PHYSICS 571 –
Master's of Science Teaching
“Electromagnetism and Light”

Lecture 3: Motors and Dipole moments

Instructor – Richard Sonnenfeld

mpsonnenfeld@gmail.com ← Homework and Questions
575-835-6434

Outline

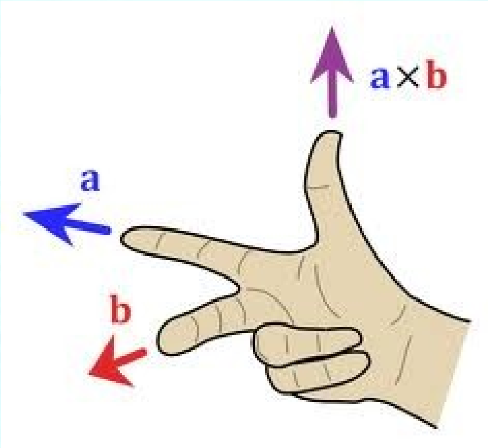
Cross Products

How motors work.

Dipole Moments

Two ways to do a cross product

I. Geometric



If $\vec{C} = \vec{A} \times \vec{B}$, then $C = A\,B \sin(\theta)$

Get direction of $\vec{C}$ from right-hand rule.

II. Algebraic

$$\begin{aligned}\vec{A} \times \vec{B} &= (A_y B_z - A_z B_y)\,\hat{\imath} \;+ \\ &\quad\; (A_z B_x - A_x B_z)\,\hat{\jmath} \;+ \\ &\quad\; (A_x B_y - A_y B_x)\,\hat{k}\end{aligned}$$

Algorithm for cross product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\begin{aligned} \vec{A} \times \vec{B} = & (A_y B_z - A_z B_y)\,\hat{\imath} + \\ & (A_z B_x - A_x B_z)\,\hat{\jmath} + \\ & (A_x B_y - A_y B_x)\,\hat{k} \end{aligned}$$

Algorithm for cross product

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 5 & 7 & -4 \end{matrix}$$

$$\begin{aligned} \vec{A} \times \vec{B} \;=\; & (A_y B_z - A_z B_y)\,\hat{\imath} \;+ \\ & (A_z B_x - A_x B_z)\,\hat{\jmath} \;+ \\ & (A_x B_y - A_y B_x)\,\hat{k} \end{aligned}$$

Algorithm for cross product

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 5 & 7 & -4 \end{bmatrix}$$

$$= \hat{i}\,[(-1)\times(-4) - (7)\times(3)]$$
$$- \hat{j}\,[(2)\times(-4) - (5)\times(3)]$$
$$+ \hat{k}\,[(2)\times(7) - (5)\times(-1)]$$
$$= \hat{i}\,[4 - 21] - \hat{j}\,[-8 - 15] + \hat{k}\,[14 + 5]$$
$$= -17\hat{i} + 23\hat{j} + 19\hat{k}$$

wikiHow

$$\begin{aligned} \vec{A} \times \vec{B} \;=\; & (A_y B_z - A_z B_y)\,\hat{\imath} \;+ \\ & (A_z B_x - A_x B_z)\,\hat{\jmath} \;+ \\ & (A_x B_y - A_y B_x)\,\hat{k} \end{aligned}$$

Worked problem

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Given: $q = 0.20\ \text{mC}$

$\vec{v} = 1.5\,\hat{i} + 3\,\hat{k}\ \text{m/s}$

$\vec{B} = 0.1\,\hat{i} - 0.2\,\hat{j} - 0.3\,\hat{k}\ \text{T}$

$\vec{F} = ?\ \text{N}$

$$\begin{aligned} \vec{A} \times \vec{B} \;=\; & (A_y B_z - A_z B_y)\,\hat{\imath} \;+ \\ & (A_z B_x - A_x B_z)\,\hat{\jmath} \;+ \\ & (A_x B_y - A_y B_x)\,\hat{k} \end{aligned}$$

Worked problem

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

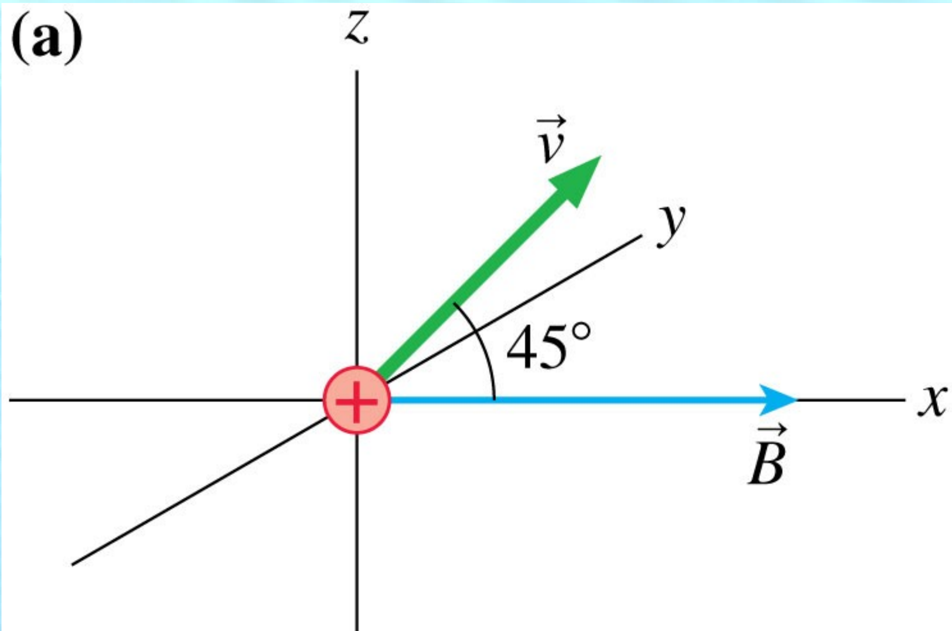
Given:

$\vec{B} = 0.5\,\hat{i}$ T

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\,\hat{\imath} + (A_z B_x - A_x B_z)\,\hat{\jmath} + (A_x B_y - A_y B_x)\,\hat{k}$$

