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A Contribution to the Electrostatic Theory of a Lightning Discharge

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Abstract. The electrostatic treatment of the field and charge distribution of a lightning discharge leads to the result that the charge distribution of a lightning stroke is composed of (1) the influence charges induced by the electric field of the thundercloud with the net charge zero, and (2) the net charge, which results from the potential difference of the lightning stroke and the ground before the lightning hits the ground. The first kind of charge distribution is that of a cloud discharge and that of the first leader of a ground discharge. The second kind of charge distribution is a feature of the main or return stroke of a ground discharge only. It is the charge distribution of a charged body and independent of the field distribution in the thundercloud. We can therefore apply to the return stroke the well-known electrodynamic theory of transients on a transmission line.

Introduction

The lightning discharge is an electrodeless spark, which grows in the electric field of the thundercloud. This electrostatic field of the cloud furnishes the energy and determines, in accordance with the laws of the electrodynamic theory and gaseous discharges, the development of the lightning flash. The relationships among the numerous physical elements involved are usually given by differential equations. In a mathematical sense, then, the theory of the lightning discharge would be the solution of a set of differential equations by certain electrostatic initial or boundary conditions. These boundary conditions would be given by the potential function ϕ_1 of the thundercloud for the time $t = 0$ just before the lightning starts, and the potential function ϕ for the time $t = T$ when the lightning stroke is completed.

The potential function ϕ can be obtained by the superposition of the original potential function ϕ_1 of the charge distribution in the thundercloud and a secondary potential function ϕ_2 of the charge distribution at the lightning channel. ϕ_2 can be calculated from ϕ_1 and the geometric form of the lightning channel by treating the lightning as a conductor in an electrostatic field.

We know, from photographic investigations with the Boys camera, that especially the lightning to the ground consists of different phases such as the stepped leader and the main or return stroke. In a multiple ground stroke this sequence may be repeated several times. As these phases, even the single steps of the stepped leader, are separated from one another by a rest period, we may be justified (at least as an approximation) in framing each phase by electrostatic boundary conditions.

It is the purpose of this paper to discuss these electrostatic boundary conditions in greater detail and to see how we can improve the existing conception of the lightning discharge. We will find that the application of the electrodynamic theory of traveling waves on transmission lines to the main stroke of the lightning discharge is justified.

Bewley [1951] has presented a fine treatment of this electrodynamic part of the problem. The present author's concept of the events in the cloud and of the leader stroke is different from Bewley's, but it is, with some modification, the same for the main stroke. Therefore the electrodynamic treatment of the main stroke will be referred to Bewley's book.

Another very interesting electrodynamic theory of the ground stroke is given by *Hill* [1957]. Even though this theory reflects very much the present concept of the lightning stroke, it deviates considerably in the treatment of the cloud as a conductor and in the charge distribution along the lightning channel from the treatment in the present paper. In opposition to Bewley's assumption of reflected waves, Hill considers the main stroke an aperiodic discharge. Finally, the calculation of the electrostatic boundary condition presented here is based on a paper by *Kasemir* [1950], read at a conference on thunderstorm electricity in 1948 at Buchan, Germany; an extract was published in Israel's *Das Gewitter,* to which the reader is referred for a more detailed discussion of the electrostatic aspect of the lightning discharge.

Discussion

The charge distribution on a conducting spheroid in an arbitary, given electric field. The development of any theory usually involves considerable mathematical calculation, which is the most laborious part of the theory. Since it is a hindrance to have the physical discussion of the results interrupted by lengthy mathematical calculations, they are not included here.

The lightning is represented by a prolonged spheroid, which is placed in the electric field of the thundercloud. The problem is assumed to be of rotational symmetry, whereby the spheroid lies in the rotational axis. The electric field of the thunderstorm is given by its potential function ϕ_1.

