Derive $\nu_i = \nu_m \left(\frac{E}{E_m}\right)^k$ with $k = \frac{5}{2}$

Begin with

$$\frac{\alpha}{p} \cong \left(5.9\times10^{-4}\sqrt{\frac{\text{cm}\,T}{V^3}}\right)\left(\frac{E}{p}\right)^{3/2}$$ B&R 2.11 p. 20

$1\ \text{cm}^{-1}\text{Torr}^{-1} = 0.75\ \text{m}^{-1}\text{Pa}^{-1}$

Define $H = 5.9\times10^{-4}\sqrt{\frac{\text{cm Torr}}{V^3}}$ (Awful!)

$\nu_i = \alpha v_e$ (definition)

B&R 2.9 p. 19
α = ionization events/cm
So ν_i = ion. freq.
Hz = events/cm cm/s

$$\frac{\nu_i}{v_e p} = H\left(\frac{E}{p}\right)^{3/2}$$

Pick a pressure (say 760 torr)

$$\frac{\nu_i}{v_e} = \frac{H}{\sqrt{p}}E^{3/2} = \frac{H}{\sqrt{760\,\text{torr}}}E^{3/2}$$

We will sweep all the constants under the rug

$$\frac{v_m}{\nu_m E_m^{3/2}}\left[\frac{\nu_i}{v_e} \sim E^{3/2}\right]$$

We want to compare E to E_m and ν_i to ν_m and v_e to v_m

$$\frac{\nu_i}{\nu_m}\frac{v_m}{v_e} \sim \left(\frac{E}{E_m}\right)^{3/2}$$

(Of course different const of proportionality)

$$\frac{v_m = -\mu_e E_m}{v_e = -\mu_e E} \rightarrow \frac{v_m}{v_e} = \frac{E_m}{E}$$

$$\frac{\nu_i}{\nu_m}\frac{E_m}{E} \sim \left(\frac{E}{E_m}\right)^{3/2} \longrightarrow \frac{\nu_i}{\nu_m} = \left(\frac{E}{E_m}\right)^{5/2}$$

Derive Equation 3-6

2.24 $$\frac{\partial n_e}{\partial t} + \mathrm{div}\left(-n_e\mu_e E - D_e \nabla n_e\right) = \nu_i n_e - \beta n_e n_+$$

Ignore diffusion (small in large fields)

small attachment compared to ionization

$$\frac{\partial n_e}{\partial t} + \mathrm{div}\left(-n_e\mu_e E\right) = \nu_i n_e$$

$\mu_e E = v_e$

$\mu_+ E = v_+$

$$\frac{\partial n_+}{\partial t} + \mathrm{div}\left(n_+\mu_+ E\right) = \nu_i n_e$$

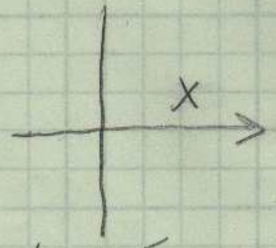
$\mathrm{div} = \frac{d}{dx}$ in 1-D

RHS same in both equations because ions & electrons are created at same rate (and the slow moving ions are not themselves causing ionization)

$$\frac{\partial n_e}{\partial t} - \nabla \cdot n_e v_e = \nu_i n_e$$

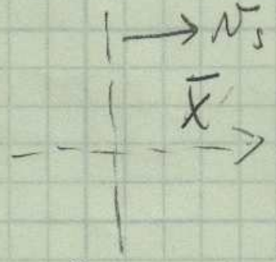
$$\frac{\partial n_+}{\partial t} + \nabla \cdot n_+ v_+ = \nu_i n_e$$