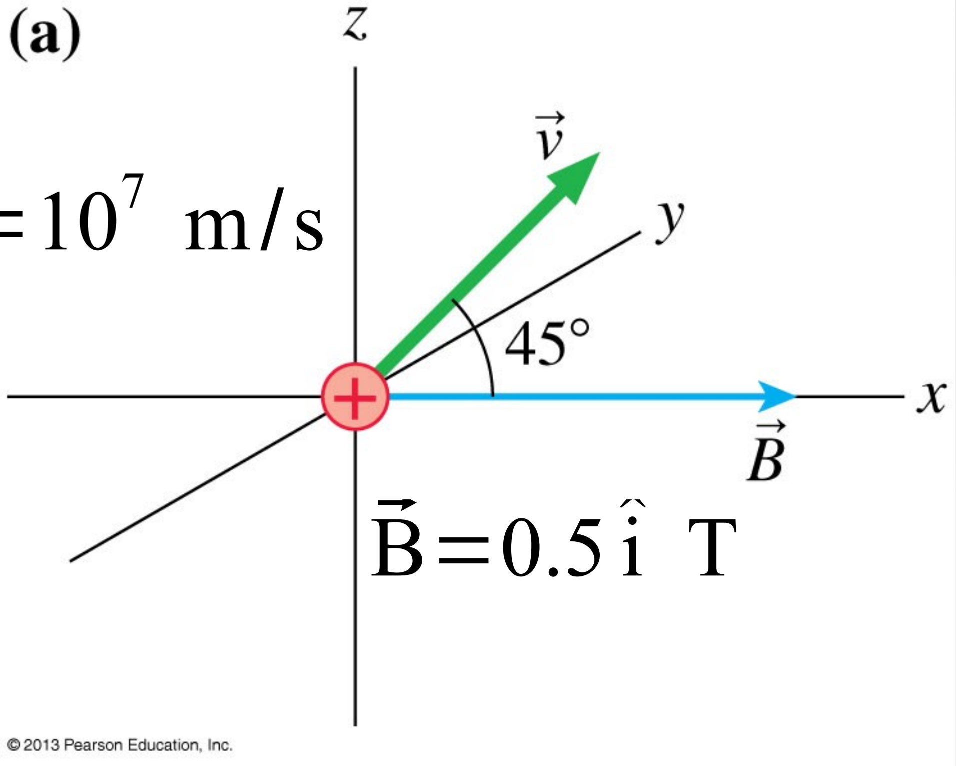
A proton moves at 10 million m/s as indicated in the figure.

$\vec{F} = ?$ N



Worked problem (do it both ways)

$$\vec{F} = q\,\vec{v} \times \vec{B}$$



$v = 10^7$ m/s

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Cross Products

How motors work.