For electrostatic considerations, the lightning stroke is a conductor. On its surface an influence charge distribution will form which generates a secondary potential function ϕ_2 of such a kind that the superposition of ϕ_1 and ϕ_2 results in a constant potential ϕ_L at the surface of the lightning stroke. The value of ϕ_2 is determined for a cloud stroke by the condition that the net charge of the stroke is zero. This condition is usually neglected in present theories, but it follows from the fact that the mobility of cloud-charge particles is far too low for them to take an active part in the lightning discharge. The charge movement in the lightning channel is supported only by free electrons produced by the ionization of collision in the channel itself. They move from one end of the lightning stroke, leaving behind a surplus positive charge, to the other end, where they form a surplus negative charge. In this way they generate the charge distribution on the channel which causes the potential function ϕ_2. For the ground stroke we do not have the condition that the net charge of the lightning channel has to be zero. In the electrostatic sense, the earth is a conductor of great capacity, and the lightning can draw any necessary charge from it. After contact with the earth the lightning will assume earth potential.

Rather than discuss the potential function ϕ_2, which is generated by the charge distribution on the lightning channel, we will deal with the charge distribution itself. With the help of several graphs we will start from simple cases and advance to the complete picture. In the graphs the lightning is always represented by a heavy vertical line, and the charge distribution on it is shown by the shaded areas. The potential function ϕ_1 of the given electric field is presented by a solid line ϕ_1. We will see that the charge distribution is proportional to ϕ_1, but of opposite polarity. Therefore, with a convenient scale factor, the envelope of the shaded areas appears to be the mirror image of the potential function ϕ_1 with respect to the lightning channel. To obtain this simple relationship, the potential ϕ_L of the lightning stroke has to be chosen as the reference potential, which means that it is convenient to set $\phi_L = 0$. From $\phi_L = \phi_1 + \phi_2 = 0$ it follows that $\phi_1 = -\phi_2$. The potential function ϕ_2 at the surface of the lightning, which is generated by the influence charge, is of the same amplitude as, but of opposite polarity from, the potential function ϕ_1 generated by the cloud charges.

In (a) of Figure 1 we see the charge distribution of a charged spheroid. The net charge is equally distributed along the channel. The charge per unit length is constant. This is also the charge distribution on a charged transmission line and, as we will see later, on the main stroke of a lightning flash to ground. The potential function ϕ_2 at the surface of the spheroid is constant, and therefore $\phi_1 = \phi_2$ is also a constant; see Figure 1(a). Figure 1(b) shows the charge distribution on the spheroid in a constant field. The charge at the mid point of the spheroid is zero and increases linearly by progressing to the upper end of the spheroid. It decreases linearly with the progress to the lower end of the spheroid. In this way we have the positive influence charge on the upper half and the negative influence charge on the lower half of the spheroid. The net charge is zero.

If the stroke starts from the base of the cloud and advances with its lower end to the ground

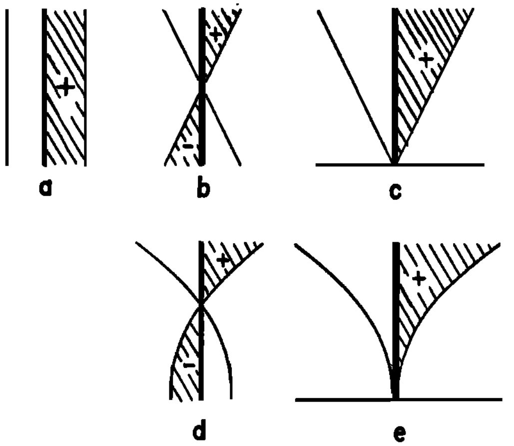

Fig. 1. Charge distribution: (*a*) on a charged spheroid; (*b*) on an uncharged spheroid in a homogeneous electric field; (*c*) on a charged spheroid in a homogeneous electric field; (*d*) on an uncharged spheroid in an inhomogeneous electric field; (*e*) on a charged spheroid in an inhomogeneous electric field.

and its upper end into the cloud, (*b*) represents the charge distribution on the leader stroke just before it makes contact with the ground. After the contact with the ground, the lightning has assumed ground potential, but the charge distribution along its channel is again proportional to ϕ_1. Because ϕ_1 at the ground is zero, the charge density at ground point of the channel also has to be zero. The new charge distribution is shown in (*c*). We can construct this charge distribution of (*c*) by superimposing that of (*a*) to that of (*b*). Hereby, (*a*) would give the charge distribution deposited on the channel by the main stroke. Before advancing to the lightning stroke in an inhomogeneous field, we should like to point out the difference between this concept and the present model of a lightning stroke.