Solve in Lab Frame & Moving Frame



Lab Frame

Neutral Gas at rest

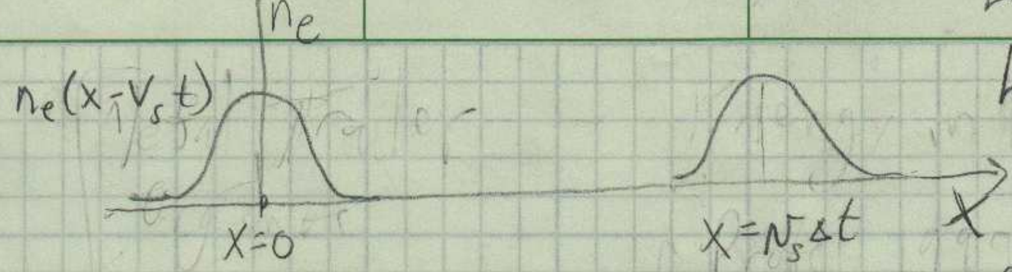


Streamer Frame

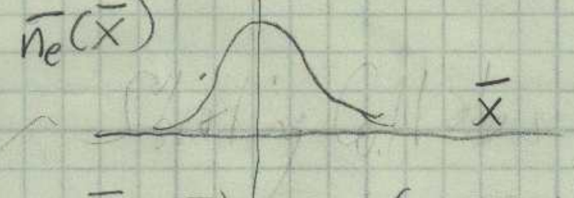
Streamer Head is at rest

$t=0$ $\qquad$ $t=\Delta t$ $\qquad$ Lecture 35-3

Lab Frame



Streamer



$$\left.\begin{aligned}
\bar{n}_e(\bar{x}) &= n_e(x-V_s t)\\
\bar{n}_+(\bar{x}) &= n_+(x-V_s t)\\
\bar{V}_+(\bar{x}) &= V_+(x-V_s t)-V_s\\
\bar{V}_e(\bar{x}) &= V_e(x-V_s t)-V_s
\end{aligned}\right\} \quad \begin{aligned}\bar{x} &\equiv x-V_s t\\ \bar{t} &= t\end{aligned}$$

$\bar{n}_e$ and n_e have same shape

$\bar{V}_e$ and V_e " " " (but differ by V_s)

In streamer frame

$$\frac{\partial \bar{n}_e}{\partial t}=0 \qquad \nabla\cdot(\bar{n}_e\bar{V}_e)=\frac{\partial}{\partial\bar{x}}(\bar{n}_e\bar{V}_e)$$

But $\frac{\partial}{\partial\bar{x}}=\frac{\partial}{\partial x}$. Also the constant V_s is constant so $\frac{\partial}{\partial x}$ does not matter.

Also $\frac{\partial\bar{n}}{\partial\bar{x}}=\frac{\partial\bar{n}}{\partial\bar{x}}\frac{\partial\bar{x}}{\partial x}=\frac{\partial n}{\partial x}$

$$-\frac{\partial}{\partial\bar{x}}\bar{n}_e\bar{V}_e=\nu_i\bar{n}_e \qquad \text{Go back to lab frame}$$

$$\frac{\partial}{\partial\bar{x}}\bar{n}_+\bar{V}_+=\nu_i\bar{n}_e$$

$$-\frac{\partial}{\partial x}n_e(V_e-V_s)=\nu_i n_e$$

$$\frac{\partial}{\partial x}n_+(V_+-V_s)=\nu_i n_e$$

$V_+ << V_s$

$$\frac{d}{d\bar{x}}\left[m_e(\nu_e-\nu_s)\right] = \nu_i n_e \qquad 3.5a$$

$$-\nu_s \frac{\partial m_+}{\partial x} = \nu_i n_e \qquad 3.5b$$

subtract 3.5b from 3.5a

$$\frac{d}{d\bar{x}}\left[m_e(\nu_e-\nu_s) + m_+\nu_s\right] = 0$$

Integrate

$$m_e(\nu_e-\nu_s) + m_+\nu_s = \text{const}$$

$$(m_+-m_e)\nu_s + m_e\nu_e = \text{const}$$

For large $\bar{x}$ $\quad m_+ - m_e = 0$ and $\nu_e = 0$

$$\boxed{(\Delta m)\nu_s = -m_e\nu_e \qquad \underline{3.6}}$$

QED

Attempt at Normalized Results. Unhappy w/ this.

$$\left(\frac{n_+}{n_m} - \frac{n_e}{n_m}\right)\nu_s = -\frac{n_e}{n_m}\nu_e$$

$$(N_+ - N)\nu_s = -N\nu_e \quad \leftarrow \text{Normalized results}$$

$$n_c \equiv n_+ + n_e \rightarrow N_c = N_+ + N$$

$$N_+ = N(\nu_s - \nu_e) = N(\nu_s - \mu_e E) = N(\nu_s - \mu_e F E_m)$$

$$N_c = N(1 + \nu_s - \nu_e)$$

$$\Delta N = N_+ - N = N(\nu_s - \nu_e - 1) = N_c - 2N$$

Want to derive $v_s = \frac{n_m \mu_e e r_m}{2\epsilon_0}$

Gauss Law $\frac{1}{r^2}\frac{d}{dr} r^2 E = \frac{e}{\epsilon_0}(n_+ - n_e)$ 3.3

$$\frac{dE}{dr} + \frac{2E_r}{r} = \frac{e}{\epsilon_0}(\Delta n)$$

where $\frac{dE}{dr} = 0 \quad E = E_m$

$$\frac{2E_m}{r_m} = \frac{e}{\epsilon_0}(\Delta n)$$

$$E = \frac{e \Delta n_m r_m}{2\epsilon_0} \quad 3,4$$

$$v_s = \frac{-n_e}{\Delta n} v_e = +\frac{n_e}{\Delta n}\mu_e E \quad \Big[\text{True at } E_m, r_m$$

$$v_s = \frac{n_m}{\Delta n_m}\mu_e E_m$$

$$v_s = \frac{n_m}{\cancel{\Delta n_m}}\mu_e \frac{e \cancel{\Delta n_m} r_m}{2\epsilon_0}$$

$$v_s = \frac{n_m e \mu_e r_m}{2\epsilon_0} \quad 3.9$$