f_{random}\theta_0$, where f_{local} and f_{random} are
768 fractional multipliers which represent the actual model input. The maximum magnitude
769 of $\delta\theta_{local}^{max}$ and $\delta\theta_{random}^{max}$ are approximated below by multiplying the respective fractional
770 multipliers by 300 K. We prescribe random temperature fluctuations with a maximum
771 magnitude of $\delta\theta_{random}^{max} = 0.3$ K to be distributed at 1 km in altitude. Similarly, setting
772 $\delta\theta_{local}^{max} = 9$ K increases the local temperature for a 3 km region centered horizontally with
773 a vertical extent of about 1 km centered at 1 km in altitude. We originally chose such
774 a large perturbation in hopes that we could get a reliable response (either developing
775 precipitation or not) from an initially dry domain.

776 A limited number of simulations which vary $\delta\theta_{local}^{max}$ and $\delta\theta_{random}^{max}$ were performed to
777 determine the sensitivity to initial conditions. While preliminary results suggest that the
778 final state of the system is not very sensitive to these perturbations, two sets of experiments
779 suggest that the initial conditions play some role in the equilibration time of the model.
780 Specifically, as noted in section 2, there were two experiments in which the free troposphere
781 moistened but convective inhibition suppressed the development of deep convection even
782 after running the simulation for a full year. In these cases, the precipitation rate remained
783 negligible compared to the corresponding rate in the equilibrium state with persistent
784 deep convection, but the NGMS remained noisy up until the last month of the simulation.
785 Repeating these experiments with slightly different initial conditions (i.e., adjusting $\delta\theta_{local}$
786 and $\delta\theta_{random}$) allowed the model to equilibrate quicker, but did not change the final state
787 of the system.

788 More investigation is necessary to conclusively determine the sensitivity of the final
789 state on the initial potential temperature perturbations. As stated in section 2, setting

790 $\delta\theta_{random}^{max} = \delta\theta_{local}^{max} = 0$ and $f = 0$ (where f is defined in equation (9)) will not generate
791 convection no matter what the boundary conditions are.

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Notes

1. A table of model parameters, including run times of each simulation is provided as a supplement for this paper. It can
also be found at <http://www.physics.nmt.edu/~sessions>
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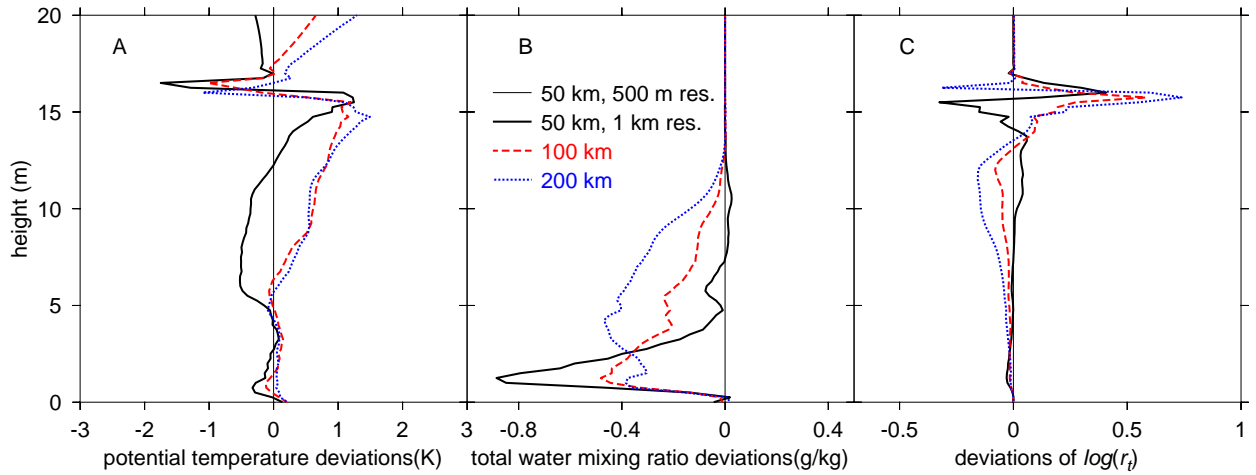


Figure 1. Comparison of RCE profiles for 50, 100, and 200 km domains (solid, dashed, and dotted lines, resp.). To emphasize the difference between the profiles characterizing the large scale environment, we plot the deviations of the profiles from those of the 50 km domain with 500 m resolution (thin solid line). A) Shows deviations in potential temperature, B) gives deviations in total water mixing ratio, and C) shows the differences in the log of the mixing ratio to emphasize the differences in moisture in the upper troposphere. The larger domains are warmer in the upper troposphere, but are also considerably drier. Note the difference in moisture distribution for the 50 km domains (line thickness denotes grid resolution).

