Physics 122 – Class #7
Date: 9/9/14

Questions?
Announcements/reminders

MP HW02 and written HW02 are due today (written HW02 is due at the beginning of the class....now)!
MP HW03 and written HW03 are due next Tuesday, 16 September.

Always check the class website to make sure of assignments and due dates.
(http://www.physics.nmt.edu/~saska/phys122.html)
Physics 122 – Class #7

Last class: Superposition, Standing Waves, ..

Today: Interference and starting Wave Optics.
Interference

\[ D_r(x, t) = D_1(x, t) + D_2(x, t) + \cdots + D_n(x, t) \]

Superposition Principle

Sinusoidal waves, same amplitude and wavelength
Interference

\[ D_1 = A \sin (kx_1 - \omega t + \phi_{10}) = A \sin \phi_1 \]

\[ D_2 = A \sin (kx_2 - \omega t + \phi_{20}) = A \sin \phi_2 \]

\[ D = D_1 + D_2 = A \left[ \sin \phi_1 + \sin \phi_2 \right] \quad - \quad - \quad - \quad - \quad - \quad (1) \]

\[ \text{use trig identity} \]

\[ \sin \alpha + \sin \beta = 2 \cos \left( \frac{\alpha - \beta}{2} \right) \sin \left( \frac{\alpha + \beta}{2} \right) \]

\[ \Rightarrow (1) \rightarrow D = 2A \cos \left( \frac{\phi_1 - \phi_2}{2} \right) \sin \left( \frac{\phi_1 + \phi_2}{2} \right) \]

\[ \text{--- (2)} \]
Interference

\[ D = 2a \cos \left( \frac{\phi_2 - \phi_1}{2} \right) \sin \left( \frac{\theta_1 - \theta_2}{2} \right) \] \hspace{1cm} (3)

\[ \Delta \phi = \phi_2 - \phi_1 = k(x_2 - x_1) + (\phi_{2o} - \phi_{1o}) = k\Delta x + \Delta \phi_0 \] \hspace{1cm} (*I)

\[ \phi_2 = kx_2 - \omega t + \phi_{2o} \hspace{1cm} \phi_1 = kx_1 - \omega t + \phi_{1o} \]

\[ \frac{\phi_1 + \phi_2}{2} = \frac{k(x_2 + x_1) - 2\omega t + \phi_{2o} + \phi_{1o}}{2} \]

\[ = kX_{aw} - \omega t + \phi_{0aw} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (\#2) \]
Interference

Sub. #1 and #2 in (3):

\[ P = 2a \cos \left( \frac{\Delta \phi}{2} \right) \sin \left( kx_{av} - \omega t + \phi_{0av} \right) \]

\[ A = \left| 2a \cos \left( \frac{\Delta \phi}{2} \right) \right| \]

Let's find when \( A = A_{\text{max}} \)

and \( A = 0 \)
Interference

\[ A = A_{\text{max}} \text{ for } \cos \left( \frac{\Delta \phi}{2} \right) = 1 \]

\[ \frac{\Delta \phi}{2} = m \pi \quad \Rightarrow \quad m = 0, 1, 2, \ldots \]

\[ \Delta \phi = m \left( 2\pi \right) \]

\[ \Delta \phi = k \Delta x + \Delta \phi_0 = \frac{2\pi}{\lambda} \Delta x + \Delta \phi_0 \]

So \( \Delta \phi \) depends on:

- \( \Delta x \) - path length difference
- \( \Delta \phi_0 \) - inherent phase difference

What were the sources doing at \( t = 0 \)?

\( \Delta \phi_0 = \phi_0 - \phi_1 = 0 \)
Interference in 2-D and 3-D

\[ D_r(r,t) = a(r) \sin(kr - \omega t + \phi_0) \]

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta r + \Delta \phi_0 \]

\[ \Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \phi_0 = m \cdot (2\pi) \leftarrow \text{max, constr.} \]

\[ \Delta \phi = -\pi \rightarrow \leftarrow (m+\frac{1}{2}) \cdot (2\pi) \leftarrow \text{perfect destructive} \]

https://www.youtube.com/watch?v=luv6hY6zsd0
Antinodal lines, constructive interference, oscillation with maximum amplitude. Intensity is at its maximum value.

Nodal lines, destructive interference, no oscillation. Intensity is zero.
\[ A = 0 \]
\[ A = 2a \cos \left( \frac{\Delta \phi}{2} \right) \]
\[ \cos \frac{\Delta \phi}{2} = 0 \quad \Rightarrow \quad (m + \frac{1}{2}) 2\pi \text{ rad} = \Delta \phi \]

\[ A = A_{\text{max}} \quad \Rightarrow \quad \text{constructive interference (IF)} \]
\[ A = 0 \quad \Rightarrow \quad \text{destructive (IF)} \]

For \( \Delta \phi_0 = 0 \), \( \Rightarrow \) identical sources.
Application: Thin films

\[ \Delta \phi = 0 \]

