

1 Convective response to changes in the
2 thermodynamic environment in idealized weak
3 temperature gradient simulations

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4 **Abstract.** We investigate the response of convection to idealized pertur-
5 bations in the thermodynamic environment in simulations which parame-
6 terize the large scale circulations using the weak temperature gradient (WTG)
7 approximation. The perturbations include a combination of modifying the
8 environmental moisture and atmospheric stability via imposing anomalies
9 in reference moisture and temperature profiles. We find that changes in at-
10 mospheric stability strongly influence the character of convection by dras-
11 tically modifying the vertical motion profile, whereas changes to atmospheric
12 moisture modulate the intensity of precipitation produced by the convection,
13 but do not qualitatively change the shape of the vertical motion profile.

14 An important question is how does horizontal moisture advection into the
15 domain affect convection? We test several different parameterizations of this
16 process; these include lateral entrainment by circulations induced by enforc-
17 ing WTG, a moisture relaxation which parameterizes the advection of mois-
18 ture by large scale non-divergent circulations, and control simulations in which
19 both of these mechanisms are turned off so horizontal advection is assumed
20 negligible compared to vertical advection. Interestingly, the most significant
21 differences resulting from the choice of horizontal moisture advection scheme
22 appear in environmental conditions which suppress—rather than support—the
23 development of deep tropical convection. In this case, lateral entrainment re-
24 lated to WTG circulations is the only parameterization which results in ex-
25 treme drying of the troposphere in environments which suppress convection.

²⁶ Consequently, this is the only parameterization which permits multiple equilibria–
²⁷ dry or precipitating steady states–in convection.

1. Introduction

28 Understanding the interaction between deep tropical convection and the large-scale en-
29 vironment benefits our knowledge of the tropical atmosphere and leads to improvements
30 in the convective parameterizations in numerical models used for weather forecasting and
31 climate prediction. This interaction is two-way: convection fuels waves that drive the
32 large-scale transport, while the large-scale circulation sets the environment for convec-
33 tion. In this work, we focus on the latter part of this interaction and investigate how the
34 characteristics of convection respond to changes in the large-scale thermodynamic envi-
35 ronment, where the large-scale environment is parameterized using the weak temperature
36 gradient approximation [*Sobel and Bretherton, 2000; Raymond and Zeng, 2005*].

37 The weak temperature gradient (WTG) approximation is based on the observation
38 that horizontal temperature gradients are small in the tropical atmosphere where gravity
39 waves act to balance convective heating and radiative cooling. Models employing the
40 WTG approximation achieve this balance by generating a domain-mean vertical velocity
41 that counteracts buoyancy anomalies produced by diabatic processes. This WTG vertical
42 velocity—and thus the modeled convection—is sensitive to changes in the reference profiles
43 of potential temperature and moisture which represent the thermodynamic environment
44 [*Mapes, 2004; Raymond and Sessions, 2007; Wang and Sobel, 2012; Emanuel et al., 2013;*
45 *Wang et al., 2013; Herman and Raymond, 2014*]. It is also sensitive to the model and the
46 specific implementation of WTG [*Daleu et al., 2012; Herman and Raymond, 2014*], as well
47 as to details of how horizontal moisture advection is parameterized [*Sobel and Bretherton,*
48 *2000; Sobel et al., 2007*].

49 The purpose of this investigation is twofold: 1) to diagnose the changes in convection
50 modeled in different thermodynamic environments using the WTG approximation, and
51 2) to determine how different choices for parameterizing horizontal moisture advection
52 affects the convection. We also consider how these influence the existence of multiple
53 equilibria in precipitation.

54 Several modeling studies have demonstrated the sensitivity of convection to the thermo-
55 dynamic environment—characterized here by atmospheric stability and humidity. *Mapes*
56 [2004] used a cloud resolving model to investigate the transient rainfall response to deep
57 vertical and vertical-dipole perturbations in potential temperature and water vapor mix-
58 ing ratio. While both of these perturbations—representing first and second baroclinic
59 mode vertical displacements, respectively—generated transient responses in rainfall, *Mapes*
60 [2004] found that the vertical-dipole perturbations enhanced the transient rainfall response
61 compared to deep vertical displacements. *Raymond and Sessions* [2007] and *Herman and*
62 *Raymond* [2014] showed that more stable environments produce more bottom-heavy con-
63 vection with increased precipitation rates, while more moist environments produce more
64 intense convection without changing the altitude of the maximum mass flux. An inter-
65 esting contrast is found in results of *Wang and Sobel* [2012], who showed that strong
66 lower tropospheric drying can reduce top-heaviness and ultimately prevent deep convec-
67 tion entirely, though this did not occur in a similar investigation when convection was
68 also parameterized [*Sobel and Bellon*, 2009].

69 The sensitivity of convection to the thermodynamic environment is not unique to WTG
70 simulations; alternate parameterizations of the large scale also produce responses broadly
71 consistent with WTG simulations. For example, *Kuang* [2010] computed linear response

72 functions based on the response of convection to temperature and moisture perturbations.
73 His results were corroborated in a parallel study by *Tulich and Mapes* [2010], who con-
74 sidered transient sensitivities of convection to sudden perturbations in temperature and
75 moisture.

76 Idealized studies which investigate how convection responds to prescribed changes in
77 the thermodynamic environment—and how the response depends on the implementation
78 of WTG—provide valuable insight for identifying mechanisms involved in convective pro-
79 cesses. These studies also provide a framework for interpreting WTG simulations which
80 incorporate observed anomalies in reference profiles of WTG simulations, such as those
81 used to study the Madden-Julian Oscillation [*Wang et al.*, 2013].

82 Previous studies have demonstrated the importance of vertical moisture advection on
83 the existence of convectively coupled waves [e.g., *Kuang*, 2008]. Another important aspect
84 of this work is to determine how the sensitivities of convection to the thermodynamic
85 environment depend on the method used to parameterize horizontal moisture advection.
86 This is potentially important for improving the representation of convection in global
87 models [*Derbyshire et al.*, 2004], as well as for improving the simulation of the Madden-
88 Julian Oscillation [*Pritchard and Bretherton*, 2014; *Zhu and Hendon*, 2015].

89 Another important application of WTG simulations is investigating whether a particu-
90 lar set of parameters support multiple equilibria in precipitation. Multiple equilibria refers
91 to the ability of a model to either sustain a dry or precipitating steady state under identi-
92 cal boundary conditions; the state realized by the model depends on the initial moisture
93 profile in the model [*Sobel et al.*, 2007; *Sessions et al.*, 2010; *Emanuel et al.*, 2013; *Herman*
94 *and Raymond*, 2014]. Previous studies indicate that the existence of multiple equilibria

95 depends on the degree to which WTG is enforced [Sessions et al., 2010], domain size
96 [Sessions et al., 2010], boundary layer depth [Herman and Raymond, 2014], how environ-
97 mental moisture is chosen to enter the domain [Sobel et al., 2007], and the background sea
98 surface temperature in which the multiple equilibria experiments are performed [Emanuel
99 et al., 2013], among other things.

100 Whether or not the thermodynamic environment or choice for horizontal moisture ad-
101 vection scheme affects the existence of multiple equilibria is important for understanding
102 the relevance of these choices in large-scale representations. For example, multiple equi-
103 libria in WTG domains is believed to be analogous to convecting and dry regions of
104 large domain radiative convective equilibrium simulations with self-aggregated convection
105 [Bretherton et al., 2005; Muller and Held, 2012; Wing and Emanuel, 2013; Emanuel et al.,
106 2013; Jeevanjee and Romps, 2013]. Wing and Emanuel [2013] and Emanuel et al. [2013]
107 demonstrated the importance of the feedback between radiative cooling and water va-
108 por in self-aggregation and multiple equilibria experiments, respectively; thus, identifying
109 parameters which influence water vapor content in these WTG experiments may help
110 identify mechanisms relevant for organizing convection.

111 This paper is organized as follows: We briefly introduce the weak temperature gradient
112 approximation and its implementation in our model in section 2. In section 3, we describe
113 the model and the series of numerical experiments used for this work. Diagnostic quantities
114 are defined in section 4, we present results in section 5, and we summarize and discuss
115 the consequences of our results in section 6.

2. Weak temperature gradient (WTG) approximation

116 The weak temperature gradient (WTG) approximation is a useful tool for investigating
 117 convection in limited domain simulations [*Sobel and Bretherton, 2000; Raymond and Zeng,*
 118 2005]. This work uses an implementation of WTG similar to that used by *Raymond and*
 119 *Zeng* [2005], but with some significant upgrades which primarily result in changes to
 120 the source terms in the equations governing the equivalent potential temperature, θ_e ,
 121 and the total water mixing ratio, r_t . For the purpose of this work, the most important
 122 changes are: different representations for parameterizing horizontal moisture advection
 123 from the environment into the model domain (“moisture treatment”); and performance
 124 improvements and bug fixes (described in the model documentation, not here). These
 125 changes are documented in *Herman and Raymond* [2014]; though we summarize those
 126 pertinent to this work here.

The thermodynamic equations for equivalent potential temperature, θ_e , and total water mixing ratio, r_t , are:

$$\frac{\partial \rho \theta_e}{\partial t} + \nabla \cdot (\rho \mathbf{v} \theta_e - K \nabla \theta_e) = \rho (S_{es} + S_{er} - S_e) \quad (1)$$

and

$$\frac{\partial \rho r_t}{\partial t} + \nabla \cdot (\rho \mathbf{v} r_t - K \nabla r_t) = \rho S_{cr} + \rho (S_{rs} - S_r) \quad (2)$$

Here, ρ is the density, \mathbf{v} is the velocity, and K is the eddy mixing coefficient. S_{es} is the source of equivalent potential temperature from surface fluxes; S_{er} is the source of θ_e from radiation. S_{rs} is the source of total cloud water from surface evaporation; S_{cr} is minus the conversion rate of cloud water to precipitation. S_e and S_r are sinks of equivalent potential temperature and total water mixing ratio due to external sources; these are a consequence

of enforcing the WTG approximation. The domain mean potential temperature, $\bar{\theta}$, is relaxed to a reference profile representing the large-scale, θ_0 . This relaxation is initiated by a potential temperature anomaly, $(\bar{\theta} - \theta_0)$, that accounts for radiative cooling and convective heating within the model domain. This modulates a potential temperature sink, S_θ :

$$S_\theta = \lambda_\theta M(z)(\bar{\theta} - \theta_0) \quad . \quad (3)$$

Here $1/\lambda_\theta$ is the time scale over which the domain mean potential temperature relaxes to the reference profile; physically it represents the time over which gravity waves would redistribute buoyancy anomalies. $M(z) = \sin(\pi z/H)$ is a masking function which modulates the relaxation. It is applied only to the vertical layer $b < z < H$, where b is the height of the boundary layer top and H is the tropopause height. Above H , M is set to zero. The temperature anomaly diagnosed in equation 3 then generates a vertical velocity that counteracts the heating via adiabatic cooling. This velocity is the weak temperature gradient vertical velocity, w_{wtg} , defined as:

$$w_{wtg} = \left(\frac{\partial \bar{\theta}}{\partial z} \right)^{-1} S_\theta \quad . \quad (4)$$

127 Strictly speaking, the WTG approximation is based on weak horizontal gradients in
 128 virtual temperature, not potential temperature. The model used in this study does not
 129 include the effects of water vapor on buoyancy. However, some simple tests (for example,
 130 defining w_{wtg} in terms of virtual potential temperature instead of potential temperature)
 131 indicate that this is a small error (well within model variability), and thus excluding
 132 buoyancy effects of moisture is unlikely to qualitatively affect the results of this work.

133 The parameterized vertical velocity in equation 4 vertically advects θ_e and moisture.
134 Since the WTG vertical velocity is assumed to satisfy the anelastic mass continuity equa-
135 tion, vertical motion can induce horizontal convergence of environmental air into the
136 model domain. This contributes to external sources, S_e and S_r in equations 1 and 2.
137 The specific form of these is given in section 3.3, where we discuss options for moisture
138 treatment. In the boundary layer, convective heating is shallow and the corresponding
139 gravity waves are slow [Bretherton and Smolarkiewicz, 1989]. Consequently, WTG is not
140 a good approximation for the boundary layer, so for $z < b$ the WTG vertical velocity is
141 linearly interpolated in height from its value at b to zero at the surface.

3. Numerical experiments

142 In this section, we describe the implementation of WTG in our model and the experi-
143 ments used in this investigation.

3.1. Model set-up

144 All numerical experiments in this study are conducted using two-dimensional geometry.
145 The horizontal dimension is 200 km with 1 km grid resolution; the vertical spans 20 km
146 with 250 m resolution. We choose to use two-dimensional domains for computational
147 efficiency; previous studies have shown that they give qualitatively similar results as their
148 three-dimensional counterparts [Wang and Sobel, 2011], and are therefore sufficient for
149 this study.

150 All simulations use a uniform SST of 303 K. The model is run in non-WTG mode until
151 the convective heating balances radiative cooling (radiative convective equilibrium, RCE).
152 The RCE profiles are calculated with interactive radiation using the toy radiation scheme

153 of *Raymond and Torres* [1998], and a mean surface wind speed of 5 m s^{-1} . The strength
154 of convection is modulated through surface fluxes which can be increased by increasing
155 sea surface temperatures (SSTs) or surface wind speeds. To investigate the characteristics
156 of convection in WTG mode, it is useful to increase the surface fluxes relative to the value
157 used in the RCE calculation so the model exhibits stronger convective heating compared
158 to radiative cooling. We choose to increase the surface wind speed to 7 m s^{-1} for most
159 simulations, up to 10 m s^{-1} for multiple equilibria experiments (see below).

160 Although the RCE simulations invoke interactive radiation, we choose to perform all
161 WTG simulations with non-interactive (static) radiative cooling. The radiative cooling
162 profile is taken as the time and domain mean of the RCE simulation, see figure 1. Static
163 radiative cooling in the WTG simulations allows us to isolate the effect of changes in the
164 thermodynamic environment and moisture treatment independent of the changes to the
165 cooling profile that would occur with radiative feedbacks. Using the RCE cooling profile—
166 rather than a cooling profile that is held constant with height in the troposphere—allows
167 the convection to respond to a cooling profile that is more representative of the model
168 environment.