Dipole Moments

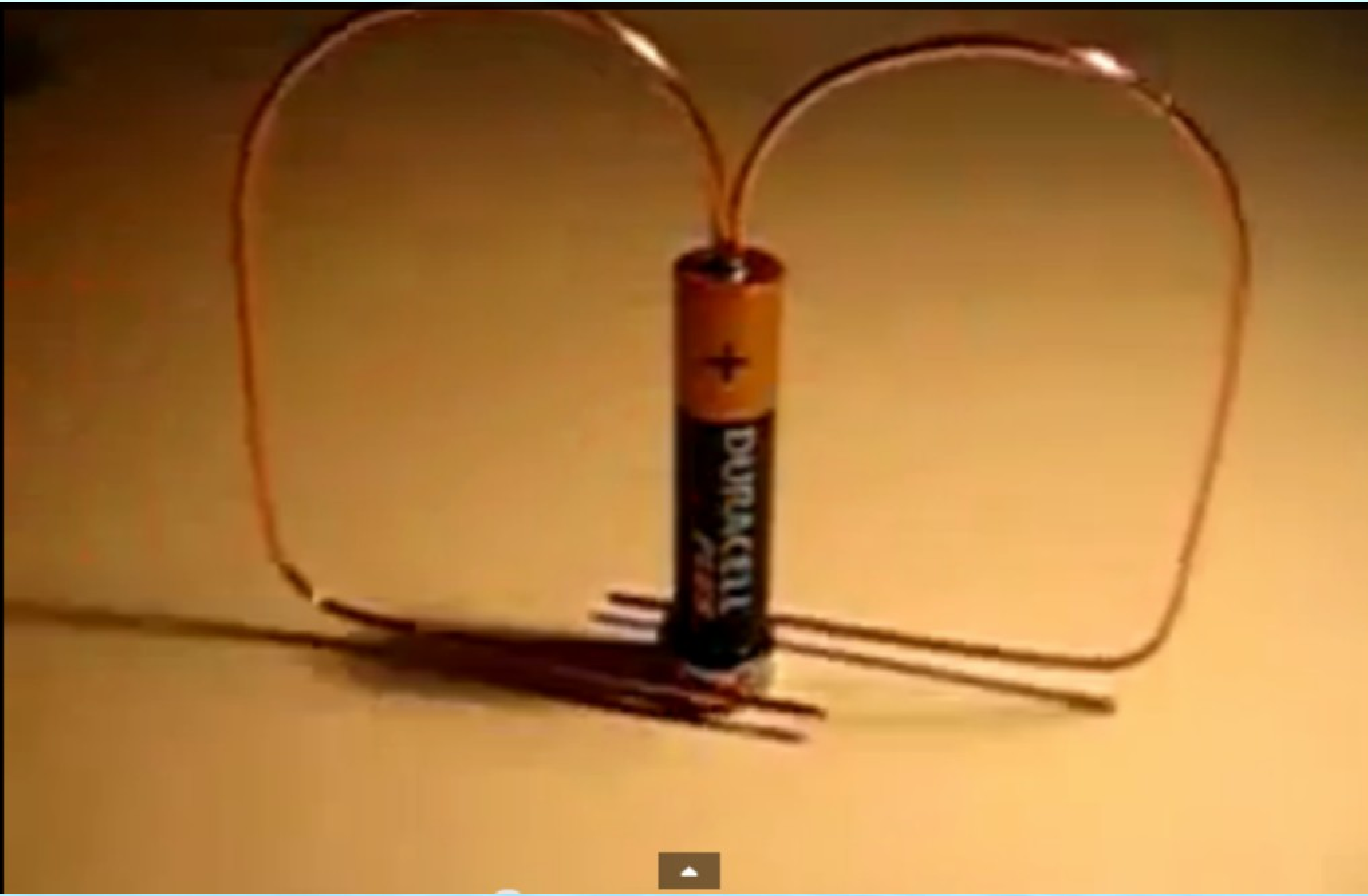
DC Electric Motor

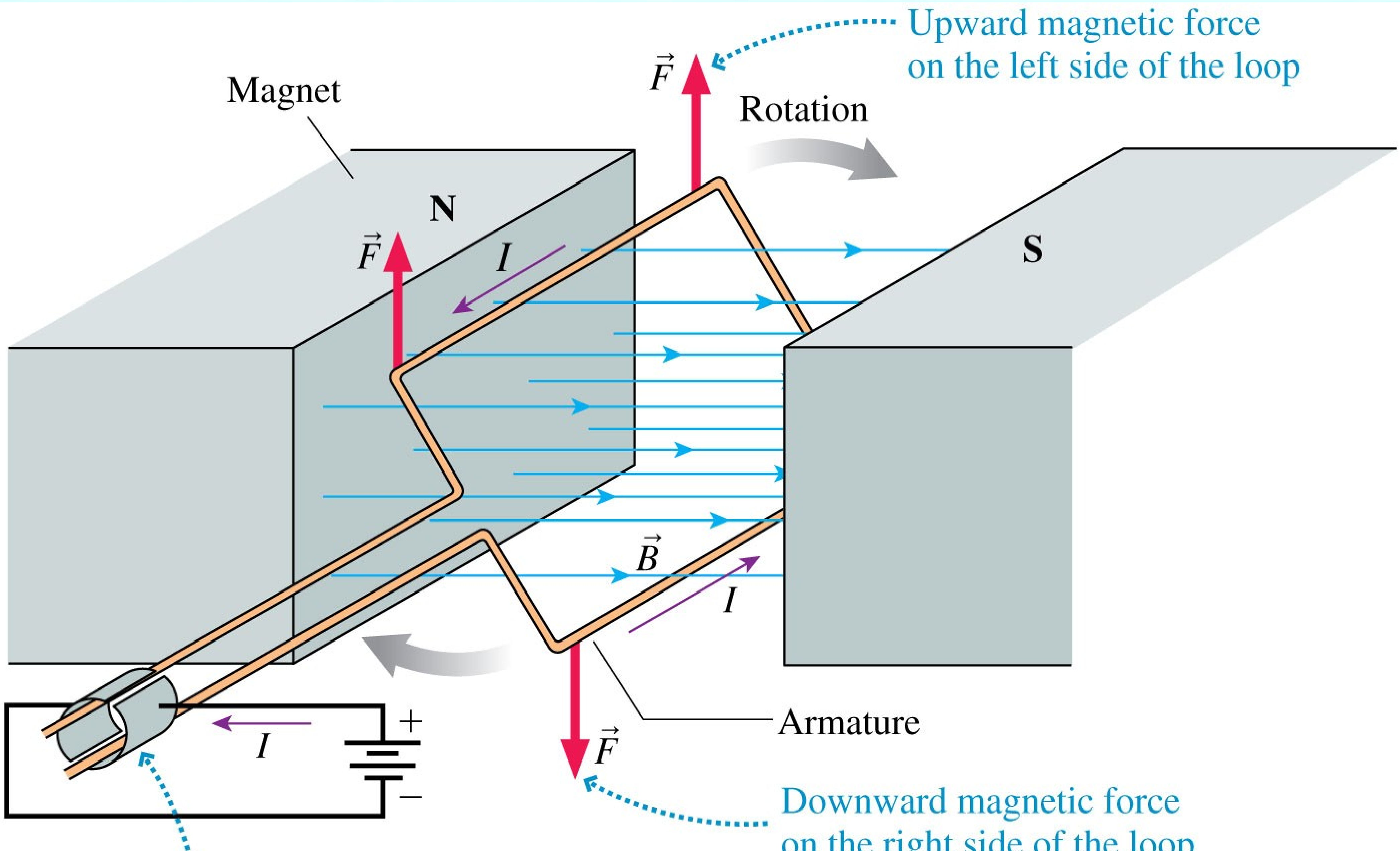
http://www.youtube.com/watch?v=FjNnRyLexNM
Http://tinyurl.com/DCMotor2