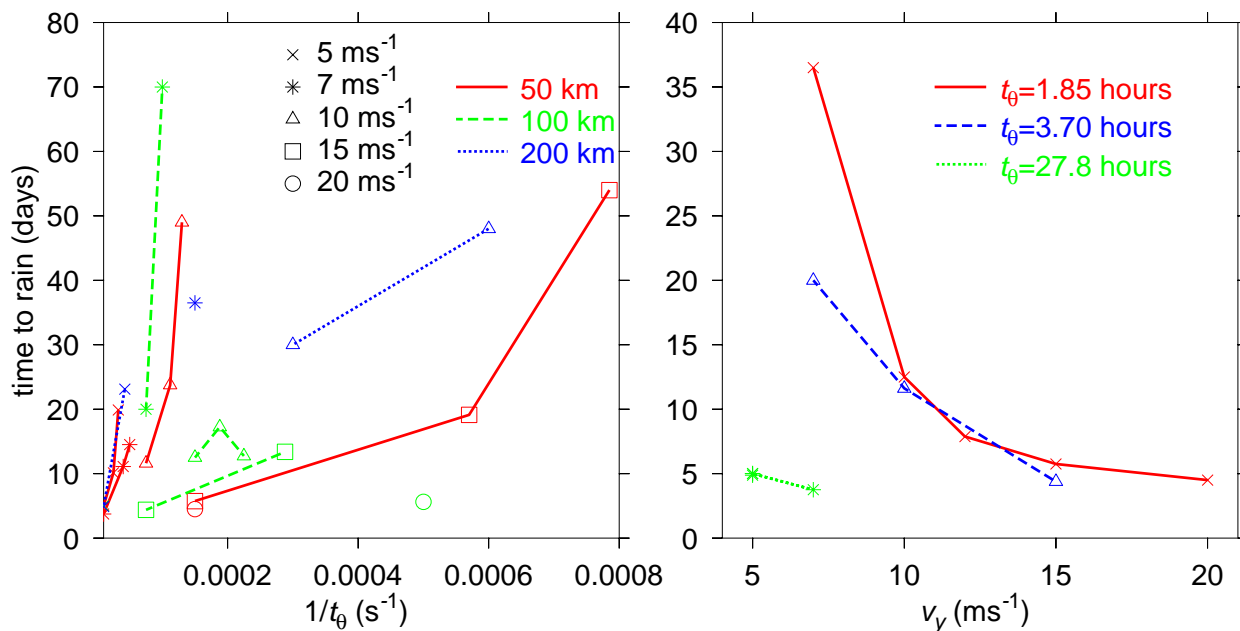


Figure 2. The amount of time needed for an initially dry domain to generate at least 1 mm day⁻¹ precipitation rate is plotted as a function of relaxation rate (t_{θ}^{-1} , left) and surface wind speed (v_y , right). In the first case, lines connect experiments with same domain size (indicated by line style) and wind speed (symbols). Experiments shown on the right panel had a common t_{θ} and thus allowed us to see how the imposed wind speed influenced the onset of precipitation (independent of domain size). Note the different scale on the vertical axes.

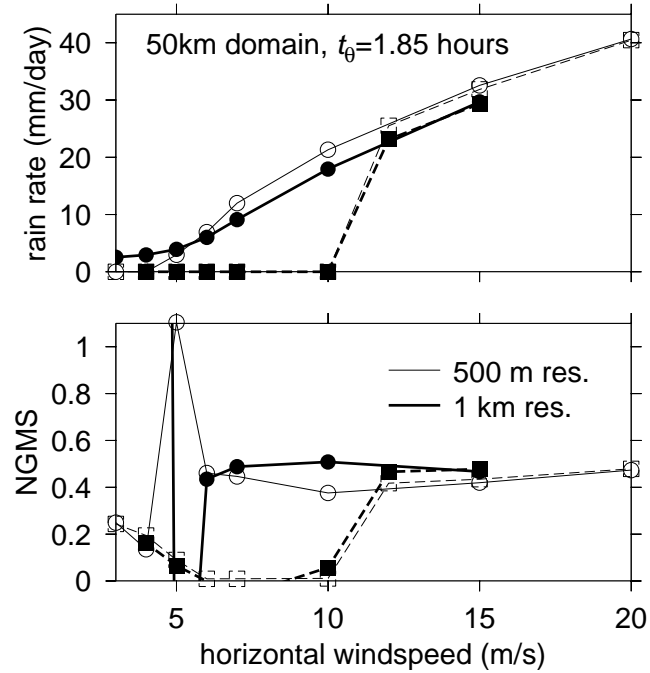


Figure 3. Precipitation (top) and NGMS (bottom) as a function of imposed horizontal wind speed. Solid lines (circles) represent simulations initiated with RCE profiles of potential temperature and mixing ratio. Simulations which were initially dry are shown with dashed lines (squares). Line thickness indicates horizontal grid resolution. In the range of 5 and 10 ms^{-1} , 2 equilibrium states exist regardless of grid resolution; above 10 ms^{-1} , only the precipitating state exists; below 5 ms^{-1} , only the dry state remains for the finer grid, while multiple equilibria persist at lower wind speeds for the coarser grid.