169 Finally, we must specify the time scale over which the domain averaged potential tem-
170 perature is relaxed to the reference profile ($1/\lambda_\theta$ in equation 3). $\lambda_\theta \rightarrow \infty$ represents
171 a strict enforcement of WTG ($\bar{\theta} = \theta_0$), while $\lambda_\theta \rightarrow 0$ turns WTG off and allows the
172 model to approach RCE. We choose a relaxation time scale of approximately 11 minutes
173 ($\lambda_\theta = 1.5 \times 10^{-3} \text{ s}^{-1}$). This is a much shorter time scale than is typical of WTG exper-
174 iments, which range from 1-3 hours [*Sessions et al.*, 2010; *Wang and Sobel*, 2011, 2012;
175 *Daleu et al.*, 2012; *Anber et al.*, 2014; *Herman and Raymond*, 2014], though *Sobel et al.*

176 [2007] used strict enforcement of WTG (0 hours). *Sessions et al.* [2010] considered a range
 177 of time scales (0.01-3.5 hours) and found that shorter time scales permit a larger range of
 178 multiple equilibria; part of the reason we choose a time scale that is much shorter than
 179 typical values is because we are considering multiple equilibria in this work. Furthermore,
 180 a stricter enforcement of WTG forces convection to be more sensitive to the thermody-
 181 namic environment, and is thus conducive for investigating the response of convection to
 182 changes in the environment. We expect that a longer relaxation time would decrease the
 183 magnitude of the response, but would not qualitatively change the results. This is sup-
 184 ported by the WTG simulations of *Romps* [2012b], who found that different time scales (5
 185 and 30 minutes) affected the magnitude—but not the overall shape—of the WTG vertical
 186 velocity profile.

187 As we mentioned previously, the WTG relaxation time scale represents the time it takes
 188 gravity waves to neutralize buoyancy anomalies. This corresponds to a distance for a given
 189 gravity wave speed, however, it is not clear what the appropriate distance is. The scale
 190 of the convective disturbance [*Romps*, 2012a, b] and the spacing between disturbances
 191 [*Bretherton and Smolarkiewicz*, 1989; *Cohen and Craig*, 2004] are possible candidates.
 192 Gravity waves traveling at 50 m s^{-1} travel 33 km in 11 minutes. Depending on what the
 193 appropriate length scale is, the time scale used in this study may be physically reasonable.

3.2. Reference profiles

194 In the WTG approximation, we must specify reference profiles of potential temperature
 195 and total water mixing ratio representative of the convective environment (θ_0 and r_{t0} in
 196 equations 3 and 6). The reference profiles are generated by running the model to RCE in
 197 non-WTG mode (i.e. $\lambda_\theta = 0$ in equation 3; and $\lambda_{adv} = \lambda_m = 0$ in equation 6). Time and

198 domain averages of potential temperature and total water mixing ratio give the reference
 199 profiles $\theta_0(z)$ and $r_0(z)$, shown in figure 2 for RCE simulations. The time average is taken
 200 over the last 30 days of a 1 year simulation.

201 In order to investigate the response of convection to changes in the reference environ-
 202 ment, we perform numerical experiments similar to *Raymond and Sessions* [2007]. *Ray-*
 203 *mond and Sessions* [2007] showed that either moistening or stabilizing the environment
 204 resulted in increased precipitation rates for given surface fluxes; increasing the reference
 205 moisture increased the magnitude of the vertical mass flux without changing the shape,
 206 while increasing the stability both increased the magnitude of the vertical mass flux and
 207 lowered the level of maximum mass flux, resulting in more “bottom-heavy” convection.
 208 As a consequence, this concentrates the convergence to low levels where the air is more
 209 moist, resulting in a higher precipitation efficiency.

Raymond and Sessions [2007] represented changes to the reference environment by
 adding idealized perturbations to either the potential temperature or the mixing ratio
 reference profiles. An increase in the atmospheric stability was produced by specifying a
 cooling of $\delta\theta = 2$ K centered at $h = 3$ km and a warming of the same magnitude centered
 at $h = 10$ km. The form of the perturbation centered at level h is given by:

$$\Delta\theta = \delta\theta \left(\frac{z}{h}\right)^2 e^{[2(1-z/h)]} \quad , \quad (5)$$

210 where z is the altitude. In addition to a more stable environment, we also explore the
 211 impact of a less stable environment with perturbations of the same magnitude but with
 212 opposite signs (warming of 2 K at 3 km with cooling of 2 K at 10 km).

213 Moistening or drying is achieved by modifying the reference mixing ratio profile with a
 214 perturbation similar to equation 5, but with $\delta\theta$ replaced by δr , where $\delta r = \pm 1.0$ g kg⁻¹

215 and $h = 3$ km. This choice is consistent with the moisture perturbations of *Raymond and*
216 *Sessions* [2007], and is similar to the lower tropospheric drying level used in *Wang and*
217 *Sobel* [2012]. In order to explore the full range of possible environments, we perform sets
218 of nine experiments which account for all combinations of perturbations to the reference
219 potential temperature and moisture profiles. These combinations are shown in figure 3.
220 The symbols in the upper right corners of each panel represent the modifications to the
221 reference profiles. Environmental stability is represented by the geometric stability of the
222 symbols:

- 223 1. the completely unperturbed RCE profiles (control, center panel) are represented by
224 a bulls-eye;
- 225 2. more stable environments (top row) are represented by upright triangles (geometri-
226 cally more stable shapes);
- 227 3. less stable environments (bottom row) are represented by inverted triangles (geo-
228 metrically unstable shapes);
- 229 4. an atmosphere with the stability of the RCE profile (middle row) is represented by
230 squares (neutrally stable shapes).

231 The symbol shading indicates a moistening or drying of the reference environment. In
232 analogy with a glass of water,

- 233 1. empty is drier;
- 234 2. half-filled is unperturbed;
- 235 3. filled is moister.

236 These symbols serve as a legend for results presented in section 5.

237 Rather than doing individual experiments for each combination shown in figure 3, ex-
238 periments are run for 90 days with perturbations imposed in 30 day increments. For each
239 combination of perturbations, two sets of 90 day experiments are run; the first month is
240 unperturbed; the second month has *either* potential temperature or moisture perturbed;
241 the third month has *both* profiles perturbed. A set of eight experiments—graphically de-
242 picted in figure 4—is required to represent all combinations of reference environments shown
243 in figure 3.

244 The time-dependence in the experimental design has several advantages compared to
245 individual experiments for each combination of perturbations:

246 1. It provides a minimum of two simulations with identical boundary conditions to
247 confirm the uniqueness of the state for the given conditions (each combination of pertur-
248 bations represented in figure 3 is repeated at least twice; the unperturbed reference state
249 is repeated 8 times).

250 2. It confirms that the state in month 3 is unique as it is reached from two distinct
251 steady states in the previous month;

252 3. It gives a sense of variability when conditions are the same;

253 4. It gives temporal information for studying the transition itself as the conditions
254 change (though this is not explicitly studied in this paper).

255 We choose 30-day increments to give enough time for the system to re-equilibrate after
256 the perturbation occurs, and enough simulation time to generate mean-state statistics.
257 Statistics are taken from domain mean time averages over the last two weeks of each 30
258 day run (minus one hour to avoid the ambiguous data at the transition). See figure 6

259 in section 5 for sample data showing precipitation rate as a function of time for the 8
 260 experiments depicted in figure 4.

3.3. Moisture treatment

The prognostic equation for total water mixing ratio (equation 2) includes an external sink, S_r , which is a consequence of enforcing WTG. This external sink is given by

$$S_r = w_{wtg} \frac{\partial \bar{r}_t}{\partial z} + \lambda_{hadv} (\bar{r}_t - r_x) \frac{1}{\rho_0} \frac{\partial \rho_0 w_{wtg}}{\partial z} + \lambda_m (\bar{r}_t - r_{t0}) \quad , \quad (6)$$

where

$$r_x = \begin{cases} \bar{r}_t & \text{if } \partial \rho_0 w_{wtg} / \partial z < 0 \quad (\text{detraining levels}) \\ r_{t0} & \text{if } \partial \rho_0 w_{wtg} / \partial z > 0 \quad (\text{entraining levels}) \end{cases} \quad . \quad (7)$$

261 The three terms on the right hand side of equation 6 represent sinks of moisture due
 262 to large scale vertical advection by the mean vertical velocity w_{wtg} , explicit lateral en-
 263 trainment from the surrounding environment, and an imposed relaxation to the reference
 264 profile, r_{t0} which is independent of the WTG velocity.

265 As long as the model is operating in WTG mode and w_{wtg} is non-zero, moisture will
 266 be vertically advected within the domain (first term, equation 6). Horizontal advection
 267 of moisture occurs either by lateral entrainment due to divergent circulations generated
 268 by enforcing mass continuity in the WTG velocity field (second term, equation 6), or
 269 from large-scale flow that deposits dry or moist air into the domain independent of WTG
 270 circulations. The latter is parameterized by relaxing the domain mean moisture profile to
 271 the reference profile, r_{t0} (third term, equation 6). Note that this relaxation occurs through
 272 the entire depth of the troposphere, not just at entraining levels. Figure 5 illustrates the
 273 difference between these processes.

274 The choice of horizontal moisture advection scheme is set by the values of λ_{hadv} and λ_m ,
275 which are specified externally. λ_{hadv} has values of either 0 or 1, to turn lateral entrainment
276 off or on. Setting this to zero assumes the change in domain moisture via horizontal
277 advection is small compared to that due to the vertical advection; a value of 1 laterally
278 entrains moisture from the reference environment according to mass continuity of the
279 WTG velocity field. $\lambda_m = 0$ assumes horizontal moisture advection is purely divergent; a
280 non-zero value relaxes the domain moisture to the reference profile over a timescale $1/\lambda_m$.
281 Both of these choices have been employed in WTG experiments. *Raymond and Zeng*
282 [2005]; *Raymond and Sessions* [2007]; *Sessions et al.* [2010]; *Wang et al.* [2013]; *Herman*
283 *and Raymond* [2014] have all implemented explicit lateral entrainment of environmental
284 moisture. Other investigations which explicitly aimed to determine the effect of moisture
285 (including drying) on convection have relaxed moisture to a specified profile [*Sobel et al.*,
286 2007; *Sobel and Bellon*, 2009; *Wang and Sobel*, 2012]. It is worth noting that *Sobel and*
287 *Bretherton* [2000] investigated the effect of horizontal moisture advection by horizontal
288 winds that were independent of WTG circulations; moisture relaxation parameterizes this
289 mechanism.

290 Since the divergent and rotational flow are decoupled, both effects may influence convec-
291 tion and we either choose one mechanism to represent the horizontal moisture advection,
292 or we can simultaneously allow both to be turned on ($\lambda_{hadv} = 1$, $\lambda_m \neq 0$) since both of
293 these mechanisms may be at work in the real environment. In principle, the source due
294 to large-scale motions associated with the direct relaxation may have a unique reference
295 profile that represents the moisture in an environment upstream from the convecting do-
296 main. Since we do not have a reference profile to represent the upstream moisture, we

297 simply assume that the reference profile represents the moisture immediately available to
298 the convective domain, and we use this for both lateral entrainment and moisture relax-
299 ation. Using this configuration, lateral entrainment and moisture relaxation will usually
300 act in concert to either increase or decrease domain moisture, but in some conditions,
301 these mechanisms may compete and result in opposite tendencies (see section 5). In ei-
302 ther case, when the WTG vertical velocity is zero or else implies divergence via equation
303 7, the entrainment is shut off.

304 Alternatively, if we assume the horizontal contributions are small compared to the
305 vertical advection of moisture, we can shut off both moisture schemes ($\lambda_{adv} = 0$, $\lambda_m = 0$).
306 This is equivalent to an implicit horizontal moisture advection where moisture is advected
307 into the domain via circulations that obey mass continuity, but they advect moisture
308 from an environment that has a moisture profile identical to that in the model domain.
309 The moisture profile of the domain is a result of a combination of surface evaporation,
310 vertical advection by the WTG vertical velocity, and evaporation of precipitation, so in
311 this case, the environmental moisture is determined by the modeled convection, and it is
312 independent of an externally specified reference moisture profile. This has been a popular
313 choice in previous studies [e.g., *Sobel and Bretherton, 2000; Sobel et al., 2007; Wang and*
314 *Sobel, 2011; Wang et al., 2013; Anber et al., 2014*]. Because this is the only moisture
315 treatment which does not depend on a reference moisture profile, we refer to this as the
316 control method.

317 For the simulations which include moisture relaxation, we choose a relaxation time scale
318 of 1.8 days. To establish the moisture relaxation time scale, we conducted experiments
319 over a range of moisture relaxation time scales and compared the modeled precipitation

320 rate to the values produced using lateral moisture entrainment. Unperturbed environ-
 321 ments were not sensitive to the relaxation time chosen, but smaller relaxation times gave
 322 higher precipitation rates for more moist or more stable environments. $1/\lambda_m = 1.8$ days
 323 represents the relaxation time that gives precipitation rates closest to those produced us-
 324 ing lateral entrainment. It is important to note that strictly enforcing the moisture profile
 325 ($1/\lambda_m = 0$) shuts off the precipitation entirely because the reference profile is unsaturated
 326 and thus cannot trigger precipitation production in our model.

327 *Wang and Sobel* [2012] performed a set of experiments that are similar to a subset
 328 of the experiments presented here. In that work, the authors simulated the response of
 329 convection to a layer of drying in the upper, middle, and lower troposphere. The drying
 330 represented horizontal advection of dry air, and the layer was relaxed to a water vapor
 331 mixing ratio of zero over a specified time scale. For drying perturbations applied to the
 332 lower troposphere—at a level comparable to that used in this work—the moisture relaxation
 333 time scale varied from 2.9 to 100 days, and they noted that time scales below this range
 334 resulted in negative moisture values. The moisture relaxation time scale used in this work
 335 is shorter—1.8 days—but we are imposing a much weaker drying (or moistening) than in
 336 *Wang and Sobel* [2012], and are thus far from this numerical limitation.