Homopolar motor *"The simplest motor of the world"*

http://www.youtube.com/watch?v=zOdboRYf1hM&NR=1
Tinyurl.com/HomoMotor1

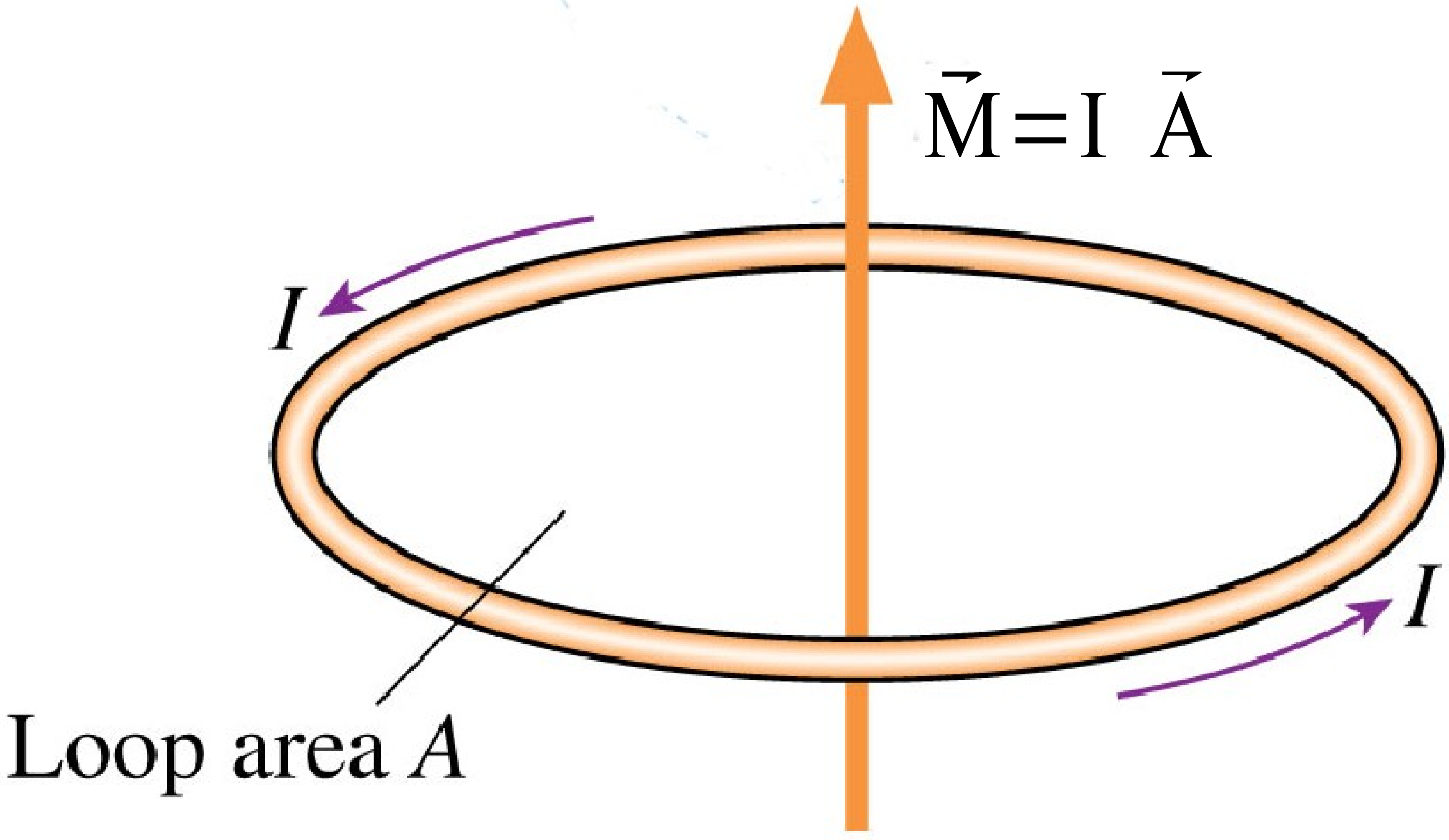
Homopolar Motor





The commutator reverses the current in the loop every half cycle so that the force is always upward on the left side of the loop.

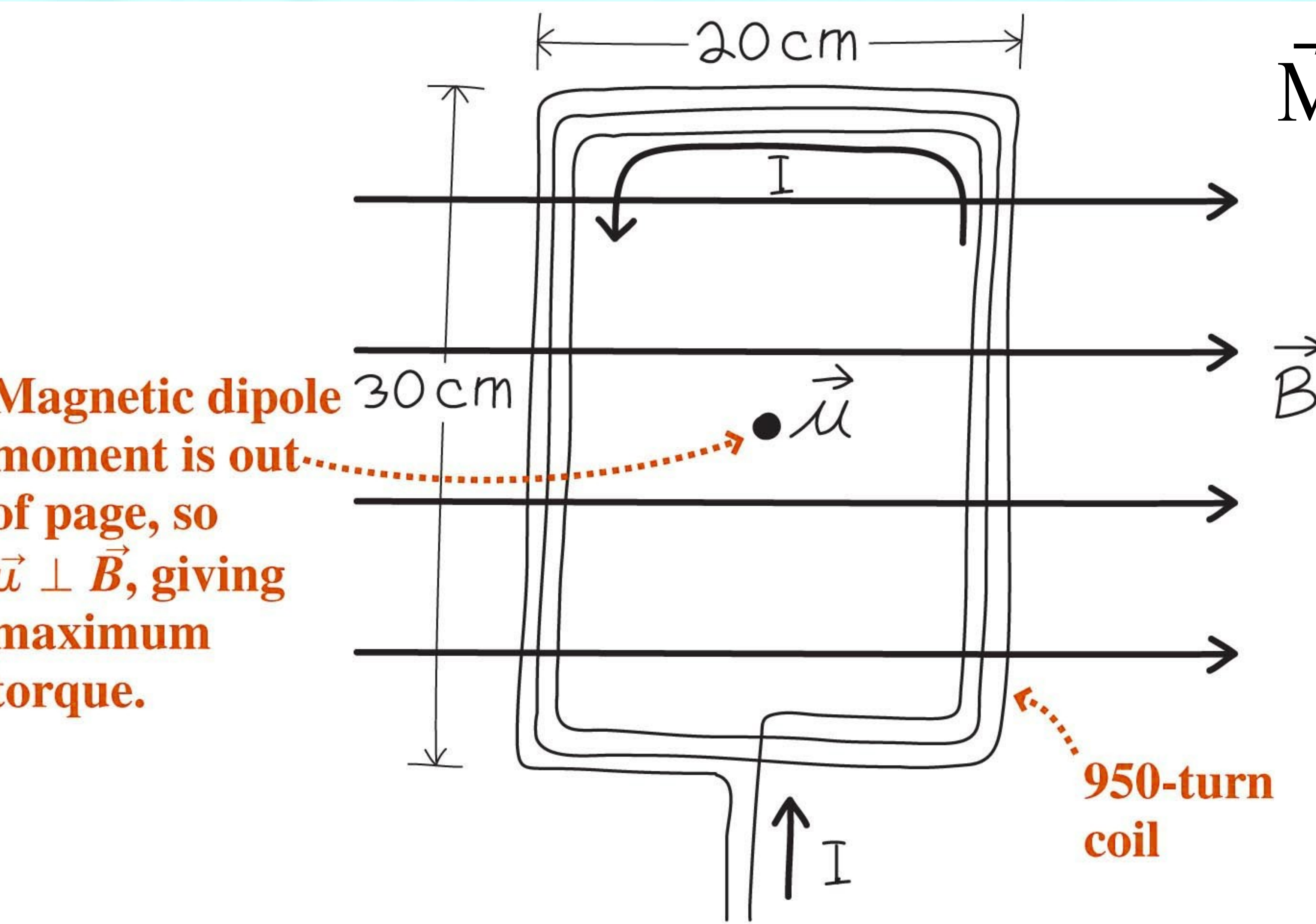
The magnetic dipole moment is perpendicular to the loop, in the direction of the right-hand rule.