In Figure 2, (*a*) shows qualitatively, in the first four successive pictures, how the leader stroke advances to the ground with the deposit of negative charge along its channel [*Schonland,* 1938]. This charge is delivered by the negative space charge center of the cloud. As soon as the leader reaches the ground the negative charge is discharged into it.

In the mathematical treatment [*Hill,* 1957], the cloud base and the earth are represented by parallel conducting planes and the leader stroke by the lower half of a conducting spheroid with its midpoint at the cloud base. In Figure 2, (*b*) and (*c*) show the charge distribution before and after contact with the ground, according to Hill. The crucial point here is the assumption of the cloud base as a conductor. We know, from conductivity measurements in clouds, that its conductivity is even less than that of clear air, i.e., the cloud is indeed a very good insulator. To overcome this discrepancy, a widespread hypothetical streamer process inside the cloud is assumed to enable the cloud charge to feed the leader strokes of the successive partial discharges of a ground flash. It may be of interest to cite here a comment from the Russian literature. The following is a somewhat free translation from one of the publications of *Trevskoi* [1954].

We cannot regard a cloud as a conductor. The conductivity of air inside the cloud is close to zero because the ions are attached to the cloud particles. For this reason, the concept that by multiple strokes the charges required for the following stroke flow to the exit area from other cloud areas must be admitted as not corresponding to reality. Of course it could be assumed that the lightning inside the cloud is so highly branched that its ramification permits a conducting connection among the charged particles, but such an assumption is highly improbable.

From a mathematical point of view, there is not much difference between the representation of the leader stroke [Figs. 1(*b*) and 2(*c*)]. We can always replace the conducting cloud base of Figure 2(*b*) by the mirror image on it and arrive at Figure 1(*b*). In this way we can avoid the introduction of the cloud as a conductor. But the transition from the leader to the main stroke is completely different in the two concepts, as can be seen from Figures 2(*c*) and 1(*c*). In Figure 1(*c*) there is a strong influence charge deposited along the lightning channel. In Figure 2(*c*) the charge of the channel is zero. It is remarkable that *Bruce and Golde* [1941] completed the qualitative pictures of Schonland in such a way as to give the main stroke a positive charge. This is shown in the last four pictures of Figure 2(*a*). This concept would come closer to the theory presented here. We will now return to the charge distribution of a spheroid in an inhomogenous field.

In (*d*) of Figure 1 is shown the charge dis-

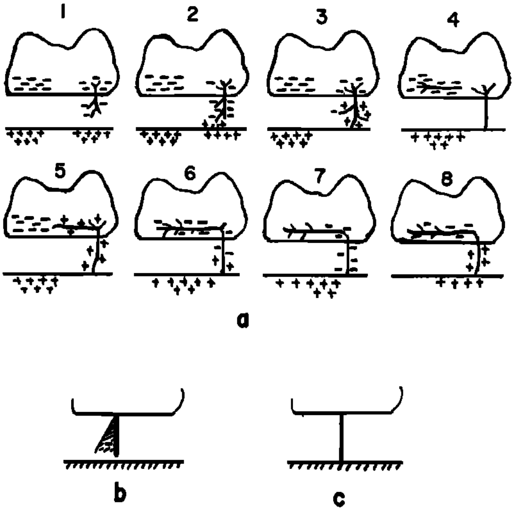

Fig. 2. (*a*) Diagram illustrating leader and main strokes to the first and subsequent strokes in a sequence of eight pictures after *Schonland* [1938] and *Bruce and Golde* [1941]. (*b*) Charge distribution of the leader stroke according to the theory of E. L. Hill. (*c*) Charge zero on the completed main stroke according to the theory of E. L. Hill.