Since our prognostic variable is θ_e rather than θ , our choices of moisture treatment also
 affect the sink of θ_e (and consequently moist entropy, see discussion after equation 10):

$$S_e = w_{wtg} \frac{\partial \bar{\theta}_e}{\partial z} + \lambda_{adv} (\bar{\theta}_e - \theta_x) \frac{1}{\rho_0} \frac{\partial \rho_0 w_{wtg}}{\partial z} + \lambda_m (\bar{\theta}_e - \theta_{e0}) \quad , \quad (8)$$

337 where the overbar indicates the domain mean, θ_x is analogous to r_x , and θ_{e0} is the reference
 338 profile of equivalent potential temperature. Both θ_0 and r_{t0} (and thus θ_{e0}) can be functions
 339 of time to permit study of convection in time-dependent situations [e.g., *Wang et al.*, 2013].

3.4. Multiple equilibria

Multiple equilibria—steady states with either persistent precipitating deep convection or a completely dry troposphere—exist in WTG simulations with identical boundary conditions [Sobel *et al.*, 2007; Sessions *et al.*, 2010; Emanuel *et al.*, 2013; Herman and Raymond, 2014]. If a set of parameters supports both equilibria, then whether the state is precipitating or dry depends on the initial moisture of the troposphere. We perform several multiple equilibria experiments to determine how the existence of multiple equilibria depends on the thermodynamic environment ($\theta_0(z)$, equation 3; $r_{t0}(z)$, equations 6 and 7) and choice of moisture treatment.

As reported in Sessions *et al.* [2010], the model used here supports multiple equilibria for a range of wind speeds in conditions similar to those used in this work. Sessions *et al.* [2010] used unperturbed RCE reference profiles with laterally entrained moisture. A potentially significant difference, however, is that interactive radiation was used in the previous work; here, radiative cooling is static. Similar multiple equilibrium experiments were performed by Herman and Raymond [2014] on an updated version of this model with a modified WTG approach which spectrally decomposed the heating to accommodate gravity wave speeds representing a set of vertical modes. In a comparison of the “spectral WTG” approach with the conventional WTG (used in this study), multiple equilibria was found to exist only when conventional WTG was applied, and in that case, the range of multiple equilibria was sensitive to the choice of boundary layer height. We do not yet understand why conventional and spectral WTG give different results for multiple equilibria, though it may be related to the treatment of the boundary layer: in spectral WTG, convection confined to the boundary layer is shallow and thus has slow

362 adjustment times [Bretherton and Smolarkiewicz, 1989]; in conventional WTG, this effect
363 is artificially imposed via a linear interpolation of the WTG vertical velocity from the top
364 of the boundary layer to zero at the surface (see discussion after equation 4).

365 In another study, Anber *et al.* [submitted] compared the existence of multiple equilibria
366 in WTG to an alternate parameterization of the large-scale [damped gravity wave, DGW;
367 see Kuang, 2008; Blossey *et al.*, 2009, for a description]. In the parameter space they
368 investigated, the WTG simulations exhibited multiple equilibria while DGW ones did not.
369 Like spectral WTG, DGW does not require special treatment in the boundary layer, and
370 the authors speculated that the existence of multiple equilibria may be an artifact of the
371 boundary layer treatment when static radiation is used. Interactive radiation produced
372 robust multiple equilibria in Sessions *et al.* [2010], so the role of radiation and boundary
373 layer treatment is not entirely clear, and is left for future work.

374 The existence of multiple equilibria is sensitive to the method of parameterizing hori-
375 zontal moisture advection. Sobel *et al.* [2007] demonstrated that multiple equilibria exist
376 over a larger range of SSTs if horizontal moisture advection is not explicitly represented
377 (equivalent to the control moisture treatment in this work) compared to when it is pa-
378 rameterized by moisture relaxation (see their figure 2).

379 The first task is to determine whether multiple equilibria exist for an unperturbed en-
380 vironment using different moisture treatments. To test this, we simply run an experiment
381 with a surface wind speed of 7 m s^{-1} and with zero initial moisture in the domain. We
382 do this for each moisture treatment to determine how different parameterizations of hori-
383 zontal moisture advection affect the existence of multiple equilibria. As in Sessions *et al.*
384 [2010], for all experiments which exhibit multiple equilibria, we repeat with a surface

385 wind speed of 10 m s^{-1} to determine the range over which multiple equilibria exist. We
 386 also repeat with a more stable and more moist environment to determine the role of the
 387 thermodynamic environment on multiple equilibria.

4. Diagnostics: characterizing convection and its environment

388 One of the most important measures of the strength of convection is the intensity of
 389 the precipitation it produces. The precipitation rate in itself—especially when averaged
 390 over space and time—is not enough to characterize the convection since different vertical
 391 and horizontal arrangements can produce the same mean precipitation rate. In order to
 392 better diagnose the convection, we compare the precipitation rates with several diagnostic
 393 quantities that we describe below.

The environmental stability is characterized by an instability index, Δs^* [*Raymond et al.*, 2011; *Gjorgjievska and Raymond*, 2014], which is defined as

$$\Delta s^* = s_{low}^* - s_{high}^* \quad , \quad (9)$$

394 where s^* is the saturated moist entropy, s_{low}^* is the mean saturated moist entropy in the
 395 level between 1 and 3 km, and s_{high}^* is the mean saturated moist entropy in the level
 396 between 5 and 7 km. Since s^* is a function of temperature and pressure only, this charac-
 397 terizes the stability of the environment: smaller values of the instability index correspond
 398 to more stable environments; larger values characterize more unstable environments.

The moisture content of the domain is characterized by the saturation fraction, which we approximate by

$$S = \frac{\int \rho(s - s_d) dz}{\int \rho(s^* - s_d) dz} \quad , \quad (10)$$

399 where the integrals are taken over the entire vertical depth of the model, $s_d = c_p \ln(\theta/T_R)$
 400 is the dry entropy ($c_p = 1005 \text{ J kg}^{-1}\text{K}^{-1}$ is the specific heat at constant pressure, and
 401 $T_R = 300 \text{ K}$ is a constant reference temperature), and s is the moist entropy (with θ
 402 replaced by θ_e in the dry entropy definition).

We define deep convective inhibition (DCIN) as a measure of how conducive or hostile the environment is to convection. As in *Raymond et al.* [2003],

$$DCIN = s_t^* - s_b \quad , \quad (11)$$

403 where the threshold entropy for convection, s_t^* , is the average saturated moist entropy
 404 over the layer between 1750-2000 m, and s_b is the boundary layer moist entropy, averaged
 405 over the lowest 1.75 kilometers of the domain. Smaller or negative values of DCIN are
 406 conducive to developing deep convection; larger values inhibit it.

The normalized gross moist stability (NGMS) provides a measure of the response of convection to its environment [*Neelin and Held*, 1987]. It is typically defined as the export of some quantity that is approximately conserved in moist processes (usually moist static energy or moist entropy) divided by some measure of the strength of the convection. As in *Raymond et al.* [2007], we choose NGMS (Γ) to be the ratio of moist entropy import to moisture export:

$$\Gamma = \frac{T_R[\nabla_h \cdot (\rho s \mathbf{v})]}{-L[\nabla_h \cdot (\rho r_t \mathbf{v})]} = \frac{T_R \int \nabla_h \cdot (\rho s \mathbf{v}) dz}{-L \int \nabla_h \cdot (\rho r_t \mathbf{v}) dz} \quad . \quad (12)$$

407 The square brackets signify a vertical integral over the troposphere and ∇_h is the horizontal
 408 divergence operator. The reference temperature, T_R , and latent heats of condensation plus
 409 freezing, L ($L = 2.833 \times 10^6 \text{ J kg}^{-1}$), are included to make Γ dimensionless. Differing
 410 environmental profiles can significantly affect the value of Γ . Stabilizing or destabilizing

411 the reference potential temperature will change the vertical profile of moist entropy which
 412 adjusts the lateral export of that quantity from the domain (numerator in equation 12);
 413 drying or moistening the environment clearly affects the import of moisture into the
 414 domain (denominator in equation 12), but it can also affect the amount of moist entropy
 415 exported or imported at given levels.

As in *Raymond et al.* [2007], we can relate the NGMS to the net precipitation rate in the steady state. To do this, we consider the vertically integrated conservation equation for specific moist entropy:

$$\frac{\partial[\rho s]}{\partial t} + [\nabla \cdot (\rho \mathbf{v} s)] = F_s - R \quad , \quad (13)$$

which is the moist entropy analog of the vertical integral of equation 1. In this case, the advection term includes advection by the grid-scale and by the parameterized large-scale WTG velocity fields. F_s is the surface moist entropy flux due to surface heat and moisture fluxes, and R is the vertically integrated entropy sink per unit mass due to radiation. Similarly, vertically integrating equation 2 gives:

$$\frac{\partial[\rho r_t]}{\partial t} + [\nabla \cdot (\rho \mathbf{v} r_t)] = E - P \quad , \quad (14)$$

where E is evaporation and P is precipitation, and the contribution by WTG circulations is included in the advection term on the left hand side. In this work, we exclusively study the statistically steady state; setting the time derivatives in equations 13 and 14 to zero and substituting into equation 12 gives a relationship between NGMS and the net precipitation [precipitation, P , minus evaporation, E ; *Raymond et al.*, 2007]:

$$\Gamma = \frac{T_R(F_S - R)}{L(P - E)} \quad . \quad (15)$$

416 Most of the experiments described hold the net entropy forcing constant: surface fluxes
 417 are fixed (F_S is constant) and a static radiative cooling profile implies R is constant.
 418 Thus, we expect $P - E \propto 1/\Gamma$, where the NGMS adjusts to account for the details of the
 419 thermodynamic environment.

We can infer much about the convective environment—as well as understand the relationship between our diagnostic quantities—by examining vertical profiles of mass flux. The vertical mass flux is calculated as the product of the density and the total vertical velocity. The total vertical velocity is the sum of the explicit velocity calculated by the model and the velocity field produced by enforcing WTG. Without WTG, mass conservation requires that the domain mean vertical velocity be zero (what comes up must go down), so the only domain mean vertical motion is that parameterized by the WTG approximation. Thus,

$$\text{mass flux} = \rho w_{wtg} \quad . \quad (16)$$

5. Results

420 In this section, we show the time evolution of precipitation and the diagnostic quantities
 421 defined in section 4 to demonstrate the effect of changes in the thermodynamic environ-
 422 ment. We also present vertical profiles of potential temperature and moisture anomalies
 423 to compare with the imposed anomalies. Vertical profiles of mass flux demonstrate how
 424 convection develops as a function of changes in the thermodynamic environment and pa-
 425 rameterization of horizontal moisture advection. Finally, we compare steady state values
 426 of precipitation and the diagnostic quantities defined in section 4 in a set of scatter-plots

427 to characterize the response of convection to changes in the large-scale thermodynamic
428 environment.

5.1. Response to changes in the thermodynamic environment for laterally entrained moisture

429 In this section, we analyze the time dependence of the diagnostic quantities defined in
430 section 4. We note that all figures showing time series data have been low-pass filtered in
431 time with a cutoff period of 1 day.

432 Figure 6 shows a time series of the precipitation rate for the experiments outlined in
433 figure 4. For convenience, we only show the time series for moisture that is parameter-
434 ized by lateral entrainment ($\lambda_{adv} = 1$, $\lambda_m = 0$ in equation 6); similar results hold for
435 other moisture advection choices, but are not shown. All simulations use unperturbed
436 RCE profiles during the first month; each of the four panels represents the four possible
437 combinations of reference profiles when both θ and r_t are perturbed; the second month
438 in each case represents either a drying/moistening OR a stabilizing/destabilizing. Each
439 case is marked with the symbol given in figure 3. Each distinct combination of reference
440 profiles (each panel in figure 3) is repeated at least twice (see figure 4). Statistics for
441 similar conditions are comparable, indicating statistically identical steady states.

442 For the perturbation magnitudes used in this study, atmospheric stability predominately
443 affects the character of convection compared to atmospheric moisture. Specifically:

- 444 1. The increase in stability-cooling of 2 K at low levels and warming of 2 K aloft-
445 produces a larger increase in precipitation rate (21 mm day^{-1}) than a 1 g kg^{-1} increase
446 in atmospheric moisture of (12 mm day^{-1}); see the second month in figure 6A.

447 2. Decreasing the moisture by 1 g kg^{-1} at 3 km reduces—but does not shut off—the
448 precipitation; whereas destabilizing the environment completely shuts off the convection,
449 even in a moister environment (compare empty squares in figure 6B,D with inverted
450 triangles in figure 6C,D).

451 3. A drier, more stable environment increases the precipitation rate compared to the
452 unperturbed RCE profile, whereas the moister, less stable environment is completely
453 devoid of precipitation (compare the third month in figures 6B and 6C).

454 These observations are specific to the magnitudes of perturbations applied to the ref-
455 erence profiles, though different choices would likely give qualitatively similar results. It
456 would be interesting to investigate how different magnitudes of drying and moistening or
457 stabilizing and destabilizing would affect the precipitation rate. *Wang and Sobel* [2012]
458 performed a series of WTG experiments to see the effect that drying a layer would have on
459 convection. In the lowest drying layer—comparable to the level that moisture perturbations
460 are applied to in these experiments—relaxing the moisture to 0% relative humidity still
461 produced convection with non-zero precipitation (though the convection became strictly
462 shallow). Thus, we do not expect moisture perturbations to have as dramatic effects as
463 perturbations in potential temperature.

464 In addition to precipitation rate, we consider several other diagnostic variables for char-
465 acterizing convection and its environment. To develop some intuition about how these
466 diagnostics behave for different convective environments, figure 7 shows time series of
467 precipitation rate, saturation fraction, instability index, NGMS, and DCIN (these are all
468 defined in section 4). The left column shows the results for the experiments which became
469 more stable and more moist (experiments 1 and 2 in figure 4, figure 6A); while those on

470 the right column evolve to less stable and drier states (experiments 7 and 8 in figure
471 4; figure 6D). The vertical axes were chosen to be the same for both columns for easy
472 comparison. As in figure 6, horizontal moisture advection is parameterized using lateral
473 entrainment.