Torque on a current loop is proportional to magnetic moment.

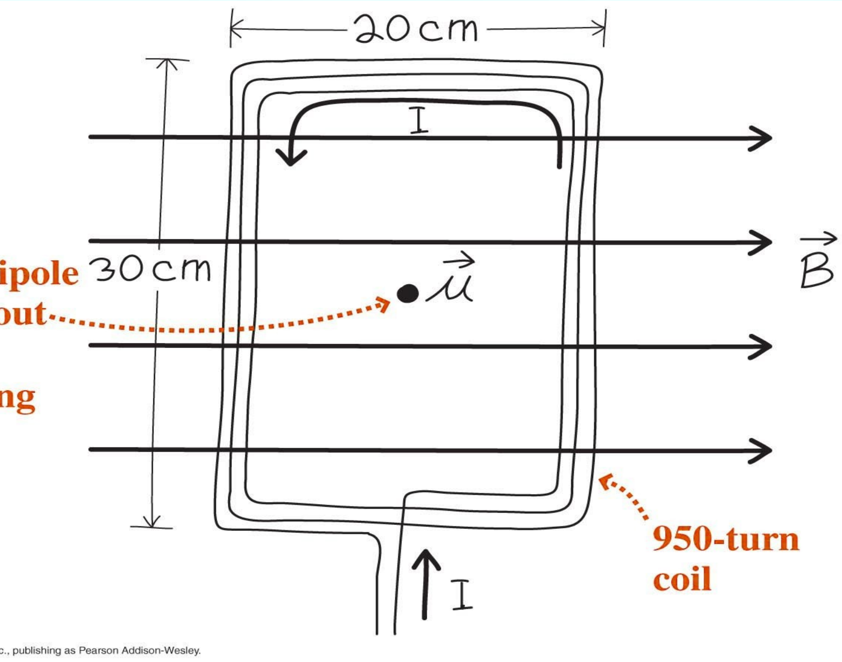
$$\vec{\tau} = \vec{M} \times \vec{B}$$

$$\vec{M} = I\ \vec{A}$$



Calculate the torque in an 0.2 T B-field

$$\vec{M} = I\ \vec{A} \qquad \vec{\tau} = \vec{M} \times \vec{B}$$



An electric motor is made from 100 turns of Copper wire in a square 10 cm on a side.
It is placed in a 10 milliTesla magnetic field.
What is the maximum force on one side of the square with a 100 amp current?

(A) 0.001 N
(B) 0.01 N
(C) 0.1 N
(D) 1 N
(E) 10 N

An electric motor is made from 100 turns of Copper wire in a square 10 cm on a side.
It is placed in a 10 milliTesla magnetic field.
What is the maximum torque with a 100 amp current?

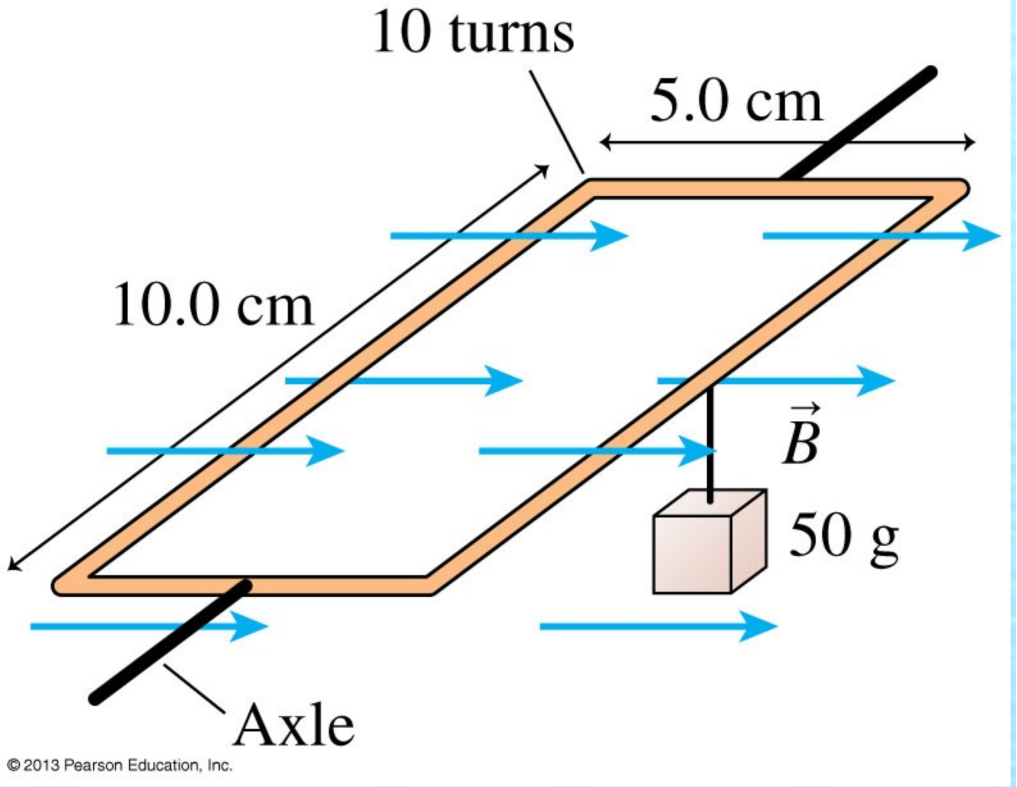
(A) 0.001 N-m
(B) 0.01 N-m
(C) 0.1 N-m
(D) 1 N-m
(E) 10 N-m

$$\vec{\tau} = \vec{M} \times \vec{B}$$

$$\vec{M} = I \vec{A}$$

Worked problem

What current is needed to balance the torque of the 50 gram weight?