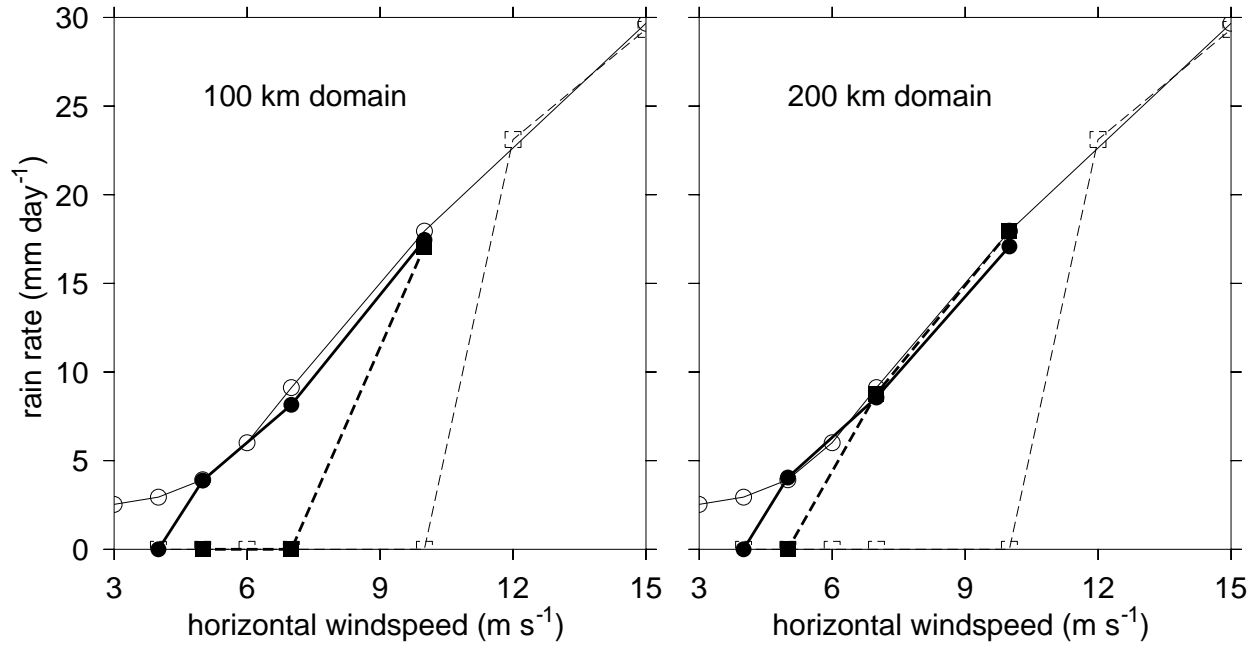


Figure 4. Same as figure 3, but for the 100 and 200 km domain experiments (left and right, respectively). The results for the 50 km domain with 1 km horizontal grid resolution have been overlaid with thin lines and open symbols for comparison. Note that the range wind speeds where multiple equilibria exist decreases as the domain size increases.

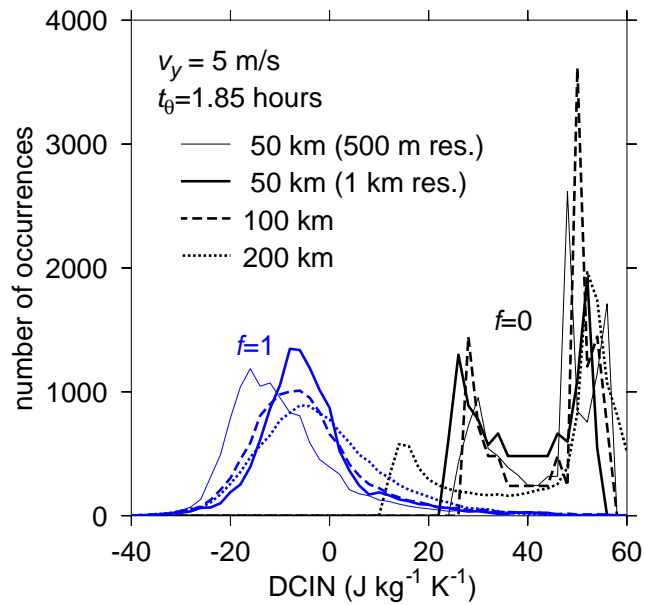


Figure 5. A histogram of values of DCIN (see text) for experiments with surface winds of 5 ms^{-1} and $t_\theta = 1.85$ hours. Line styles differentiate domain sizes and resolution. For these boundary conditions, all simulations which were initially dry remained dry ($f = 0$, on the right), and runs initially moist sustained convection ($f = 1$, on the left). Note that the $f = 0$ curve for the 200 km domain has a considerable number of low-DCIN points compared the the other domains.

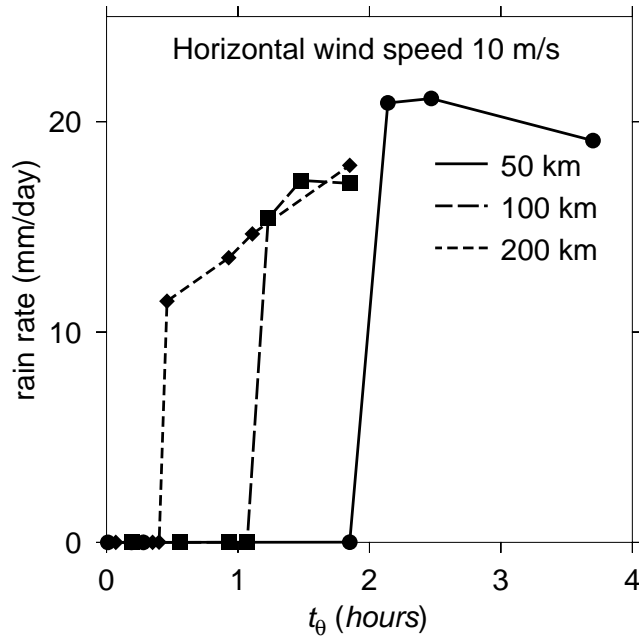


Figure 6. Equilibrium precipitation as a function of potential temperature relaxation time, t_θ , for 50, 100, and 200 km domains. The horizontal imposed wind is 10 ms^{-1} . Symbols represent numerical experiments, all of which are initially dry.