474 From figure 7, we note several features of the diagnostic quantities. First, saturation
475 fraction seems to adjust relatively quickly to changes in moisture and to an increase in
476 stability, but it takes the domain a long time to adjust to a decrease in stability (figure 7D).
477 The slow adjustment is primarily due to the relatively slow radiatively-driven subsidence
478 rate, which determines the steady-state in absence of active convection. This only happens
479 when horizontal moisture advection is parameterized using lateral entrainment for reasons
480 described later in this section. Because we calculate mean quantities from the last two
481 weeks of each month long segment, the long adjustment time for saturation fraction does
482 not give the actual equilibrium value for the statistics calculated in this work. However, the
483 error in the mean is much smaller than the difference between saturation fraction values
484 for precipitating and non-precipitating states, so we simply make note of the difference
485 and interpret the diagnostics accordingly.

486 The instability index—shown in figure 7E,F—is calculated from the saturated entropy.
487 Since this is constrained by the enforcement of WTG, it quantifies “more stable” (small
488 values) and “less stable” (large values) environments. It adjusts quickly to changes in θ
489 profiles, but is not sensitive to changes in the reference moisture profile.

490 Depending on the atmospheric conditions, NGMS can be a highly variable quantity
491 (figure 7G,H). It is defined as the ratio of lateral moist entropy export to lateral mois-
492 ture import (equation 12). As convection evolves in the domain, these quantities can

493 alternate between import and export. This is especially true if conditions are close to
494 RCE: since the system is nearly in balance, there should be no net lateral import and
495 export from the domain and these quantities alternate across the zero value. This results
496 in large fluctuations, and in these conditions, NGMS is not a good diagnostic quantity.
497 Because our simulations are performed in two-dimensions, there is more intermittency
498 in convection which results in greater fluctuations between import and export compared
499 to three dimensions [*Wang and Sobel, 2011*]. Even in a more stable environment (days
500 30-60 in figure 7G) where moisture import exceeds export, convection is intermittent and
501 significant fluctuations generate considerable variability in NGMS. On the other hand, for
502 conditions which are not close to RCE—when either import or export is dominant—NGMS
503 provides important information about the relationship between convection and the con-
504 vective forcing. For example, the last month in figures 7G,H show steady, positive values
505 of NGMS. In the more stable case with non-zero precipitation (figure 7G), the domain
506 is importing moisture and exporting moist entropy, and the precipitation rate is related
507 to the value of NGMS according to equation 15. In the less-stable environment (figure
508 7H), precipitation is suppressed, moisture is exported from the domain, and there is weak
509 import of moist entropy. This is explained in more detail in section 5.3.

510 We can gain considerable insight to the response of convection to different thermody-
511 namic environments by understanding the behavior of DCIN. Figure 7I,J shows the time
512 evolution of DCIN, and figure 8 also shows the time series of the components of DCIN: the
513 threshold saturated moist entropy (s_t^*) is shown in red, the boundary layer moist entropy
514 (s_b) is in blue. These are plotted for the experiments where the θ profile is perturbed

515 first (solid lines in figure 8C,D), and the r_t profile is perturbed first (dotted line in figure
 516 8E,F). There are three important observations:

517 1. Moisture perturbations have very little impact on either s_t^* or s_b (with the exception
 518 of increasing s_t^* in a more stable environment as seen at day 60 in figure 8C). This makes
 519 sense since s_t^* is a function only of temperature, and although WTG isn't directly enforced
 520 in the boundary layer, s_b is more sensitive to $\theta_{ref}(z)$ than $r_{t,ref}(z)$ (see figure 10); boundary
 521 layer moisture anomalies are fairly uniform in different moisture environments but are
 522 strong functions of stability. The boundary layer is drier in a more stable environment
 523 and moister in a less stable one.

524 2. Changing atmospheric stability affects both s_t^* and s_b , so the significant variations
 525 in DCIN are related to the direct change in s_t^* (which is calculated near the level of the
 526 perturbation) and an indirect change in moisture.

527 3. s_t^* and s_b rapidly adjust to changes in the reference potential temperature profile
 528 with one important exception: the boundary layer moist entropy, s_b , exhibits a very slow
 529 response to a decrease in atmospheric stability.

530 The last observation deserves some explanation. Recall that s_b is the mean moist entropy
 531 in the lowest 1.75 km—which includes a thin layer just above the 1 km nominal boundary
 532 layer. Immediately following the decrease in stability, DCIN increases trivially (figure 8B,
 533 DCIN has a maximum of about $5 \text{ J kg}^{-1}\text{K}^{-1}$ at day 30) as a result of the rapid increase
 534 in s_t^* (the response time is less than a day, and is noted by the slight lead in increase in s_t^*
 535 compared to s_b at day 30 in figure 8D). After the initial increase, DCIN decreases sharply
 536 over a period of about 3 days; boundary layer fluxes rapidly increase s_b . This is because
 537 deep convection is suppressed due to the stable layer in the lower troposphere. Surface

538 fluxes eventually reach a steady state while radiatively-driven subsidence continues to
539 stifle convection of surface parcels and even acts to reduce boundary layer entropy. This
540 occurs over a period of about 25 days, after which DCIN finally reaches a steady state.
541 This mechanism also explains the gradual decline in saturation fraction in figure 7.

542 It is important to note that this slow response only occurs when lateral entrainment
543 is the choice for moisture treatment ($\lambda_{adv} = 1$, $\lambda_m = 0$); all other choices result in a
544 rapid adjustment to any change in the thermodynamic environment (not shown). The
545 long adjustment time for the lateral entrainment only treatment is likely a result of the
546 linear interpolation of the WTG vertical velocity to zero in the boundary layer (first two
547 terms in the right hand side of equations 6 and 8). This constraint implies that lateral
548 entrainment vanishes near the surface, so boundary layer entropy may only be reduced by
549 slower subsidence processes. When lateral entrainment is turned off ($\lambda_{adv} = 0$), or when
550 moisture relaxation is turned on ($\lambda_m \neq 0$), the boundary layer entropy can quickly adjust
551 to the reference profile, thus reducing the transition time.

5.2. Vertical profiles

552 In order to interpret the mean diagnostics, it is helpful to compare vertical profiles
553 of θ and r_t perturbations to the imposed perturbations; it is also useful to analyze the
554 vertical motion that arises as a consequence of these anomalies and of the different param-
555 eterizations for horizontal moisture advection. It is important to note that the vertical
556 resolution throughout the troposphere—including the boundary layer—is 250 m. While this
557 is sufficient for most of the troposphere, it is too coarse for the boundary layer and thus
558 limits the extent to which we can make physical interpretations about the behavior in

559 the boundary layer. Nevertheless, it is useful for making qualitative comparisons and
560 explaining the response of convection to different thermodynamic environments.

561 As discussed in section 3.3, the choices for parameterizing horizontal moisture advection
562 are entirely captured in the values for λ_{adv} and λ_m in equation 6. Table 1 summarizes
563 the values used in this study and identifies the abbreviations used for the results of this
564 section.

565 We expect the θ profile to be very close to the reference profile— independent of the
566 moisture treatment— simply as a consequence of enforcing the WTG approximation (see
567 equation 3). Figure 9 shows that this is indeed the case: the model’s θ anomalies are
568 very close to the imposed profiles, with the exception of the boundary layer where WTG
569 is not enforced. The largest deviation from the free tropospheric reference profile occurs
570 in the environment which is both moister and more stable (figure 9C); the domain mean
571 is slightly warmer in the lower troposphere, and the effect is slightly exaggerated in all
572 cases where horizontal moisture advection is explicitly parameterized.

573 In contrast, there are significant differences in the moisture anomalies generated by the
574 model compared to the imposed anomalies in the reference profile. A careful compari-
575 son of the moisture anomalies in figure 10 for each distinct environment suggests that
576 the reference moisture seems to play a supporting role for the convection rather than a
577 dominant one. This is illustrated by noting that the shape of the moisture anomalies
578 are more consistent with the perturbations applied to the θ profiles than to the mois-
579 ture profiles. For example, in the control case where moisture is only advected vertically
580 ($\lambda_{adv} = \lambda_m = 0$), there is no sensitivity to changes in the reference moisture profiles—by
581 design—but there is dependence on the stability of the reference θ profile. The stronger

582 dependence on environmental stability is also seen when horizontal moisture advection is
583 parameterized; for example, the top row of figure 10(A-C)—corresponding to more stable
584 environments—shows more moist mid-tropospheres, even in a drier environment. In these
585 cases, the lowest few kilometers are significantly drier, which is likely a consequence of
586 weak descent in that layer (as seen in the vertical mass flux, figure 11A-C).

587 An important observation is that less stable environments (figure 10G-I) produce drier
588 free tropospheres, even if the environment itself is moister (figure 10I). This is especially
589 true if horizontal moisture advection is parameterized by lateral entrainment. In this case,
590 radiatively driven subsidence results in an extremely dry anomaly—up to -9 g kg^{-1} —at an
591 altitude of 2 km. No other moisture treatment reduces the tropospheric moisture by this
592 amount.

593 When used simultaneously, moisture relaxation and lateral entrainment usually work
594 together to contribute either to an overall drying or moistening of the environment. An
595 exception occurs in a less stable environment. In this case, lateral entrainment contributes
596 to an extreme drying compared to the other parameterizations; when used in combination
597 with moisture relaxation, the reference profile is moister than the domain mean vertical
598 moisture profile, and the relaxation counters the extreme drying that occurs when lateral
599 entrainment is used exclusively.

600 Comparing vertical mass flux profiles for the different moisture treatments in different
601 environments can shed light on the behavior of convection in these simulations. With a few
602 exceptions, the most important factor in determining the shape of the vertical mass flux
603 profile is environmental stability. Changing the reference moisture primarily modulates
604 the magnitude of the mass flux profile, but does not change the shape. This is in contrast

605 to results presented by *Wang and Sobel* [2012] who found that extreme drying of a layer
606 in the lower troposphere produced a more bottom heavy convective profile. It is possible
607 that a larger magnitude of drying would do so here, but that study is outside the scope
608 of this paper.

609 More stable environments— independent of moisture or moisture treatment— have
610 stronger, more “bottom-heavy” convective profiles than unperturbed or less stable en-
611 vironments (compare rows in figure 11). Conceptually, we can visualize buoyant parcels
612 accelerating faster in the low-level cool anomaly, and becoming less buoyant in the warm
613 anomaly aloft, thus producing a bottom-heavy profile. This is supported by the results of
614 *Bretherton and Smolarkiewicz* [1989]. On the other hand, less stable environments inhibit
615 convective development; consequently, radiative cooling produces subsidence throughout
616 the free troposphere, though weak updrafts persist in the boundary layer. Environments
617 with decreased stability effectively suppress convection— independent of the environmen-
618 tal moisture— with one exception: If horizontal moisture advection is turned off so that
619 moisture transport within the domain is dominated by vertical advection (control case,
620 black line), there is upward motion above 5 km, with slightly stronger descent between the
621 boundary layer and 5 km. In this case, the cooling aloft accelerates the buoyant parcels
622 upward while the warm anomaly below results in descent. This strict response to changes
623 in the atmospheric stability is modified significantly if horizontal moisture advection is
624 explicitly parameterized and environmental moisture is permitted to enter the domain.
625 In this case, drier environmental air (represented by the reference profile) inhibits con-
626 densation of lifted moisture— and evaporates any condensation— which cools the parcel and
627 results in descent. The overturning of boundary layer air, necessitated by surface fluxes,

628 is amplified by moister environmental air so this effect monotonically increases with the
629 amplitude of the imposed moisture anomaly (figure 11G-I).

630 The most significant difference in mass flux profiles when comparing different parame-
631 terizations of horizontal moisture advection occurs in less stable (non-precipitating) en-
632 vironments: the mass flux profile differs significantly when horizontal advection is not
633 explicitly parameterized (control) compared to when it is (via lateral entrainment and/or
634 moisture relaxation). Aside from this, there aren't many significant qualitative differences
635 in mass flux profiles for different moisture treatments with one exception: in the absence of
636 θ -perturbations, the convection is more sensitive to the choice for parameterizing moisture
637 advection, especially in a dry environment (see figure 11D). In this case, there is almost no
638 vertical motion if moisture relaxation is used (green line); weak upward motion develops
639 if moisture is laterally entrained (blue); but there is weak descent if both mechanisms are
640 employed (red). The moisture relaxation case is consistent with the findings of *Wang and*
641 *Sobel* [2012].

5.3. Diagnosing convection

642 Now that we have some insight as to how the shape and strength of convection de-
643 pends on atmospheric stability, environmental moisture, and choice for parameterizing
644 horizontal moisture advection, we investigate the relationship between precipitation and
645 the diagnostic quantities defined in section 4. This allows us to quantify the impact of the
646 thermodynamic environment on the convection itself. Figure 12 shows scatter plots of rain
647 as a function of saturation fraction, instability index, NGMS, and DCIN. Each symbol
648 represents time and domain averages of the last two weeks (minus one hour to avoid the
649 ambiguous data at the transition) of each one month segment of the simulations. The

650 symbols themselves identify the reference environment—the environmental moisture and
 651 stability—according to the legend embedded in the top left panel (this symbol-only legend
 652 corresponds to the perturbations shown in figure 3). Colors indicate moisture treatment
 653 used; table 1 gives a simple legend for abbreviations and gives the values of λ_{hadv} and λ_m
 654 which determine the moisture treatment according to equation 6.