Outline

Cross Products
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Dipole Moments

Why is magnetic dipole moment a useful concept?

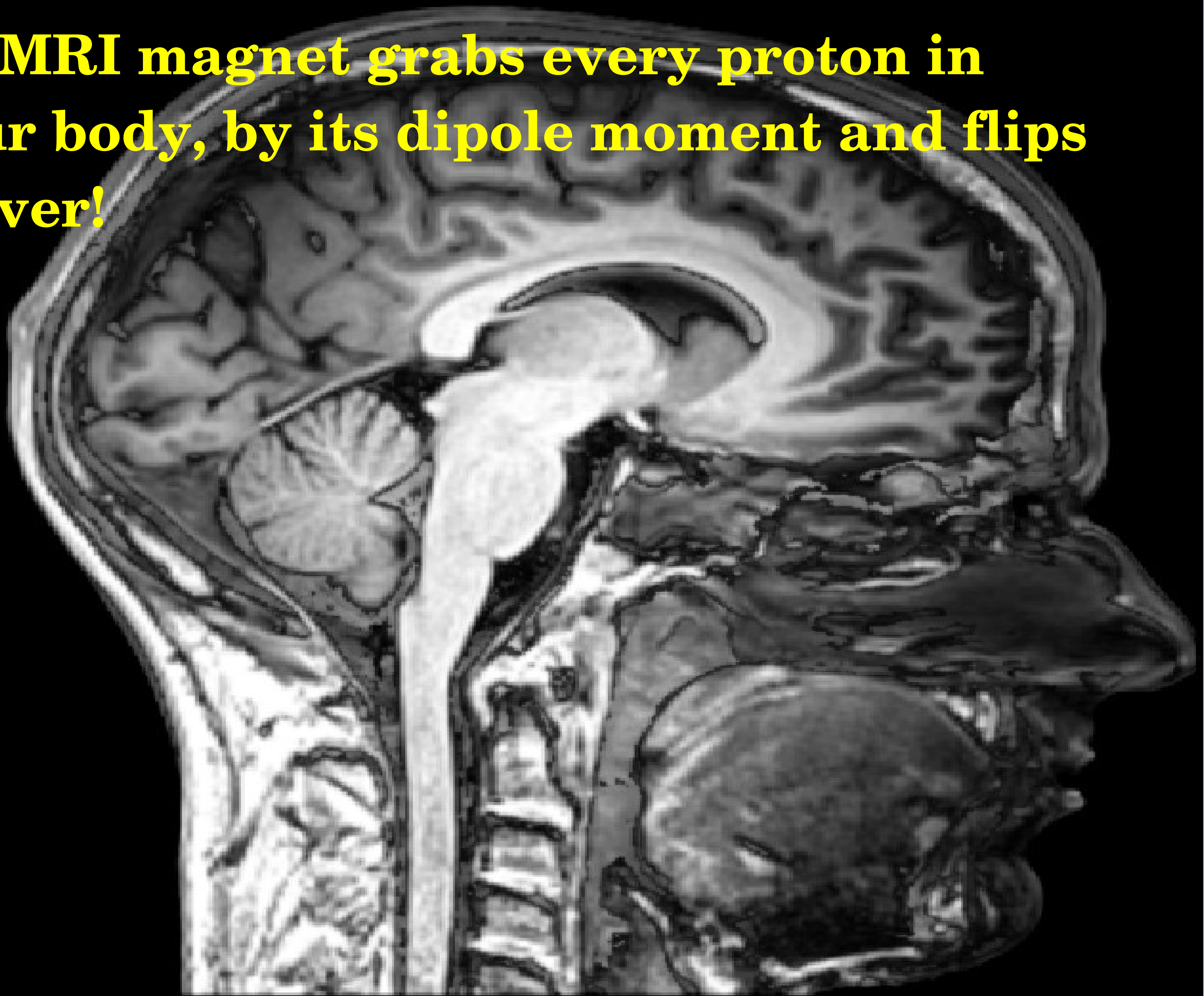
It's good for understanding motors.

It can be used to quantify the strength of a magnet.

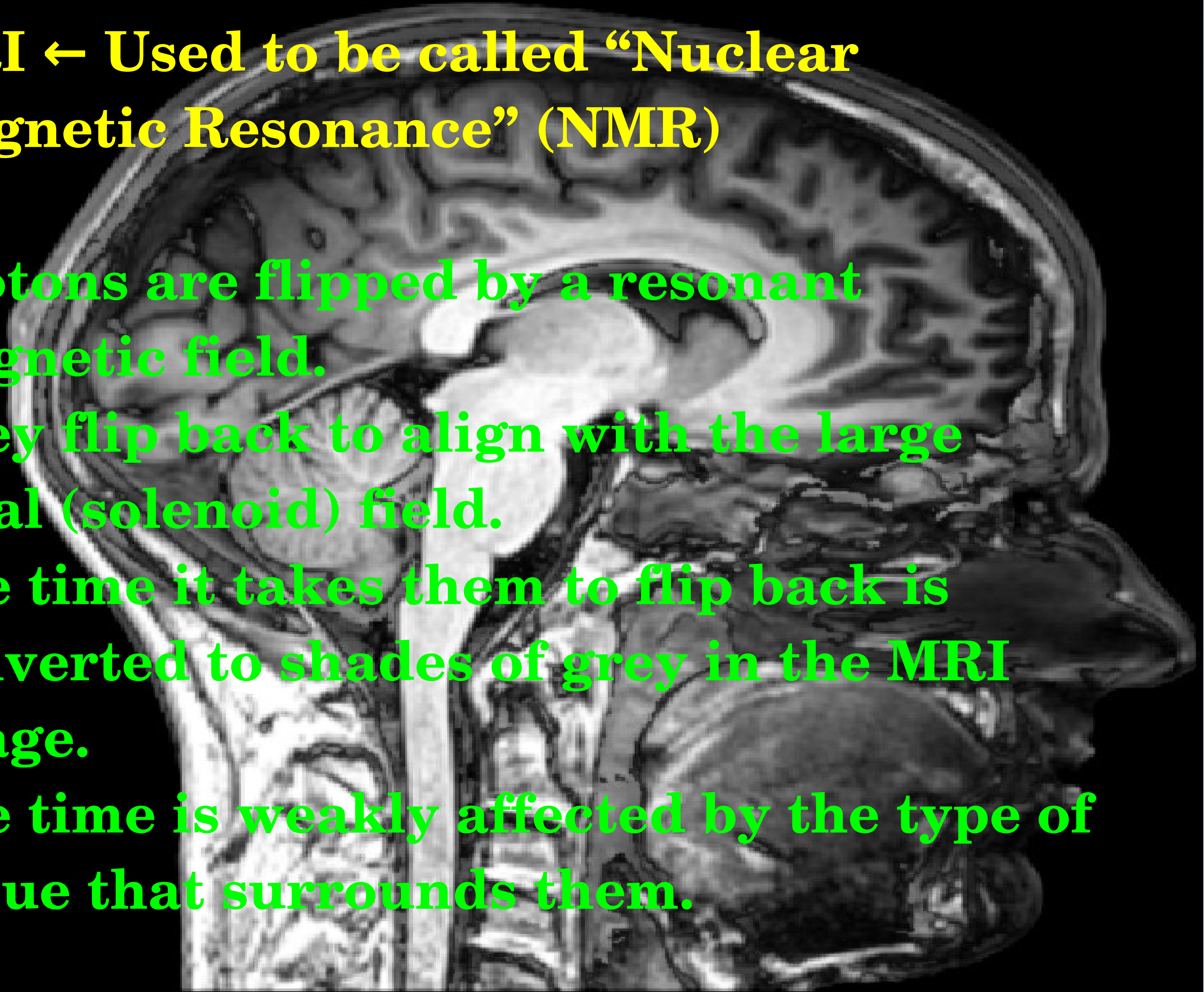
Protons and electrons have dipole moments.