tribution on a spheroid for an inhomogeneous field of the potential function ϕ_1. Again we see that the charge distribution is given by the mirror image of the potential curve, but here this is true only as an approximation. The error becomes small for very slim spheroids, and, because the lightning stroke is very long indeed compared with its diameter, the approximate mirror-image relationship between the potential function ϕ_1 and the charge distribution is more justified than the representation of a real lightning stroke with its many bends and branches by a prolonged spheroid. The net charge is again zero. In Figure 1, (*d*), analogous to (*b*), would represent the charge distribution of a cloud stroke in an inhomogeneous field. To obtain the charge distribution of a ground stroke we have to superimpose again a uniform charge along the channel as shown in (*a*), so that the charge density and the potential of the contact point with ground is zero. This will result in a charge distribution as shown in (*e*). The laws of charging a long transmission line to a constant potential, therefore, hold also for the main stroke of lightning in an inhomogeneous field.

The charge distribution on a lightning stroke in a thundercloud. In the preceding paragraph we discussed the charge distribution for an arbitrary potential function ϕ_1. We will now apply the results to the potential function ϕ_1 of a thundercloud, basing our discussion on the potential function of the old Simpson-Wilson thunderstorm model. This consists of three spheres vertically arranged, one at the top of the other and filled with homogeneously distributed space charge [Fig. 3(*a*)] [*Kasemir*, 1950]. The top sphere, with positive charge, represents the main positive space charge in the top of the thundercloud; the middle sphere, with negative charge, represents the main negative space charge center in the lower part of the cloud; and the lowest sphere represents the positive charge pocket at the base of the cloud. Figure 3(*b*) shows the potential function ϕ_1 at the central axis *z* resulting from such an arrangement. The heavy vertical line represents a cloud stroke. The shaded areas again give the charged distribution on the lightning channel. With the right scale factor, the envelope of these areas is again the mirror image of the potential curve ϕ_1 with

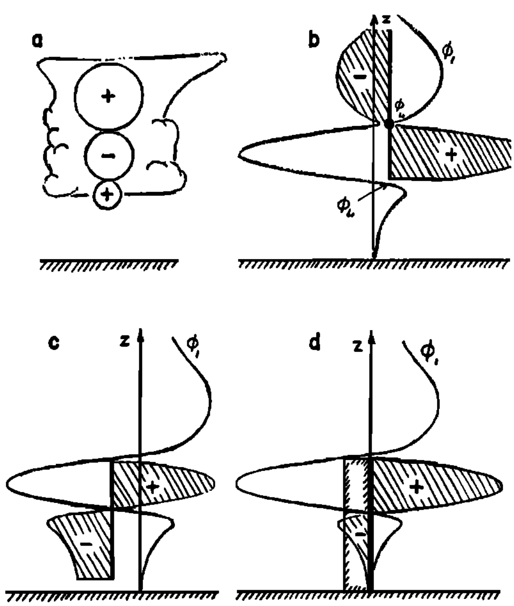

Fig. 3. Charge distribution: (*a*) in the thunderstorm model of Simpson Wilson; (*b*) on an intracloud stroke; (*c*) on a leader stroke of a cloud to ground discharge; (*d*) on the ground discharge after completion of the main stroke.