655 There are several observations to make from figure 12. First, consistent with observa-
 656 tions [Bretherton *et al.*, 2004; Peters and Neelin, 2006; Masunaga, 2012; Gjorgjievska and
 657 Raymond, 2014] and other modeling studies [Derbyshire *et al.*, 2004; Sobel and Bellon,
 658 2009; Wang and Sobel, 2012], we see that precipitation is a strong function of saturation
 659 fraction and instability index (figure 12A,B). More moist and more stable environments—
 660 as indicated with smaller instability indices, higher saturation fractions, and filled upright
 661 triangles—produce the highest precipitation rates. The former is expected; the latter is
 662 a consequence of the bottom-heavy convective profile associated with more stable envi-
 663 ronments (figure 11). The bottom-heavy convection vertically advects moister low level
 664 air which increases the precipitation efficiency, even in drier environments. This effect is
 665 enhanced in simulations with explicit lateral moisture entrainment, where $\lambda_{hadv} = 1$ and
 666 $\lambda_m = 0$ (blue), and $\lambda_{hadv} = 1$ and $\lambda_m = 1$ (red).

667 Less stable environments—as indicated with inverted triangles—inhibit precipitation in
 668 most cases (figure 12B; the exception being the control moisture treatment, $\lambda_{hadv} =$
 669 $\lambda_m = 0$); they have lower saturation fractions and higher values of DCIN (figures 12A,D,
 670 respectively). The warm anomalies in the lower troposphere inhibit moist parcel ascent
 671 in general, and result in negative vertical mass fluxes throughout the troposphere. Note
 672 the extremely low saturation fractions observed with lateral entrainment (blue symbols

673 in figure 12A). In this case, radiatively-driven subsidence down the moisture gradient
674 with no source of moisture to offset the drying (either explicitly via moisture relaxation
675 or implicitly at entraining levels as in the control case, figure 11G-I) results in severe
676 drying of the troposphere (figure 10G-I). This has important consequences for multiple
677 equilibria—and convective self-aggregation—as discussed in section 5.4.

678 There are a few cases where there is no precipitation despite having saturation fractions
679 above 0.7; this occurs in a less stable environment when moisture relaxation is applied,
680 either as the only treatment or in conjunction with lateral entrainment (figure 12A). In
681 this case, moisture relaxation is moistening a 5 km layer above the surface (with heavy
682 moistening in the boundary layer, see figure 10G-I) which results in a relatively high value
683 of saturation fraction. However, the temperature anomalies are still generating descent
684 throughout the free troposphere which inhibits precipitation (compare mass flux profiles
685 in figure 11H,I).

686 According to equation 15, static radiative cooling rates and fixed surfaces fluxes should
687 produce an inversely proportional relationship between precipitation and NGMS. Figure
688 12C demonstrates this beautifully for all moisture treatments with non-zero precipita-
689 tion rates. We should note that NGMS is a poor diagnostic in conditions close to RCE
690 since the system is nearly in balance and the net import/export of moisture and moist
691 entropy is near zero, resulting in large variations in NGMS as a result of averaging zero
692 over zero (in these simulations, values of NGMS > 1 represent poor diagnostic values).
693 Non-precipitating simulations all have small values of NGMS. In these cases, moisture is
694 exported from the system while moist entropy is weakly imported due to circulations in
695 the boundary layer. Note that there are several black symbols (control simulations) with

696 negative values of NGMS. These simulations do not explicitly parameterize horizontal
697 moisture advection ($\lambda_{hadv} = \lambda_m = 0$)—the convection is insensitive to the reference mois-
698 ture profile—and, as discussed in the previous section, they exhibit a drastically different
699 convective profile compared to the other moisture treatments in unstable environments
700 (figure 11G-I). Rather than descent through the entire free troposphere, there is ascent
701 from 6 km to the tropopause which vertically advects moisture and produces a non-zero
702 precipitation rate. In terms of the contribution to NGMS, however, the vertical motion
703 in the lower troposphere— ascent in the boundary layer and descent between the top of the
704 boundary layer and 5 km—gives net import (sources) of both moisture and moist entropy,
705 which results in $NGMS < 0$.

706 More stable environments exhibit small or negative values of DCIN, and thus represent
707 thermodynamic conditions most conducive for developing deep convection. We expect
708 unstable environments to be associated with larger DCIN; this is the case for some exper-
709 iments (figure 12D), though some show negative DCIN despite descent through the free
710 troposphere (compare figure 11G-I). These cases have more moisture in the layer below
711 1.75 km as a consequence of relaxing the domain mean moisture profile to the reference
712 profile; this increases s_b and thus decreases DCIN in these cases.

713 It is interesting that the highest precipitation rates don't occur for the most negative
714 values of DCIN, but rather for values that are near zero. We can understand this behavior
715 by re-examining figures figures 7A,I and 8A,C. If the environment becomes more stable
716 (e.g., day 30 in 8C), both s_t^* and s_b decrease as a direct consequence of the applied cooling
717 in the lower troposphere; this has a greater effect on s_t^* , which results in a negative DCIN
718 (indicating an environment conducive to developing deep convection, see discussion in

719 section 5.1). When moisture is then added to the lower troposphere (day 60 in figure 8C),
720 s_t^* increases slightly and DCIN becomes approximately zero (figures 8A,C and 12D). One
721 possible explanation for the increase in s_t^* is that a more moist environment will entrain
722 less dry air which results in less evaporative cooling, and a slightly higher temperature. In
723 contrast, a drier environment will experience more evaporative cooling and more negative
724 DCIN values (compare empty and filled upright triangles in figure 12D). Since DCIN is
725 approximately equal to negative lower tropospheric convective available potential energy
726 (CAPE), dry parcels require more negative values of DCIN to ascend.

727 We can further understand the factors controlling the characteristics of convection by
728 considering relationships between the diagnostic quantities themselves. Figure 13 shows
729 scatter plots which compare saturation fraction, instability index, NGMS, and DCIN.

730 Figure 13A clearly demonstrates that the more stable the environment, the higher the
731 saturation fraction [this is consistent with results of *Gjorgjievska and Raymond, 2014*].
732 For a given reference moisture profile (denoted by line style), the relationship is nearly
733 linear for most moisture treatments. The exception to this is the extreme drying in
734 unstable environments when horizontal moisture advection is parameterized by lateral
735 entrainment. This reinforces the notion that the important difference between moisture
736 treatments is not what happens when it is precipitating (precipitation rates and mass
737 flux profiles are fairly consistent), but what happens to the domain when it is not pre-
738 cipitating. This may be especially relevant for interpreting results of WTG simulations
739 which impose observed data in time-dependent reference profiles, or for understanding
740 conditions permitting multiple equilibria.

741 Figure 13B shows the relationship between saturation fraction and NGMS. For precip-
742 itating environments in conditions where NGMS is a good diagnostic, smaller values of
743 NGMS correlate to larger saturation fractions (see inset, figure 13B), which is consistent
744 with the precipitation-NGMS relation of figure 12. In non-precipitating cases, NGMS is
745 small as a consequence of weak import (or export) of moist entropy near the top of the
746 boundary layer.

747 There is not a significant relationship between NGMS and DCIN (figure 13C). This is an
748 interesting result that is consistent with the theories posited by *Raymond and Fuchs* [2007]
749 and *Raymond and Fuchs* [2009]. Together, these papers developed a highly simplified
750 model of the interaction between the large-scale and tropical oceanic convection. Their
751 analytic model identifies two types of convectively coupled waves: moisture modes in which
752 convection acts to increase—rather than decrease—the saturation fraction (this happens
753 when NGMS is negative), and another mode which is destabilized by convective inhibition.
754 An example of the latter is convectively coupled Kelvin waves, and recent modeling results
755 by *Fuchs et al.* [2014] demonstrated the role of DCIN in destabilizing the two-dimensional
756 analog of convectively coupled Kelvin waves. This simplified picture suggests that either
757 NGMS or DCIN is the control for destabilizing the environment, depending on the nature
758 of the interaction between convection and the large-scale. In reality, the dynamic processes
759 are much more complicated due to the inherent nonlinearity of the atmosphere, so we
760 do not expect an obvious relation between NGMS and DCIN, despite good correlations
761 between other convective diagnostics.

762 Similarly, there is also no obvious overall correlation between DCIN and saturation
763 fraction (figure 13D). Here, the primary observation is that more stable environments—

764 upright triangles—experience small or negative DCIN, which is indicative of an environment
765 conducive to convection. As explained above, DCIN in these environments becomes less
766 negative for more moist environments (indicated with filled upright triangles and higher
767 saturation fractions) because less dry air is entrained, evaporative cooling is diminished,
768 and the threshold entropy increases. Also noteworthy is that the highest values of DCIN
769 accompany the lowest saturation fractions, and these occur only with laterally entrained
770 moisture, and only in the most hostile environment for convection: more unstable and
771 drier.

772 To summarize figures 12 and 13, we note the following features:

773 1. The precipitation rate is highly sensitive to both the saturation fraction and the
774 atmospheric stability (as measured by the instability index).

775 2. Stable environments are conducive to precipitating states: they are moist, sport
776 small or negative values of DCIN, and give the highest precipitation rates.

777 3. Environmental moisture serves to modulate the precipitation by entraining more or
778 less moisture as available, but in the current implementation of WTG, it doesn't seem to
779 overcome the atmospheric stability. In other words,

780 (i) Unstable environments have greatly diminished moisture and precipitation; moist-
781 ening the environment doesn't change this.

782 (ii) More stable environments are very conducive to precipitation. Drying the envi-
783 ronment reduces—but does not eliminate—the precipitation in the domain.

784 4. NGMS—which summarizes our ignorance about the relationship between convection
785 and the convective forcing—is strongly related to the precipitation rate. In the steady state
786 with approximately constant entropy forcing, we expect—and we observe—an inversely pro-

787 portional relationship between precipitation rate and NGMS in precipitating states. The
788 relationship between NGMS and other diagnostics, however, is not as straight-forward:

789 (i) There is only a slight correspondence between NGMS and atmospheric stability,
790 which is stronger for moister environments and nearly absent for drier environments. Most
791 likely, the biggest impact of atmospheric stability is an indirect result of modifying the
792 vertical mass flux profile which controls lateral entrainment and detrainment of moist
793 entropy and moisture.

794 (ii) For the precipitating states, and for environments which are sufficiently different
795 from RCE, there is an inverse relation between NGMS and saturation fraction. Dry states,
796 on the other hand, all seem to exhibit small and sometimes negative values of NGMS [in
797 agreement with *Sessions et al.*, 2010].

5.4. Multiple Equilibria

798 One important application of WTG experiments relates to the analogy between the
799 smaller domain WTG simulations which exhibit multiple equilibria—either a persistent
800 precipitating steady state or a completely dry subsiding troposphere—and the dry and
801 moist regions of a larger domain RCE simulation with self-aggregated convection. Thus,
802 we consider the effect of different reference environments and moisture treatments on
803 multiple equilibria. Insight in this context may help elucidate the behavior of convection
804 in self-aggregation simulations.

805 Whether or not a particular set of conditions exhibit multiple equilibria is determined
806 by performing a set of parallel experiments in which all parameters are identical with
807 the exception of the initial tropospheric moisture content: one experiment is initialized
808 with the reference moisture profile, while the other is initially completely dry. If the

809 initially moist experiment maintains persistent precipitating convection while the initially
810 dry experiment remains dry with zero precipitation, then the set of parameters exhibits
811 multiple equilibria. If, on the other hand, the initially dry profile develops precipitating
812 convection—or if the initially moist profile evolves to and maintains a dry steady state—then
813 there is a single equilibrium. We hypothesize that parameters which affect the existence
814 of multiple equilibria in WTG experiments are also important for self-aggregation in large
815 RCE simulations.

816 As demonstrated in *Sessions et al.* [2010], the model used in this experiment supports
817 multiple equilibria in conditions similar to those used in this work. Using lateral entrain-
818 ment of moisture and interactive radiation, *Sessions et al.* [2010] found multiple equilibria
819 to exist for a significant range of wind speeds with unperturbed RCE reference profiles. In
820 an updated version of the model, *Herman and Raymond* [2014] showed multiple equilibria
821 occurs with static, non-interactive radiation (though not when a spectral form of WTG
822 is implemented).

823 The first task is to determine whether the existence of multiple equilibria in this model
824 depends on the parameterization of horizontal moisture advection. *Sobel et al.* [2007]
825 demonstrated that states of multiple equilibria are sensitive to how moisture advection
826 is parameterized; here we test this systematically with different horizontal moisture ad-
827 vection treatments. Specifically, we run experiments initialized with zero tropospheric
828 moisture, using unperturbed reference profiles, for each moisture treatment. All other
829 parameters are identical to the experiments reported in previous sections (including sur-
830 face wind speeds of 7 m s^{-1}). Of all the moisture treatments, the only one to maintain a
831 dry equilibrium state over 30 days was lateral moisture entrainment ($\lambda_{adv} = 1$, $\lambda_m = 0$).

832 That multiple equilibria exist for lateral entrainment in these experiments is undoubtedly
833 a consequence of the extreme drying of the free troposphere that only occurs with this
834 choice (figure 10G-I). The extreme drying is conducive to maintaining a dry state and
835 supporting multiple equilibria. These results are summarized in table 2.

836 To determine how robust multiple equilibria are with laterally entrained moisture, we
837 repeated the experiment with zero initial tropospheric moisture, but with a surface wind
838 speed of 10 m s^{-1} . In this case, the experiment began to precipitate and only a single
839 equilibrium state exists. This is an important result: with static radiative cooling, multiple
840 equilibria exists over a range of wind speeds from $5\text{-}10 \text{ m s}^{-1}$ only if horizontal moisture
841 advection is parameterized with lateral entrainment. Figure 14 shows the precipitation
842 rate for the multiple equilibria experiments performed with laterally entrained moisture.