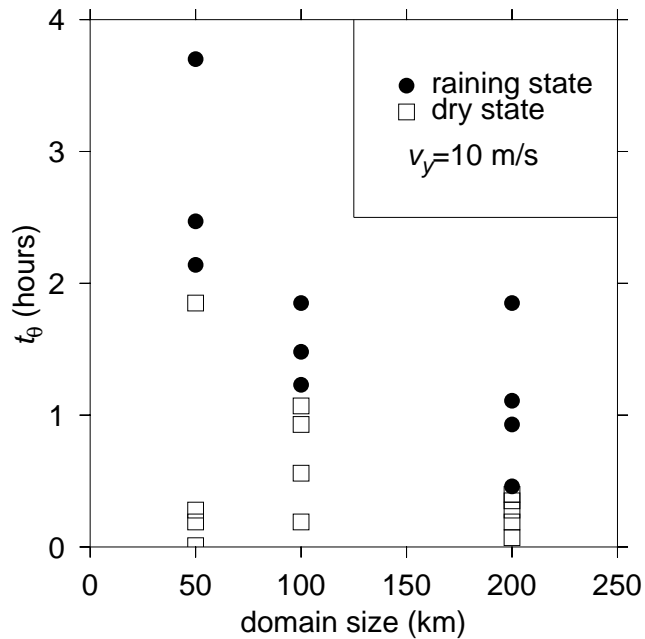


Figure 7. All symbols correspond to simulations initiated with a dry moisture profile and with fixed surface wind speeds (v_y) of 10 ms^{-1} . Solid circles represent experiments that eventually precipitated, open squares represent experiments which remained dry.

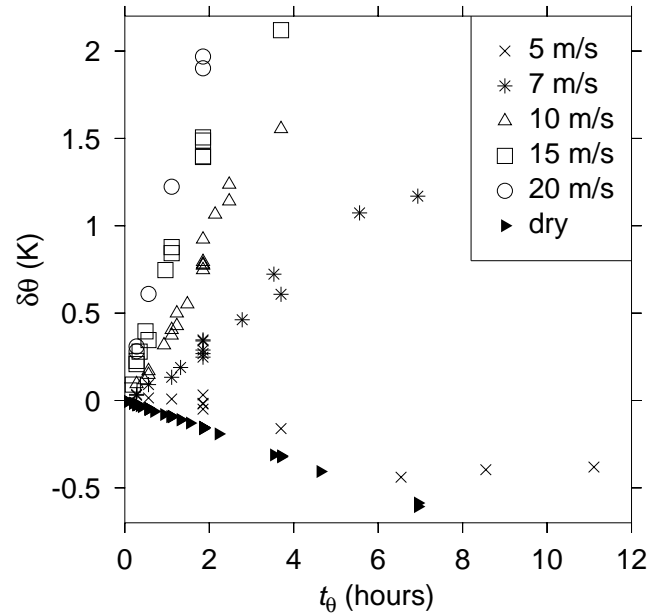


Figure 8. $\delta\theta$ is the time and domain averaged deviation of potential temperature from RCE averaged over 4-12 km in height. In this range, $\delta\theta$ is nearly constant. Black filled triangles represent experiments which resulted in the dry equilibrium, while all other symbols correspond to experiments which evolved to the precipitating equilibrium, independent of initial moisture. For the precipitating states, the different symbols indicate the magnitude of the surface winds. For a given wind speed, the relationship between $\delta\theta$ and t_θ is nearly linear. There is also a linear relation for the non-precipitating states.

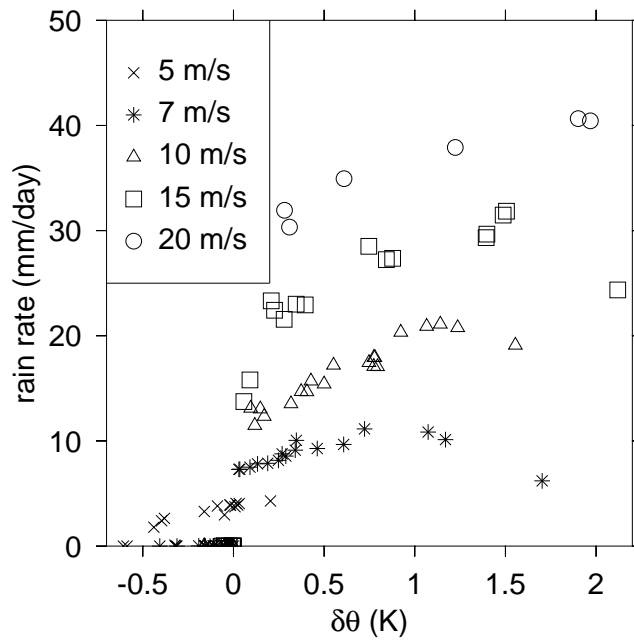


Figure 9. Precipitation rate as a function of $\delta\theta$, with the same key as in figure 8. Non-precipitating equilibrium states are obvious and thus have not been separately identified.