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta x = m \cdot (2\pi) \text{ rad} , \quad m=0,1,2, \ldots \]

\[ \Delta x = m \cdot \lambda \]

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta x = \left( m + \frac{1}{2} \right) (2\pi) \text{ rad} \]

\[ \Rightarrow \Delta x = \left( m + \frac{1}{2} \right) \lambda \]

Maximum constructive IF

Perfect destructive IF
**Application: Thin films**

We desire perfect destruct. If

\[ \Delta \phi = \frac{2 \pi}{\lambda} \Delta x + \Delta \phi_0 = (m + \frac{1}{2}) 2 \pi \text{ rad} \]

\[ 2d = \Delta x = \left( m + \frac{1}{2} \right) \lambda_{\text{coat}} \]

\[ \Rightarrow d = \left( m + \frac{1}{2} \right) \frac{\lambda_{\text{coat}}}{2} = \left( m + \frac{1}{2} \right) \frac{\sqrt{n_1 n_2}}{2} \]
Example 21.10

**EXAMPLE 21.10  Two-dimensional interference between two loudspeakers**

Two loudspeakers in a plane are 2.0 m apart and in phase with each other. Both emit 700 Hz sound waves into a room where the speed of sound is 341 m/s. A listener stands 5.0 m in front of the loudspeakers and 2.0 m to one side of the center. Is the interference at this point maximum constructive, perfect destructive, or in between? How will the situation differ if the loudspeakers are out of phase?

**MODEL** The two speakers are sources of in-phase, spherical waves. The overlap of these waves causes interference.

\[
\begin{align*}
d &= 2.0 \text{ m} \\
\Delta \phi &= 0 \\
f &= 700 \text{ Hz} \\
v &= 341 \text{ m/s} \\
x_e &= 5.0 \text{ m} \\
y_e &= 2.0 \text{ m}
\end{align*}
\]
Example 21.10

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta \Gamma + \Delta \phi_0 = m \ (2\pi) \]

\[ \Gamma_1 = \Delta \Gamma = r_2 - r_1 = \sqrt{25+9} \ m - \sqrt{25+1} \ m = 0.76 \ m \]

\[ \nu = \frac{\lambda f}{c} \Rightarrow \lambda = \frac{\nu}{f} = 0.487 \ m \]

1st case:

\[ \frac{2\pi}{\lambda} \Delta \Gamma = m \ (2\pi) \Rightarrow \frac{\Delta \Gamma}{\lambda} = 1.5 \text{ destr.} \nu \]

2nd case:

\[ \frac{2\pi}{\lambda} \Delta \Gamma + \Delta \phi = m \ (2\pi) \Rightarrow \frac{\Delta \Gamma}{\lambda} + \frac{1}{2} = m \ \ m = \frac{2}{c} \text{ const.} \]
Example
Example
WAVE OPTICS
Difraction & interference

The light spreads out behind the slit.

Viewing screen

0.1-mm-wide slit in an opaque screen

Incident laser beam
Double-slit expt.

1. A plane wave is incident on the double slit.

2. Waves spread out behind each slit.

Top view of the double slit

3. The waves interfere in the region where they overlap.

4. Bright fringes occur where the antinodal lines intersect the viewing screen.

$m = 0$ Central maximum
$m = 1$
$m = 2$
$m = 3$
$m = 4$
Double-slit expt.
Double-slit expt.
Double-slit expt.
Double-slit expt.
A laboratory experiment produces a double-slit interference pattern on a screen. If the screen is moved farther away from the slits, the fringes will be

A. Closer together.
B. In the same positions.
C. Farther apart.
D. Fuzzy and out of focus.
Diffraction Grating
Diffraction Grating
Diffraction Grating
Reflection Grating

A reflection grating can be made by cutting parallel grooves in a mirror surface. These can be very precise, for scientific use, or mass produced in plastic.
Example 22.3

**EXAMPLE 22.3** Measuring wavelengths emitted by sodium atoms

**VISUALIZE** This is the situation shown in the figure. The two fringes are very close together, so we expect the wavelengths to be only slightly different. No other yellow fringes are mentioned, so we will assume these two fringes are the first-order diffraction (\(m = 1\)).
Example
Single-slit diffraction
Single-slit diffraction

Each point on the wave front is paired with another point distance $a/2$ away.

These wavelets all meet on the screen at angle $\theta$. Wavelet 2 travels distance $\Delta r_{12} = (a/2) \sin \theta$ farther than wavelet 1.
Circular aperture diffraction
Example 22.4
Example 22.4

**EXAMPLE 22.4** Diffraction of a laser through a slit

Light from a helium-neon laser ($\lambda = 633$ nm) passes through a narrow slit and is seen on a screen 2.0 m behind the slit. The first minimum in the diffraction pattern is 1.2 cm from the central maximum. How wide is the slit?

**MODEL** A narrow slit produces a single-slit diffraction pattern. A displacement of only 1.2 cm in a distance of 200 cm means that angle $\theta_1$ is certainly a small angle.
Ray vs Wave Model of Light
Acustical interferometer