respect to the lightning channel. The lightning itself has the potential ϕ_L, which is given by the crosspoint of the vertical line and the potential curve ϕ_1. The cloud stroke may start from this point marked by a black circle in Figure 3(*b*), and grow to both sides until it crosses the potential curve ϕ_1 again. At this point it will stop, because there is no potential difference between the tips of the stroke and the surrounding air. Consequently, the electric field at the ends of the lightning stroke will drop to zero. The lightning will stop growing even before these points are reached, namely, when the field strength at the tips drops below the breakdown field strength necessary for ionization. This and many other features are pointed out by *Kasemir* [1950] and will not be repeated here, but we may mention that, at the upper end of a cloud stroke which reaches into the positive space charge center of the cloud, negative influence charge is accumulated, and positive influence charge is accumulated at the lower end of the cloud. This gives the cloud stroke a negative polarity. The opposite is true for the ground stroke, as we can see from (*c*) and (*d*) in Figure 3. Positive influence charge at the upper end and negative at the lower end gives the ground stroke positive polarity. This polarity rule holds for the vast majority of lightning strokes, according to the experimental data from several workers in this field: for example, *Pierce* [1955], *Workman and Brook* [1958], and *Kasemir* [1956].

Figure 3(*c*) shows the charge distribution of the leader stroke just before the contact with the ground is made, and (*d*) shows that of the main stroke. The difference between these charge distributions, indicated by the hatched rectangle in (*d*), is the contribution of the main stroke. The distribution of the influence charge deposited by the leader remains unchanged by the main stroke. This leads us to two conclusions that may be very valuable for the evaluation of field records of lightning strokes: (1) The charge distribution on the cloud stroke and on the leader of a ground stroke reflects very closely the potential function, and with this the charged distribution of the thundercloud. (2) The charge distribution of the main stroke of a ground discharge can be separated from the influence charge of the leader and is uniform along the lightning channel. It is independent of the inhomogeneous field of the thundercloud, and the net amount of this charge distribution is given only by the capacity, i.e., the length of the lightning channel and the potential difference of the leader stroke and the ground. As a result of this uniform charge distribution we may apply the calculus of transients on transmission lines to the main stroke of a ground discharge. As this theory is well known and already applied to the lightning flash by *Bewley* [1951] we can confine our remarks to the following. The mechanism of charging an open transmission line is that of reflected waves. If the conductivity of the lines is very high, the damping is very low and the wave travels almost with the speed of light. With decreasing conductivity, the traveling speed decreases and the damping increases until the process changes to an aperiodic form. The conductivity of the lightning channel is certainly much lower than that of a metallic wire. It is also a function of the current that has flowed through the channel, and therefore is not a constant but a function of time. Furthermore, we have to consider a loss of energy by corona discharge along the lightning channel, which makes the mathematical solution of this problem very complex and difficult. Therefore we will mention here only some experimental evidence that may point to the concept of reflected waves. Some photo-

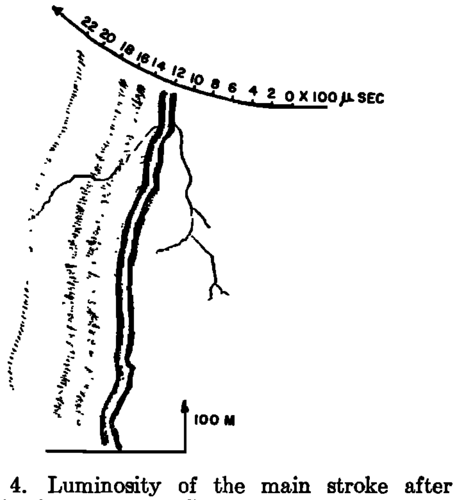

Fig. 4. Luminosity of the main stroke after *Schonland, Malan, and Collens* [1935]; dark strips indicate reflected waves.

graphs with the Boys camera show dark, regularly spaced strips imbedded in the bright band of the main stroke [*Schonland, Malan, and Collens,* 1935, Fig. 4]. These strips could indicate the alternating weakening and strengthening of the current flow by the successive reflected waves.

Another check would be the frequency of the radiated electromagnetic wave. One cycle would be completed by the time necessary for the electric surge to travel once up and down the channel. The traveling speed of the main stroke is in the average 6×10^9 cm/sec. If we assume the length of the lightning channel to be 3 km, we would arrive at a frequency of 10 kc/s, in good agreement with the experimental data.

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