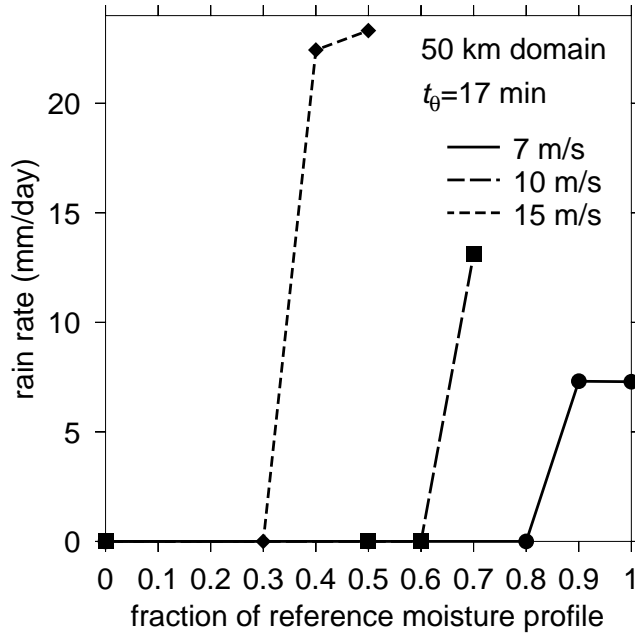


Figure 10. All symbols correspond to simulations on a 50 km domain with $t_{\theta} = 17$ min. Symbols and line styles represent the sustained surface wind speeds, and the location of the bullets indicate the fraction of the RCE mixing ratio profile which was used to initiate the simulation (i.e., f in equation (9)). Here we see that in some cases, the dry state is maintained even with considerable initial moistening of the free troposphere.

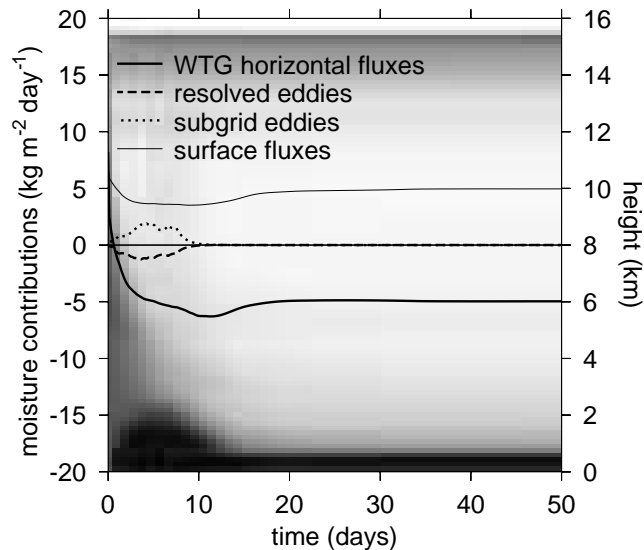


Figure 11. Image plot of the relative humidity for the 50 km domain initiated with 80% of the moisture of the surrounding environment (7 ms^{-1} surface wind, $t_{\theta} = 17 \text{ min}$). White indicates zero, while black indicates unity. The initial moisture in the free troposphere is removed by subsidence and implied lateral export, resulting in a dry steady state. Lines indicate contributions to the total mixing ratio tendency in the free troposphere by the parameterized lateral flux of moisture and eddies. The contribution by eddies is taken as the flux through a surface at 2000 m. The thin line shows the moisture tendency due to surface fluxes (which act to moisten the boundary layer). In the steady state, horizontal fluxes remove boundary layer moisture. Contributions to the mixing ratio have been low-pass filtered in time with a cutoff period of one day.

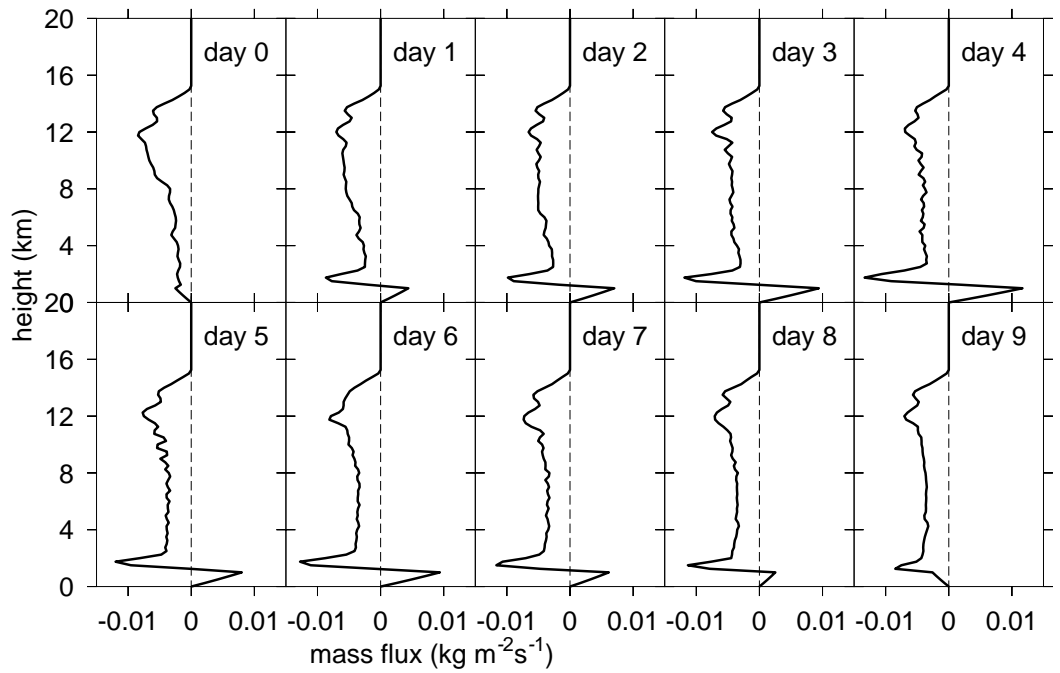


Figure 12. The vertical mass flux profile for the first 10 days of the simulation initiated with 80% of the RCE moisture profile (50 km domain, surface wind speed 7 ms^{-1} , and $t_{\theta}=17 \text{ min.}$). The mass flux profile changes very little in the time after day 9.