This is particularly surprising since electrons have zero size. Even at the quantum level, particles act like spinning balls of charge!

An MRI magnet grabs every proton in your body, by its dipole moment and flips it over!

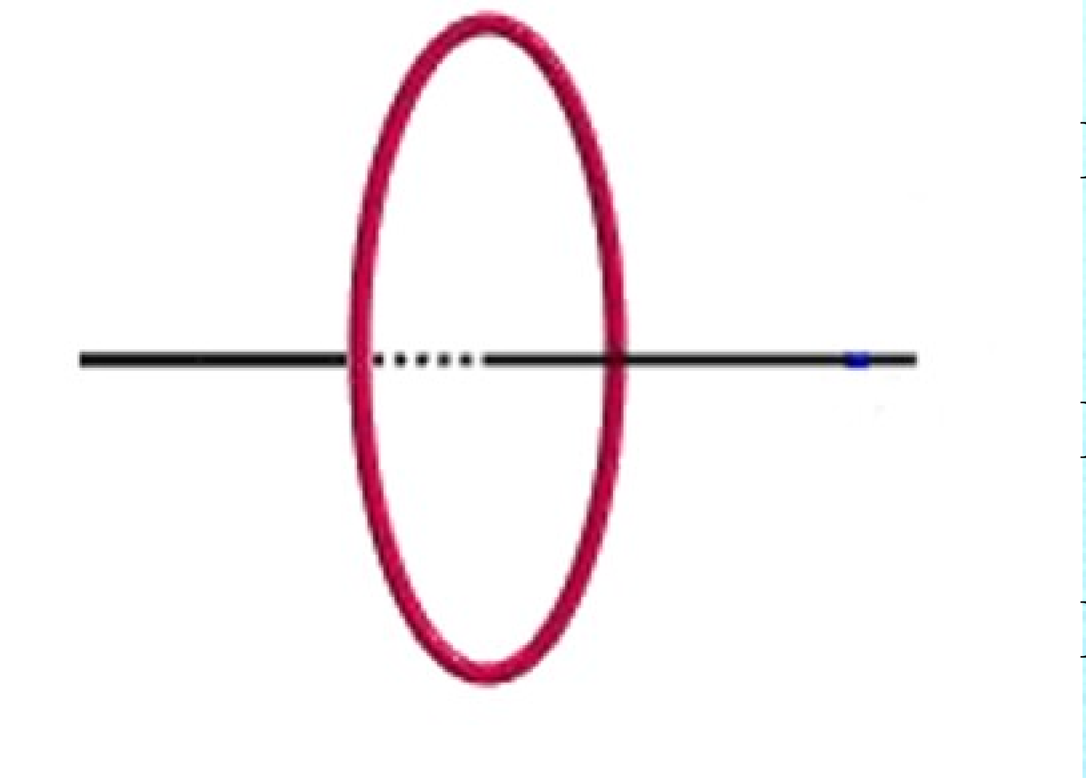


MRI ← Used to be called "Nuclear Magnetic Resonance" (NMR)



Protons are flipped by a resonant magnetic field.
They flip back to align with the large axial (solenoid) field.
The time it takes them to flip back is converted to shades of grey in the MRI image.
The time is weakly affected by the type of tissue that surrounds them.