843 It is interesting to compare the results of this section with the multiple equilibria re-
844 sults of *Sobel et al.* [2007] and *Herman and Raymond* [2014]. *Sobel et al.* [2007] found that
845 multiple equilibria exist for a large range of SSTs with experiments that did not explicitly
846 parameterize horizontal moisture advection (similar to our control method); parameter-
847 izing large-scale moisture advection via a moisture relaxation reduced the range of SSTs
848 which permitted multiple equilibria in this model. This demonstrates that different mod-
849 els, differences in implementing WTG, or differences in parameterizing horizontal moisture
850 advection can produce different results with respect to multiple equilibria. *Herman and*
851 *Raymond* [2014] tested multiple equilibria in the conventional WTG (as in this work) and
852 in a version of WTG which spectrally decomposes heating (with lateral entrainment and
853 static radiation). It is important to note that in the results of *Herman and Raymond*
854 [2014], their model only exhibited multiple equilibria for the conventional WTG approach

855 (as used in this work), but not in the spectrally modified implementation, and further-
856 more, multiple equilibria depended on the height of the boundary layer. The existence of
857 multiple equilibria may also depend on many other model details, including domain size
858 or the degree to which WTG is enforced [*Sessions et al.*, 2010], details of the implemen-
859 tation of WTG [e.g., *Daleu et al.*, 2012], or background SST [*Emanuel et al.*, 2013]. How
860 each of these factors affects the existence of multiple equilibria is not fully understood;
861 experiments such as this are aimed to improve the overall understanding, and especially
862 determine which factors are representative of physical processes in the atmosphere.

863 Finally, to determine the sensitivity of multiple equilibria to changes in environmental
864 stability and moisture, we performed two more experiments with lateral moisture entrain-
865 ment and an initially dry troposphere: the first in a more stable environment, the second
866 in a more moist environment. In both cases, the model produced precipitating convection
867 and multiple equilibria were not sustained.

6. Summary

868 We used a cloud system resolving model on a two-dimensional domain with the large-
869 scale parameterized by the weak temperature gradient (WTG) approximation to inves-
870 tigate the response of convection to changes in the thermodynamic environment. The
871 thermodynamic environment was initially set by vertical profiles of potential temperature
872 and moisture in radiative convective equilibrium (RCE), and we added perturbations to
873 change the environmental stability and moisture. For the magnitudes of perturbations
874 explored in this work, we found that atmospheric stability dominates changes in the char-
875 acter of convection by prescribing the vertical motion in the domain:

876 1. more stable environments produce bottom heavy convection with higher precipita-
877 tion rates than unperturbed profiles—even in drier environments.

878 2. less stable environments shut off precipitation by generating descent throughout the
879 free troposphere.

880 On the other hand, the environmental moisture modulates precipitation rates according
881 to the amount of moisture available for precipitation—they can amplify or weaken vertical
882 motion—but in general they don't change the shape of the convective profile.

883 Convection is characterized by a set of diagnostics that includes precipitation rate,
884 vertical mass flux, an instability index (a measure of instability), saturation fraction,
885 normalized gross moist stability (NGMS), and deep convective inhibition (DCIN). The
886 shape of the vertical mass flux directly affects budgets of moisture and moist entropy in
887 the domain, which sets the values of the diagnostic quantities. Our results show that in
888 environments which support precipitating convection, the precipitation rate is a sensitive
889 function of saturation fraction, and is inversely proportional to NGMS. Atmospheric sta-
890 bility also plays an important role in the relationship between diagnostics: more stable
891 environments—characterized by smaller instability indices—correlate with higher satura-
892 tion fractions. These relationships hold independent of the perturbations applied to the
893 reference environments.

894 Horizontal moisture advection plays an important role in the interaction between con-
895 vection and the large-scale circulations. We investigate alternate parameterizations of
896 this process, which include lateral entrainment by divergent circulations induced by en-
897 forcing WTG, a moisture relaxation which represents a parameterization of horizontal
898 moisture advection by non-divergent circulations, a combination of both of these, and

control simulations which assume horizontal advection is negligible compared to vertical advection (so lateral entrainment and moisture relaxation are both turned off). In thermodynamic environments which support precipitating convection, there is little difference in the characteristics of convection—as determined by precipitation rate, saturation fraction, DCIN, NGMS and vertical profiles of mass flux—for different moisture treatments (except that precipitation rate is insensitive to changes in reference moisture if horizontal moisture advection is not explicitly parameterized via lateral entrainment or a relaxation to a reference profile). The most significant difference between moisture treatments is seen when the environment does not support convection (less stable environments). The most significant effects are:

1. A drastic decrease in free tropospheric moisture when horizontal moisture advection is parameterized by lateral entrainment.
2. If both lateral entrainment and moisture relaxation are turned off—so the domain is not sensitive to changes in environmental moisture—the model generates ascent in the upper troposphere which supports light precipitation. In this case, moisture and moist entropy are both imported, and NGMS is negative.

Multiple equilibria—dry or precipitating states in identical boundary conditions—are of particular interest because of the hypothesized relationship to dry and moist regions in larger domain RCE simulations where convection has self-aggregated. In this work, we investigated the sensitivity of multiple equilibria to changes in the thermodynamic environment and different parameterizations of horizontal moisture advection. Using static (non-interactive) radiative cooling, we found that the existence of multiple equilibria is sensitive to both the thermodynamic environment and choice of moisture treatment. For

922 the parameters used in this work, our model only exhibited multiple equilibria for laterally
923 entrained moisture in an unperturbed reference environment. Other moisture treatments
924 exhibited only a single equilibrium, and imposing either a more stable or more moist
925 environment destroyed the dry equilibrium state even when moisture was laterally en-
926 trained. To the extent that multiple equilibria are analogous to dry and moist regions in
927 a self-aggregated RCE simulation—and to the extent that the MJO can be depicted as a
928 manifestation of self-aggregation—these results may be significant for improving simula-
929 tions of the MJO [*Pritchard and Bretherton, 2014; Zhu and Hendon, 2015*].

930 Our results are important not only for understanding the physics of tropical convec-
931 tion, but also for interpreting other studies which implement WTG. As far as mechanisms
932 governing the development of deep convection, our results suggest that convection is very
933 sensitive to the thermodynamic environment. Other large-scale forcing mechanisms, in-
934 cluding radiative cooling, surface fluxes, or the propagation of atmospheric waves, may
935 affect convection indirectly by modifying the thermodynamic environment. For example,
936 easterly waves generate virtual temperature anomalies—similar to those idealized in this
937 work—that enhance or suppress convection [*Reed and Recker, 1971; Raymond and Ses-*
938 *sions, 2007; Gjorgjievska and Raymond, 2014*]. We are not suggesting that there are no
939 direct influences on convection by these mechanisms, only that this work provides strong
940 evidence that there is also an indirect effect which acts via a modification of the ther-
941 modynamic environment. This is significant insight given the growing use of the WTG
942 approximation to understand different aspects of tropical convection, including tropical
943 cyclogenesis [*Raymond and Sessions, 2007*] and the Madden-Julian Oscillation [*Wang*
944 *et al., 2013*].

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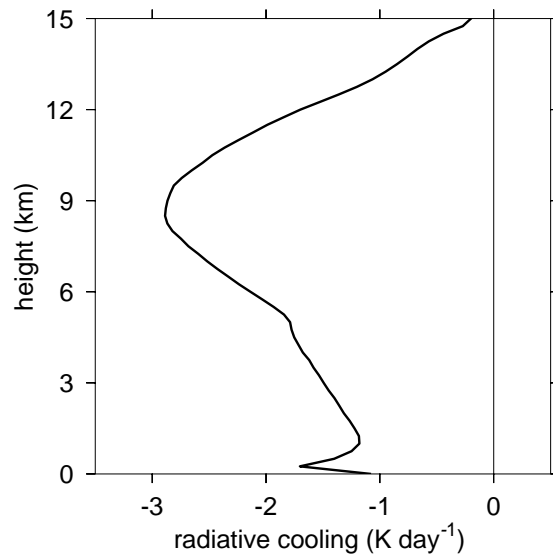


Figure 1. Mean radiative cooling profile from a radiative convective equilibrium (RCE) simulation. This cooling profile is the prescribed static cooling for all experiments in this work.

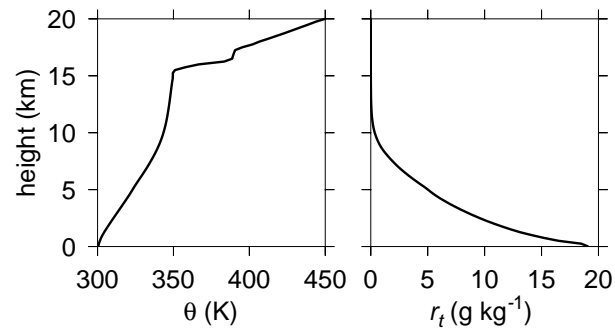


Figure 2. Radiative convective equilibrium (RCE) profiles of potential temperature (left) and total water mixing ratio (right) used as unperturbed reference profiles in WTG calculations. RCE is calculated over a uniform SST of 303 K, with surface wind speed of 5 m s^{-1} and interactive radiation on a 2D, 200 km horizontal domain.

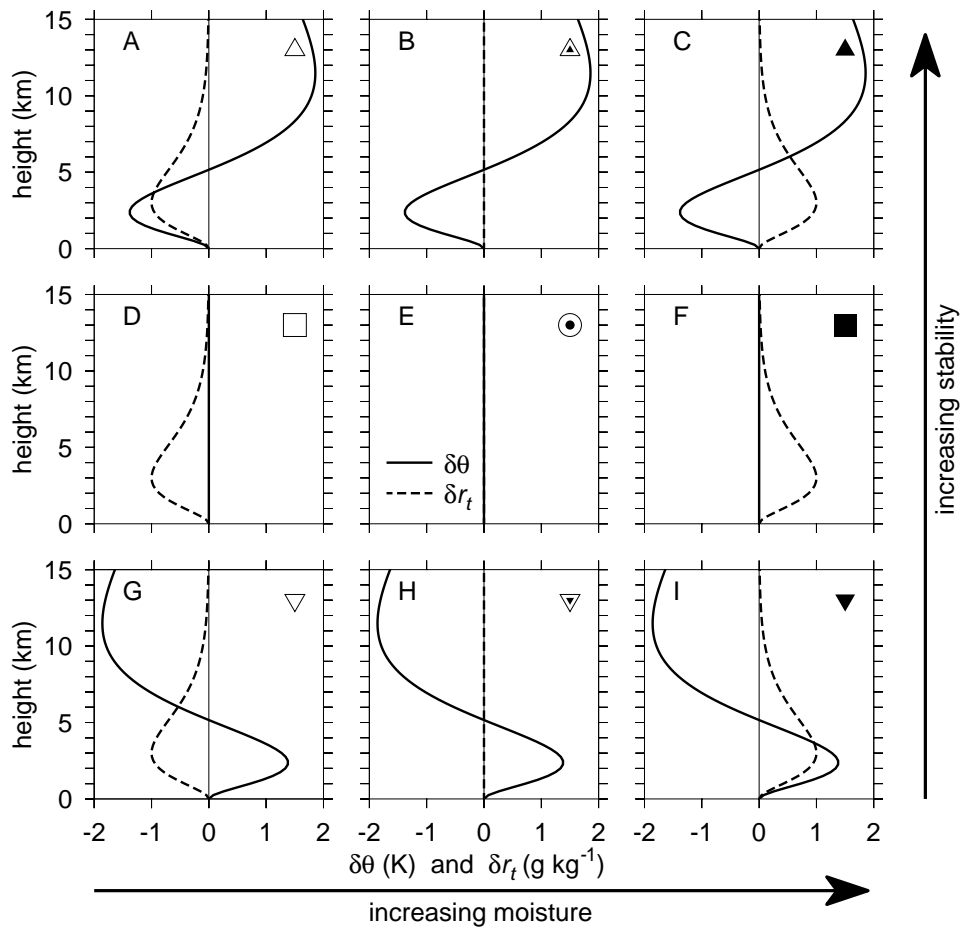


Figure 3. Perturbations added to the RCE reference profile. Solid lines represent perturbations to the potential temperature profiles, dashed lines give mixing ratio perturbations. The center panel is the unperturbed RCE reference state. The middle row has unperturbed reference potential temperature profiles, the top row has perturbations representing more stable environments, the bottom row represents less stable environments. Similarly, the middle column has no perturbations added to the reference moisture environment, the left column is drier, the right column, moister. The symbols in the upper right of each panel represent the reference environment. The shading represents the moisture perturbation: empty symbols are drier, full symbols are moister, half-filled symbols have unperturbed moisture profiles. The squares are unperturbed θ profiles; more stable environments are represented by upright triangles (geometrically more stable shapes); less stable environments are represented by inverted triangles. In order to easily distinguish the unperturbed RCE profiles, we choose bulls-eyes to represent these simulations. This figure serves as a symbol legend for results presented in section 5.

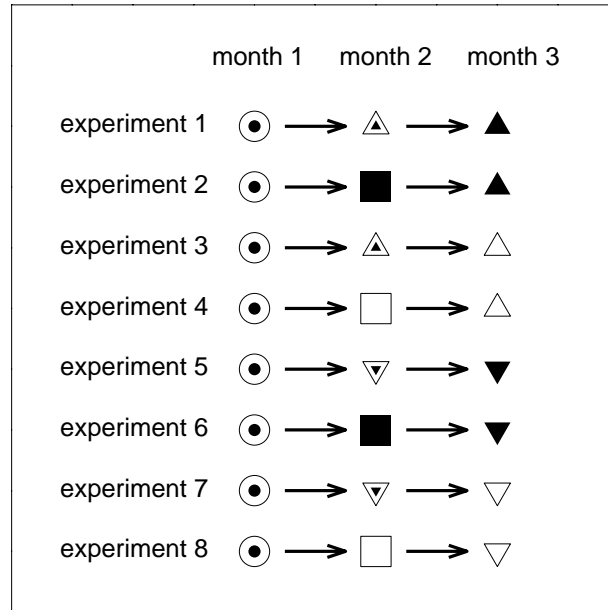


Figure 4. Graphic showing the sequence of perturbations applied in each experiment. Symbols are the same as in figure 3: bulls-eyes are unperturbed profiles; squares indicate no change in stability; triangles indicate change in stability (upright are more stable); amount of filling represents environmental moisture perturbation with empty being drier and filled being moister.