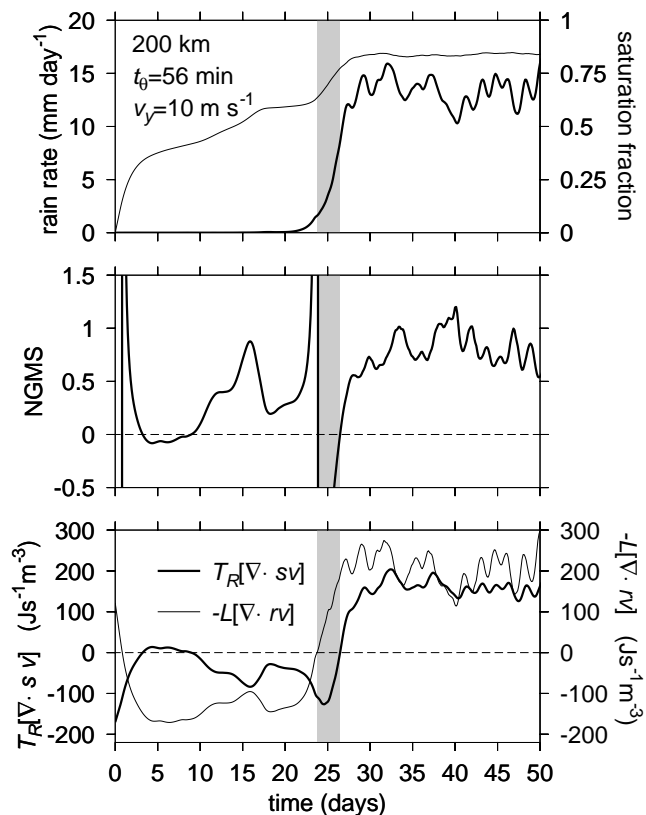


Figure 13. Top: Precipitation rate (thick) and saturation fraction (thin) as a function of time. Middle: NGMS. Bottom: Moist entropy divergence (thick) and moisture convergence (thin). Initiated with a dry atmosphere. The transition period with negative NGMS (days 24-26) is emphasized with grey shading. See text for discussion. Data shown has been low-pass filtered in time with a cutoff period of one day.

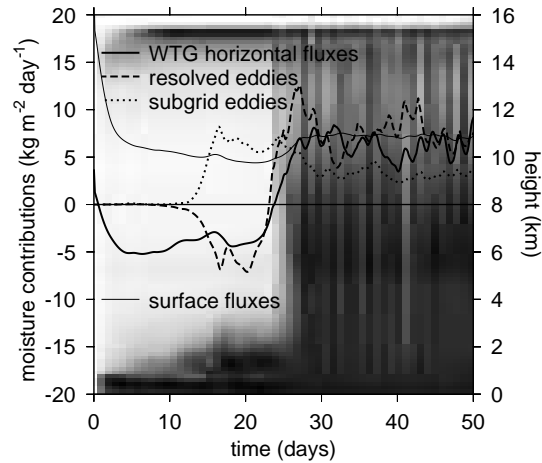


Figure 14. Domain averages of relative humidity (white indicates zero while black is 100%) and the contribution of mixing ratio to the free troposphere by eddies and the parameterized lateral flux of moisture. The eddy contributions are taken to be fluxes through a surface above the boundary layer (2000 m). The thin line is the mixing ratio tendency into the boundary layer by surface fluxes. The large initial tendency due to surface fluxes shows a rapid moistening of an initially dry boundary layer. Contributions to mixing ratio have been low-pass filtered in time with a cutoff period of one day.