Magnetic Dipole Moment and Magnet Strength



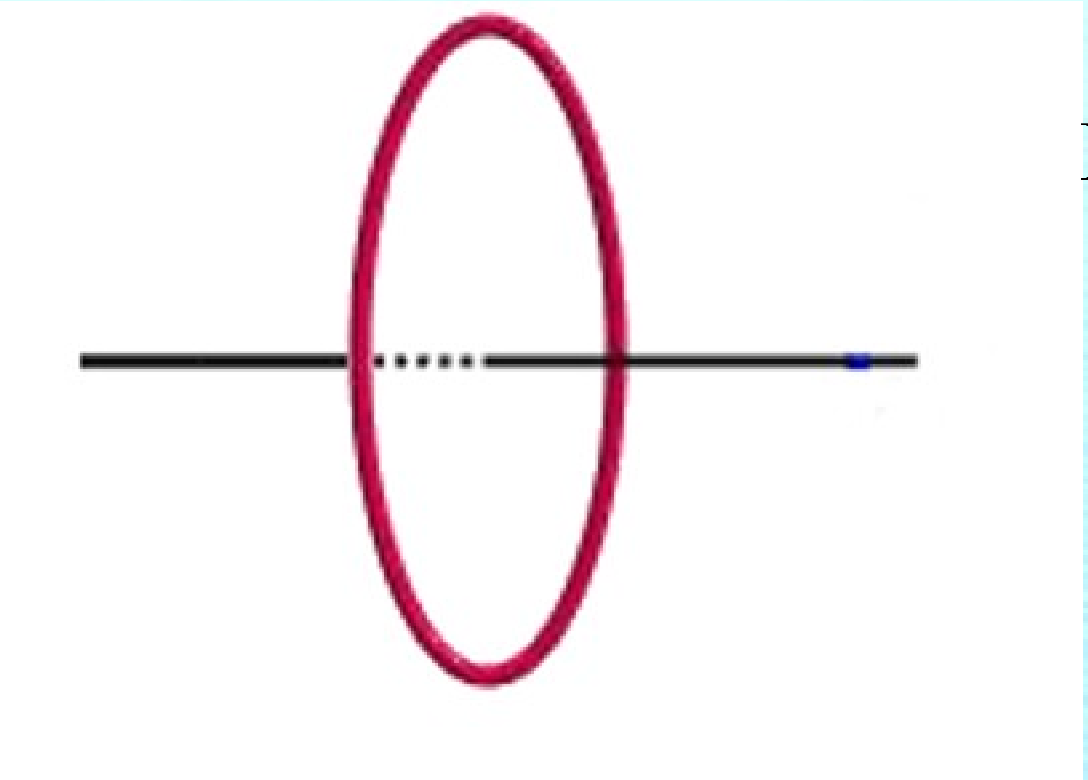
$$B_{loop} = \frac{\mu_0}{2} \frac{IR^2}{\left(x^2 + R^2\right)^{(3/2)}}$$

$$B\left(x=0\right) = \frac{\mu_0}{2} \frac{IR^2}{\left(0 + R^2\right)^{(3/2)}}$$

$$B\left(0\right) = \frac{\mu_0}{2} \frac{I}{R}$$

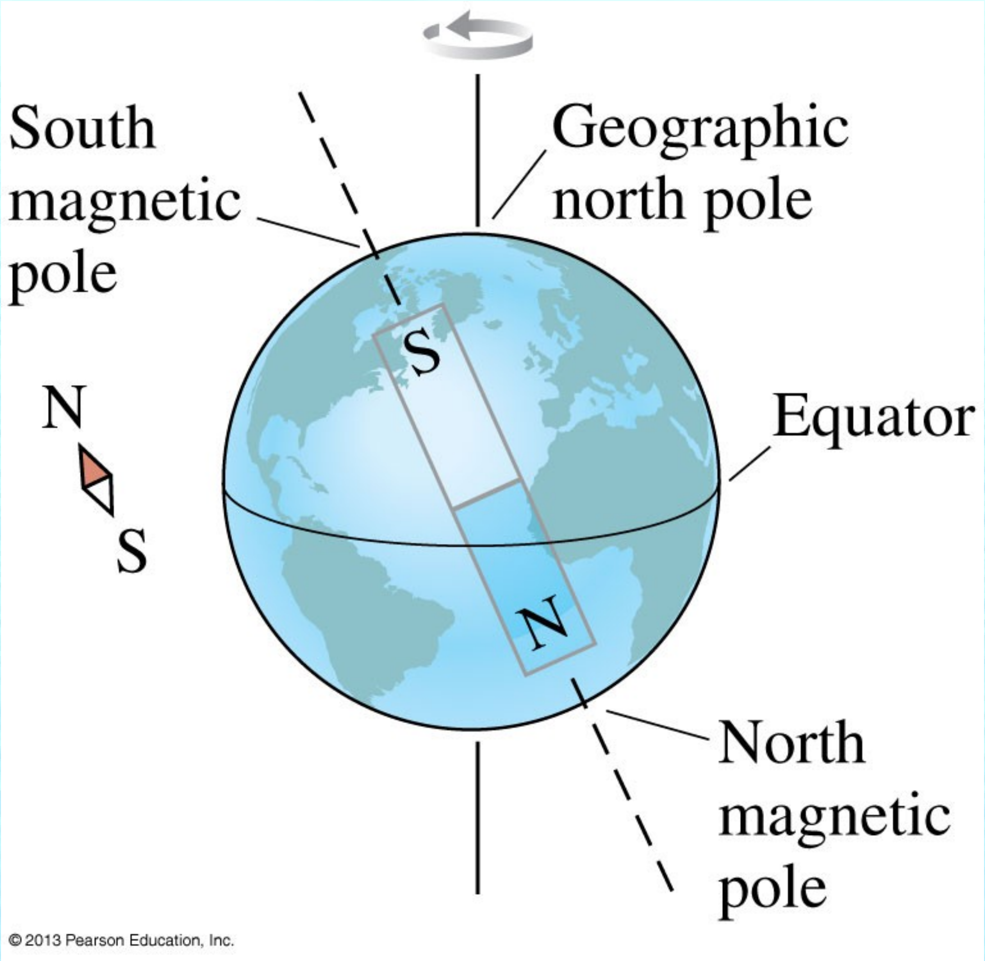
$$B\left(x \gg R\right) = \frac{\mu_0}{2} \frac{IR^2}{\left(x^2 + 0\right)^{(3/2)}} = \frac{\mu_0}{2\pi} \frac{I\pi R^2}{x^3} = \frac{\mu_0}{2\pi} \frac{IA}{x^3} = \frac{\mu_0}{2\pi} \frac{M}{x^3}$$

Magnetic Dipole Moment and Magnet Strength



$$B_{loop} = \frac{\mu_0}{2} \frac{IR^2}{\left(x^2 + R^2\right)^{(3/2)}}$$

What is the dipole moment of the Earth?



How large is the current flowing inside?

What have we learned?

Another way to calculate cross product
How to calculate a dipole moment
How to calculate field from moment
How to calculate motor torque