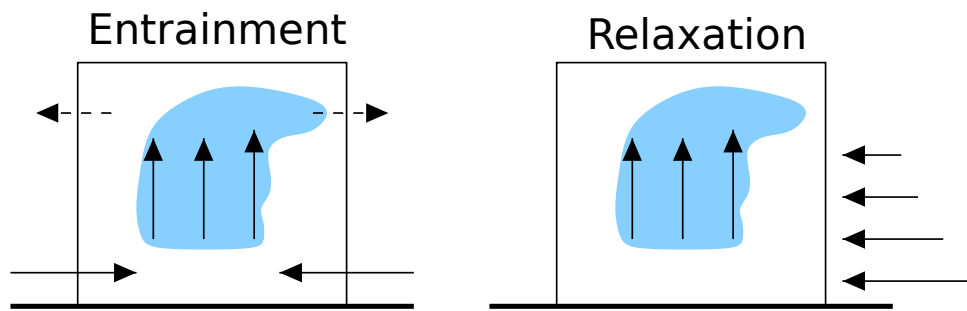


Figure 5. Cartoon representations of the physical processes captured by the different parameterizations of horizontal moisture advection. In each case, the box represents the domain of the CRM. Arrows pointing up represent the WTG vertical mass flux (ρw_{wtg}). The outside of each box represents the environment and therefore the reference profiles used in the WTG experiments. The left panel shows the lateral entrainment of the reference moisture at low levels which results from convergence via mass continuity in the WTG velocity field. The dashed arrows indicate the detrainment that would occur in the real atmosphere due to divergence in a layer where buoyancy decreases with height. Since detrainment of intrinsic quantities doesn't alter the modeled environment, there is no change in the moisture due to this mechanism (see equation 7). The right panel illustrates how moisture might enter the domain from large-scale circulations that are independent of those induced by WTG; this process is parameterized by directly relaxing the domain mean moisture profile to the reference profile.

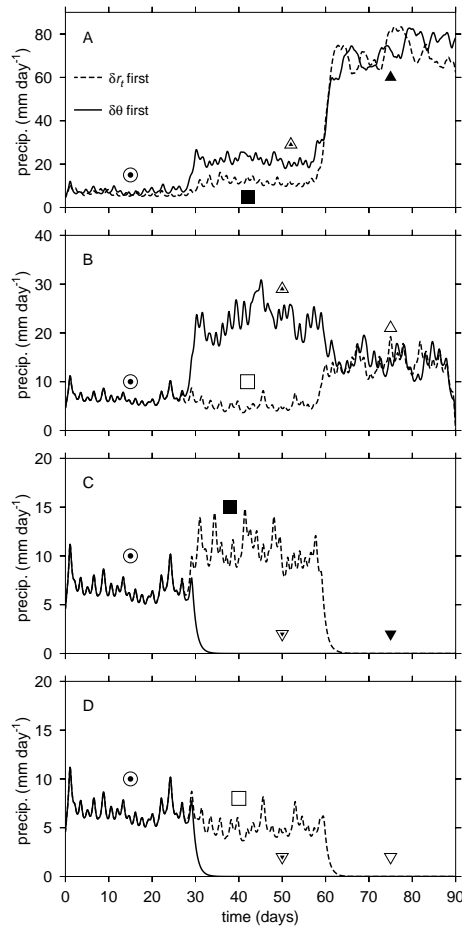


Figure 6. Three month time series of precipitation rate for the eight WTG experiments graphically described in figure 4. a) Experiments 1 and 2; b) experiments 3 and 4; c) experiments 5 and 6; d) experiments 7 and 8. The symbols indicate the perturbations of the reference profile for the one month segment, the symbol legend is given in figure 3. Solid and dashed lines indicate whether the reference θ or reference r_t profiles, respectively, were perturbed first (these indicate the perturbed profile during the second month of the experiments). Data in this figure have been low-pass filtered in time with a cutoff period of 1 day.

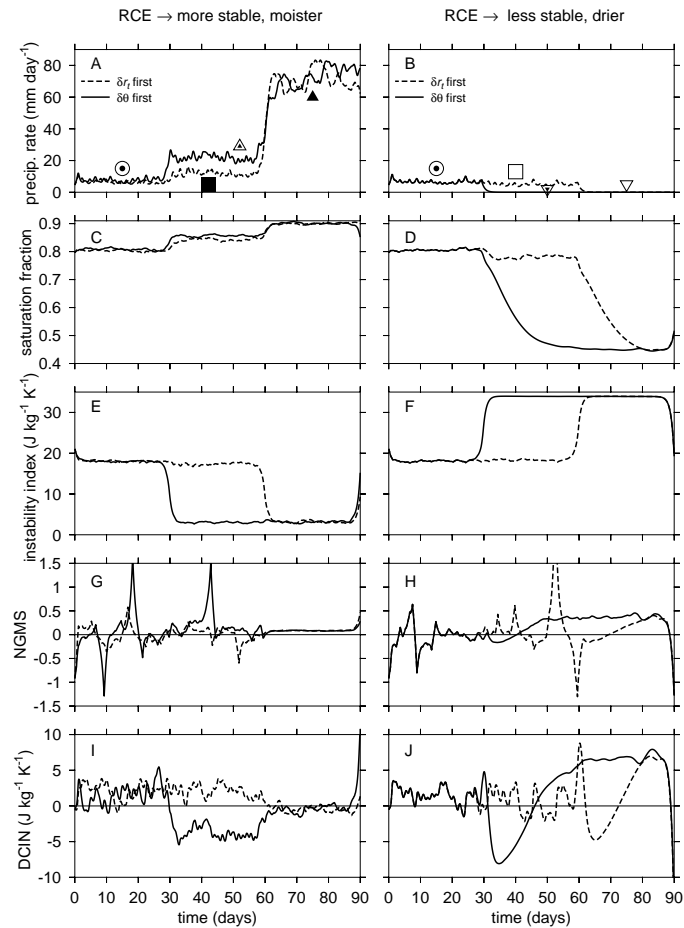


Figure 7. Time series showing precipitation rate (a,b), saturation fraction (c,d), instability index (e,f), NGMS (g,h), and DCIN (i,j) for experiments which became more stable and moister (left column, experiments 1 and 2 in figure 4), and those which became less stable and drier (right column, experiments 7 and 8 in figure 4). Data in this figure have been low-pass filtered in time with a cutoff period of 1 day.

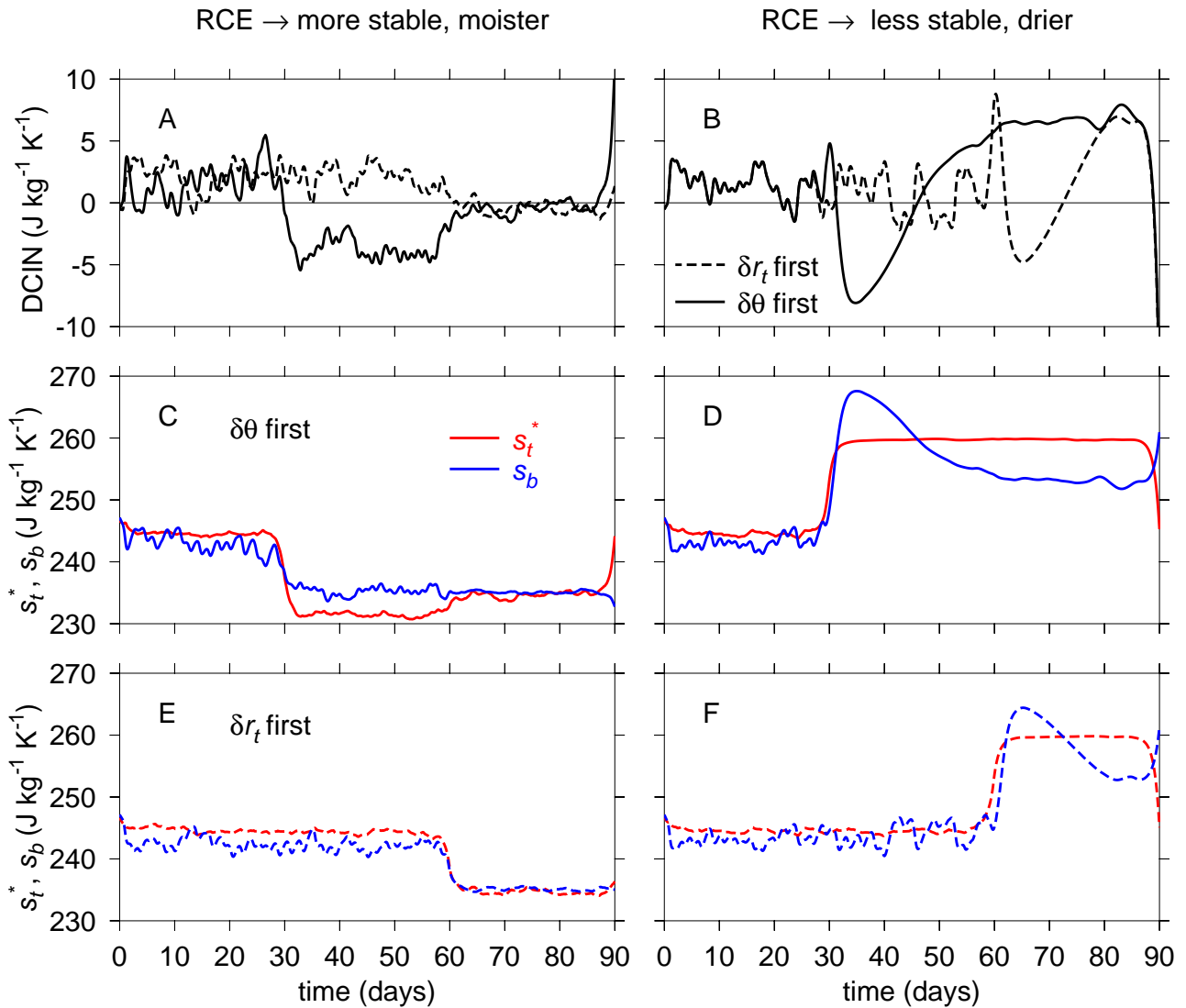


Figure 8. Time series of DCIN (a,b), and DCIN components, s_t^* and s_b (c-f). The solid lines represent experiments where the θ profile was perturbed first (c,d), while dashed lines represent experiments where moisture perturbations are imposed first (e,f). As in figure 7, the left column represents experiments 1 and 2 while the right column shows results for experiments 7 and 8 (see figure 4). Data in this figure have been low-pass filtered in time with a cutoff period of 1 day.

	$\lambda_{hadv} = 0$	$\lambda_{hadv} = 1$
$\lambda_m = 0$	control	lat ent
$\lambda_m = 1/1.8 \text{ days}^{-1}$	m-relax	both

Table 1. Abbreviations for the different combinations of moisture treatment. The values of λ_{hadv} and λ_m (equation 6) determine the choice for parameterizing horizontal moisture advection. This is the key for identifying each method: lateral entrainment (lat ent), moisture relaxation (m relax), both (lat ent & m relax). Choosing $\lambda_{hadv} = \lambda_m = 0$ disconnects the modeled convection from the reference moisture profile; this is the control.

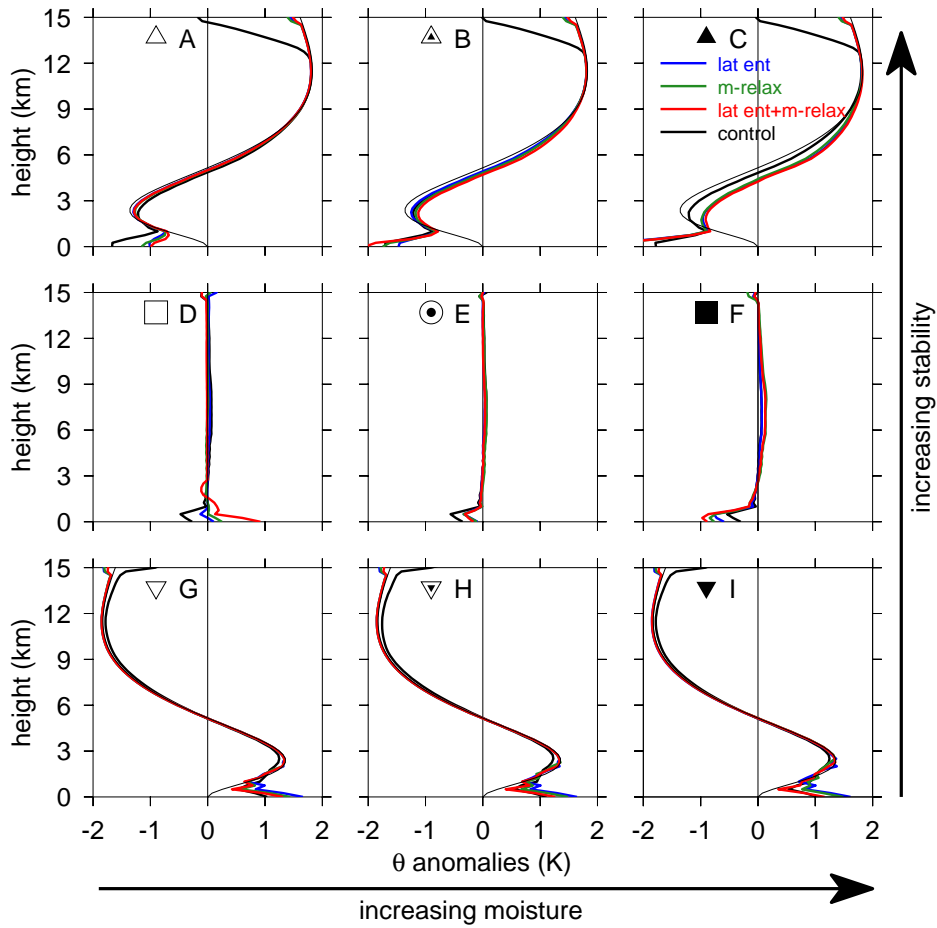


Figure 9. Modeled θ anomalies for each distinct thermodynamic environment (represented symbolically as in figure 3). Colors represent moisture treatment: lateral entrainment is blue; moisture relaxation is green; red uses both lateral entrainment and moisture relaxation; black uses neither. For reference, the thin black lines show the anomalies imposed on the reference profile (see figure 3).