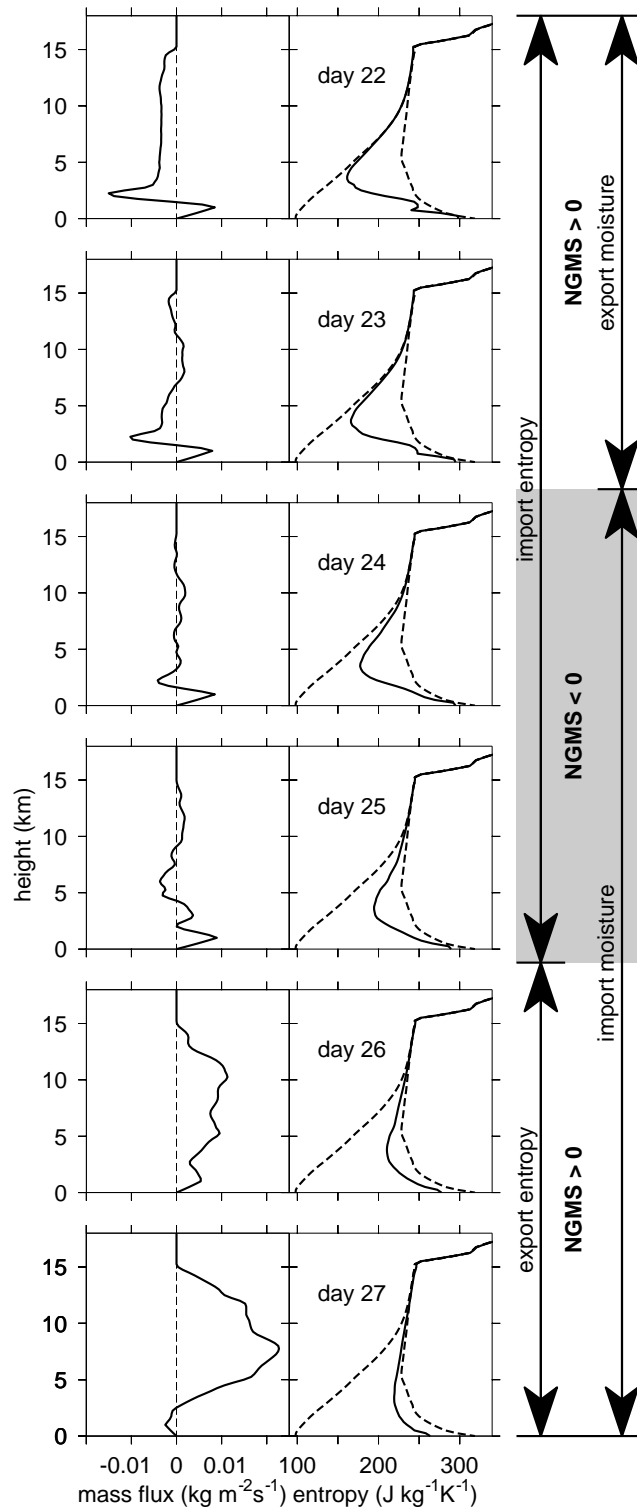


Figure 15. Vertical mass flux profiles (left) and moist entropy (right) for days 22-27 of the simulation shown in figure 13. Dotted lines in the entropy panel are the dry (left) and saturated (right) entropy profiles averaged over days 22-27. Days 24-25 represent time periods where the NGMS is negative. Note the rapid transition in moist entropy during this time.

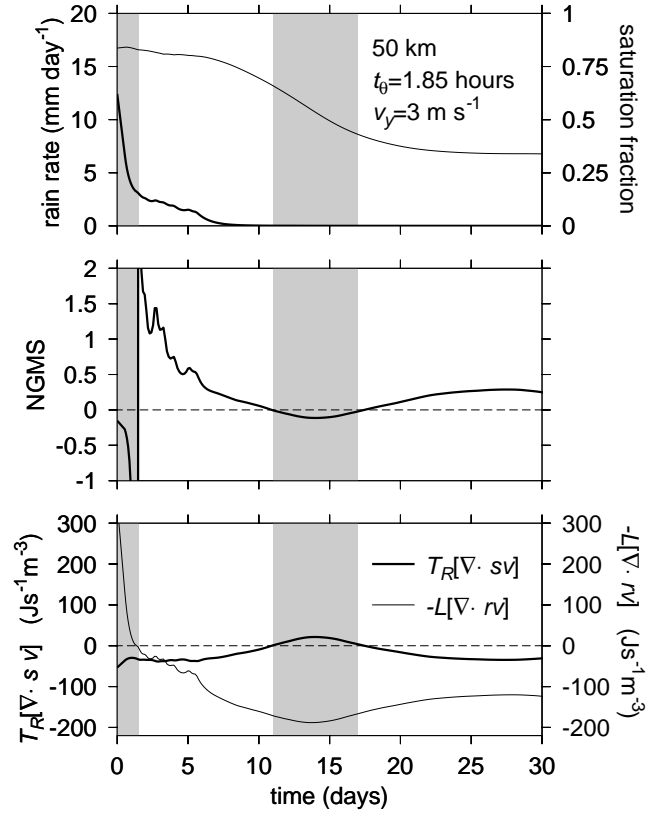


Figure 16. Top: Precipitation rate (thick) and saturation fraction (thin) as a function of time. Middle: NGMS. Bottom: Moist entropy divergence (thick) and moisture convergence (thin). Initiated with moisture profile of surrounding atmosphere. See text for discussion. Data shown has been low-pass filtered with a cutoff period of one day.

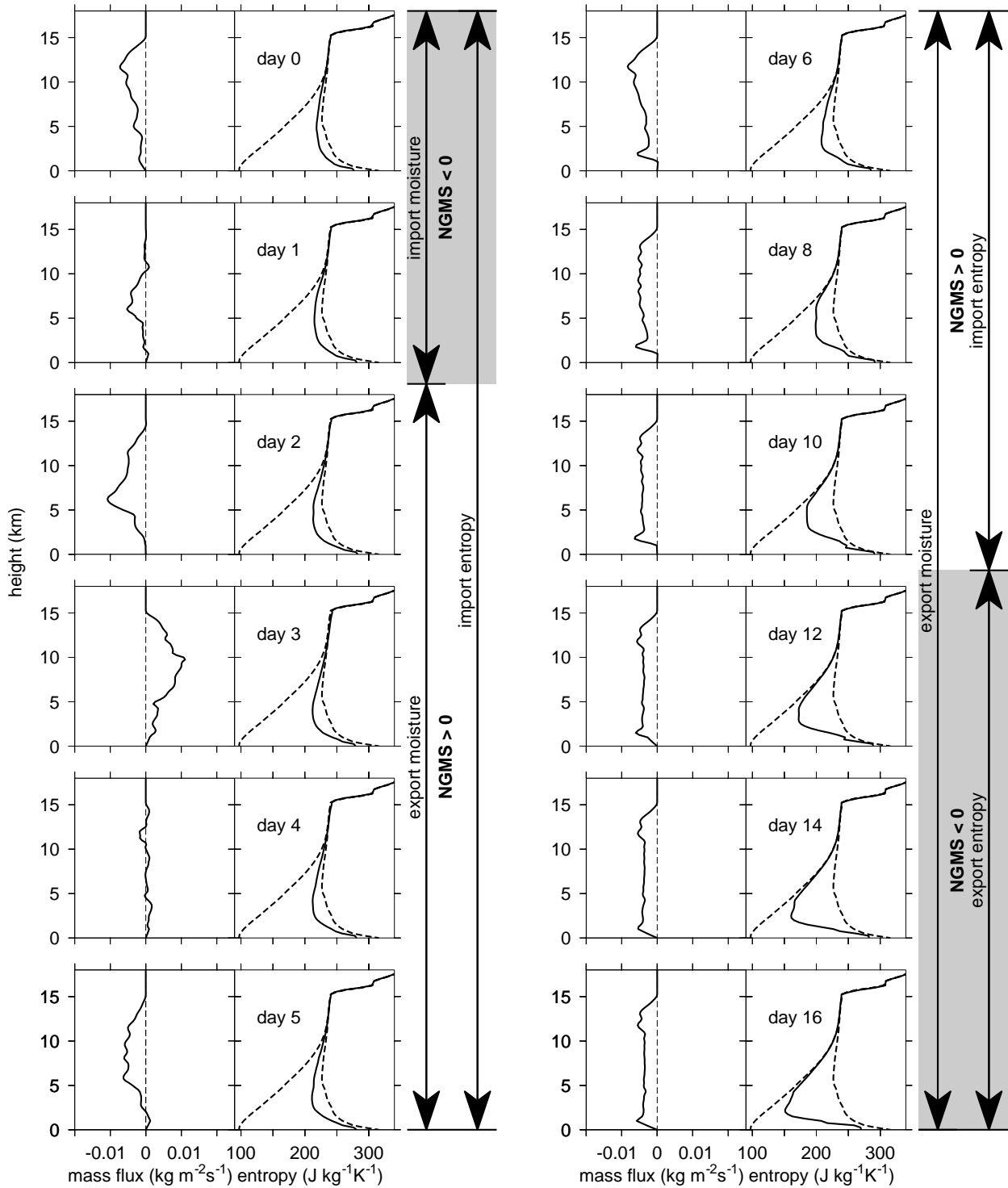


Figure 17. Vertical mass flux profiles (left) and moist entropy (right) for days 0-16 of the simulation shown in figure 16. Dotted lines in the entropy panel are the dry (left) and saturated (right) entropy profiles averaged over days 0-16. Grey shaded regions emphasize the time periods where the NGMS is negative.

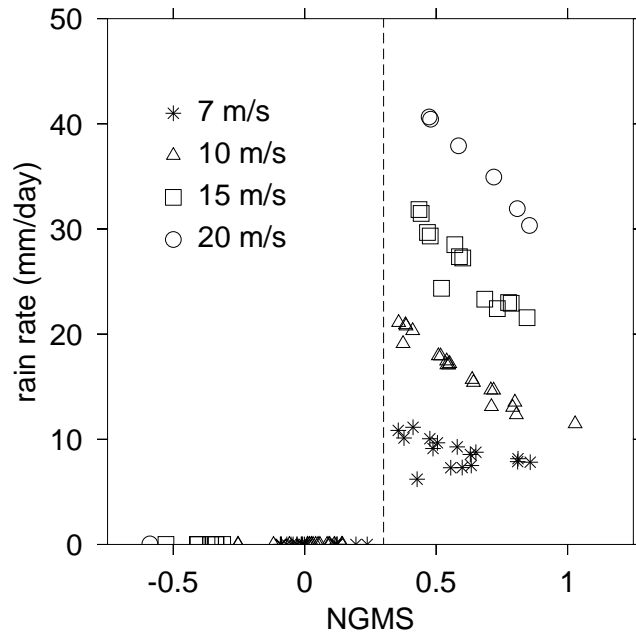


Figure 18. Symbols show the steady state rain rate and NGMS for numerical experiments with prescribed surface winds greater than 5 ms^{-1} . The different symbols represent the strength of the surface winds. A vertical line at $\Gamma = 0.3$ clearly separates the experiments with a dry equilibrium (zero precipitation) from the precipitating equilibrium (precipitation greater than zero).

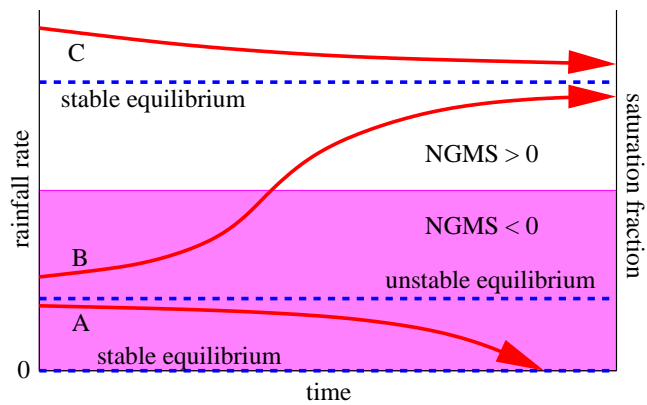


Figure 19. Two stable equilibria, one representing a precipitating state with $\text{NGMS} > 0$, and one with zero precipitation and $\text{NGMS} < 0$. An unstable equilibrium separates these two, and the state ultimately realized by the system will depend on the initial conditions. Previously published as figure 8 in *Raymond et al.* [2009].