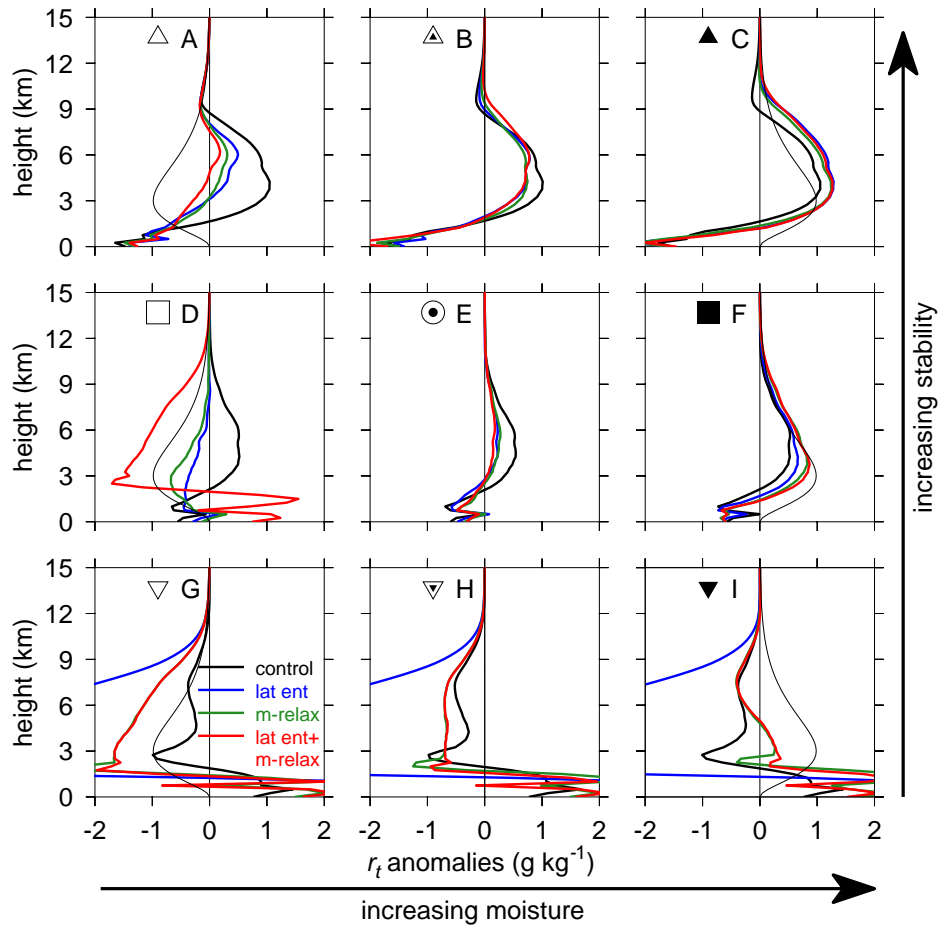


Figure 10. Moisture anomalies for different thermodynamic environments using different moisture treatments (denoted by color; see table 1 for a legend of abbreviations). The thin black line shows the imposed moisture perturbation for reference (same as figure 3). The dry anomaly for lateral entrainment (blue) in panels g-i has a minimum value of nearly -9 g kg^{-1} at an altitude of about 2 km.

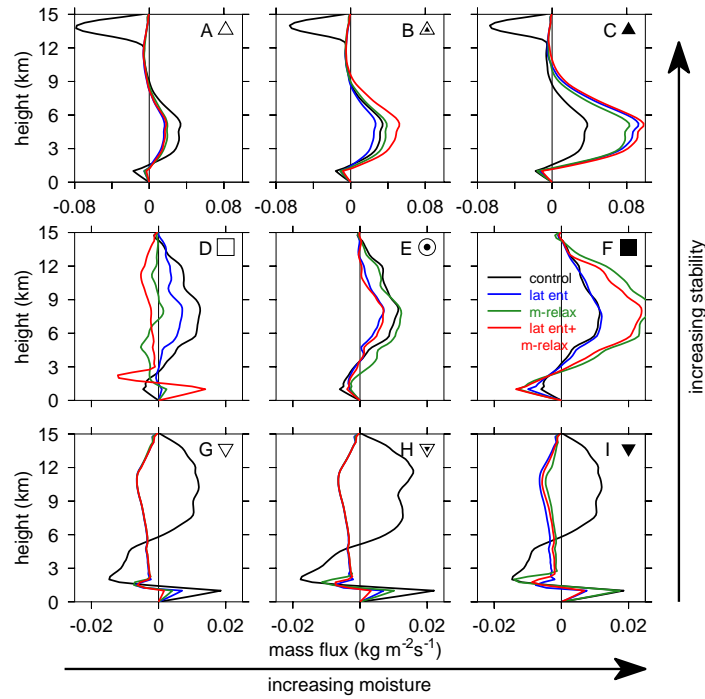


Figure 11. Vertical profiles of vertical mass flux (equation 16) for each environmental profile. Colors represent the moisture treatment used. Note the different horizontal scale in the top row figures compared to the other rows. Each tick mark on the horizontal axes in the top row represents $0.04 \text{ kg m}^{-2}\text{s}^{-1}$, while those in the middle and bottom rows represent $0.02 \text{ kg m}^{-2}\text{s}^{-1}$. More stable environments exhibit much stronger vertical mass fluxes.

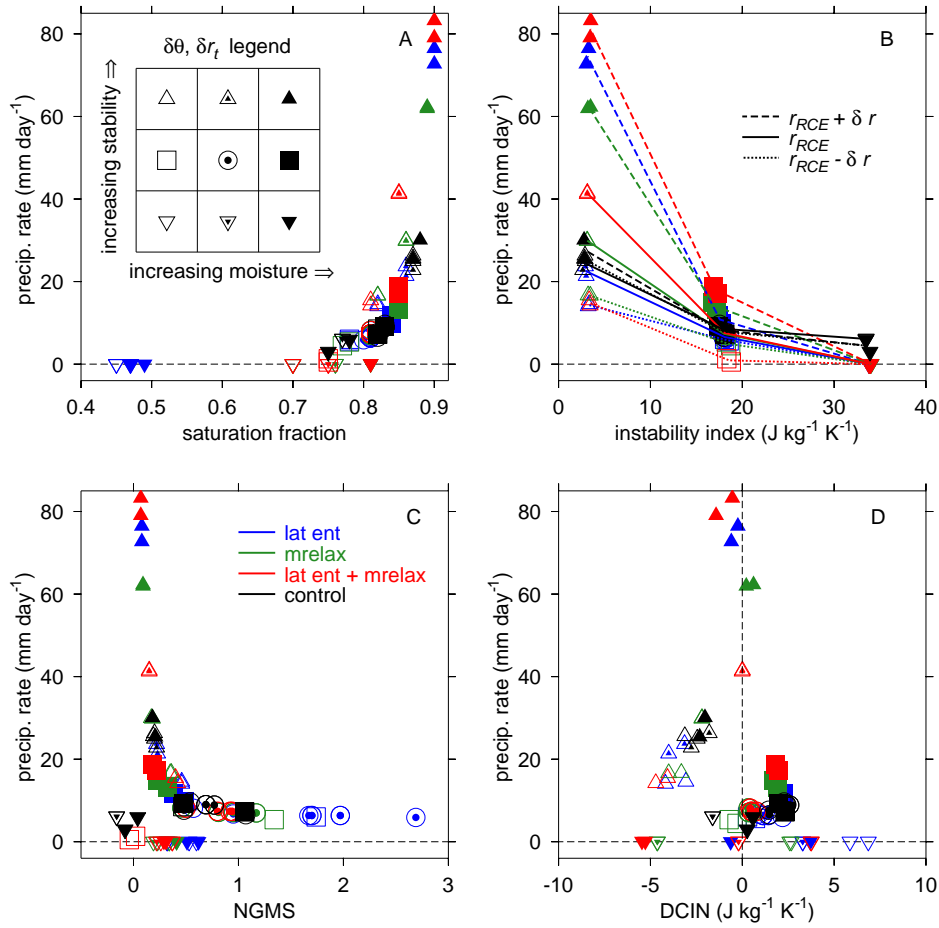


Figure 12. Scatterplots of precipitation as a function of (a) saturation fraction, (b) instability index, (c) NGMS, and (d) DCIN. Each shape represents a domain and time average for a given set of environmental conditions (see legend inset in panel a, and corresponding perturbations in figure 3). Colors represent parameterization choices for horizontal moisture advection according to table 1: blue indicates explicit lateral entrainment; green is moisture relaxation; red indicates both are used, and black is the control (no explicit parameterization). The lines in panel (b) connect experiments with identical reference moisture profiles: solid lines have unperturbed moisture profiles (r_{RCE}), dashed are more moist ($r_{RCE} + \delta r$), dotted are drier ($r_{RCE} - \delta r$).

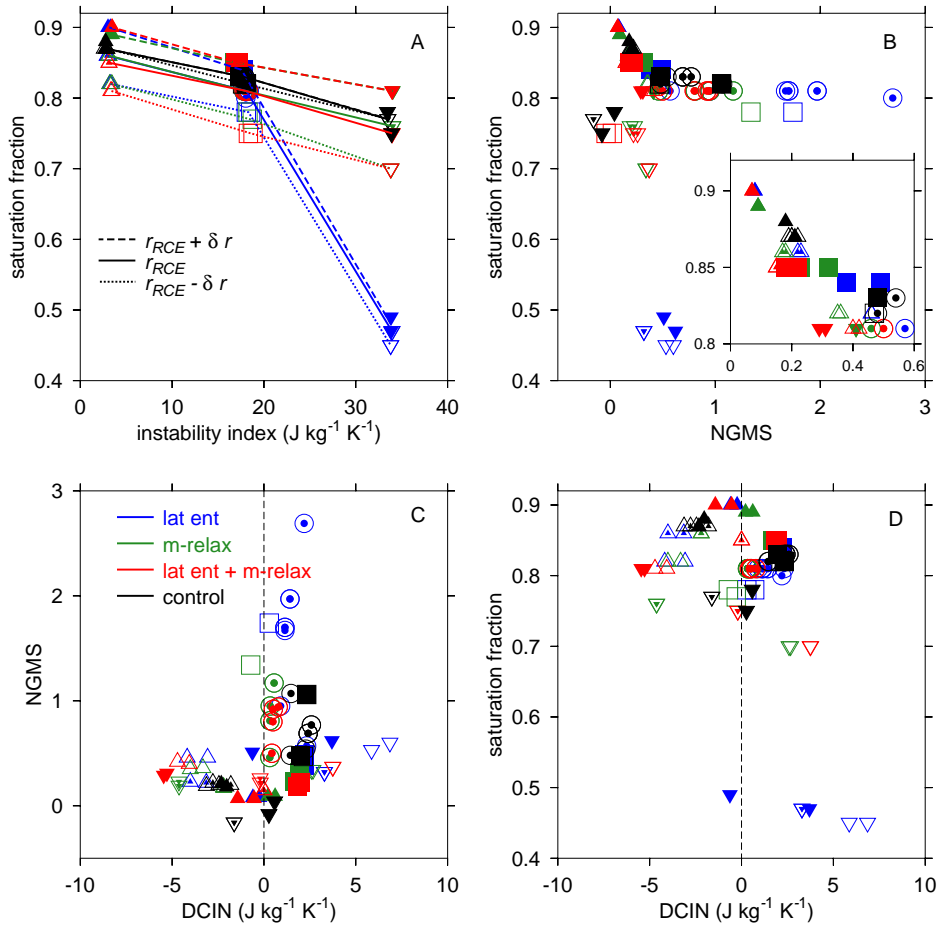


Figure 13. Relationships between diagnostic quantities: (a) saturation fraction vs. instability index, (b) saturation fraction vs. NGMS, (c) NGMS vs. DCIN, and (d) saturation fraction vs. DCIN. Colors indicate choice for horizontal moisture advection, while shapes indicate environmental stability and moisture according to the symbol legend defined in figure 12. Note the strong relationship between saturation fraction and instability index. As in figure 12, lines in panel (a) connect experiments with identical reference moisture profiles.

	$\lambda_{hadv} = 0$	$\lambda_{hadv} = 1$
$\lambda_m = 0$	NO	YES
$\lambda_m \neq 0$	NO	NO

Table 2. Table identifying which moisture treatments exhibit multiple equilibria with surface wind speed of 7 m s^{-1} . “YES” means that a dry state is maintained if initiated with a dry troposphere; “NO” means that precipitation developed in spite of an initially dry troposphere. With fixed radiation, the only moisture treatment that maintains multiple equilibria is lateral entrainment.

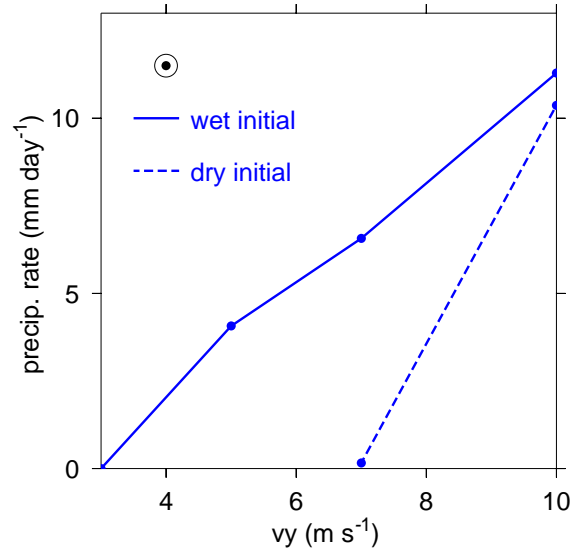


Figure 14. Precipitation rate as a function of surface wind speed for simulations which are initialized either with the reference moisture profile (solid line), or with a completely dry troposphere (dashed line). Moisture is laterally entrained in all experiments, and there is a range of wind speeds which exhibit multiple equilibria. The bulls eye in the upper left indicates unperturbed reference profiles (see figure 3).