

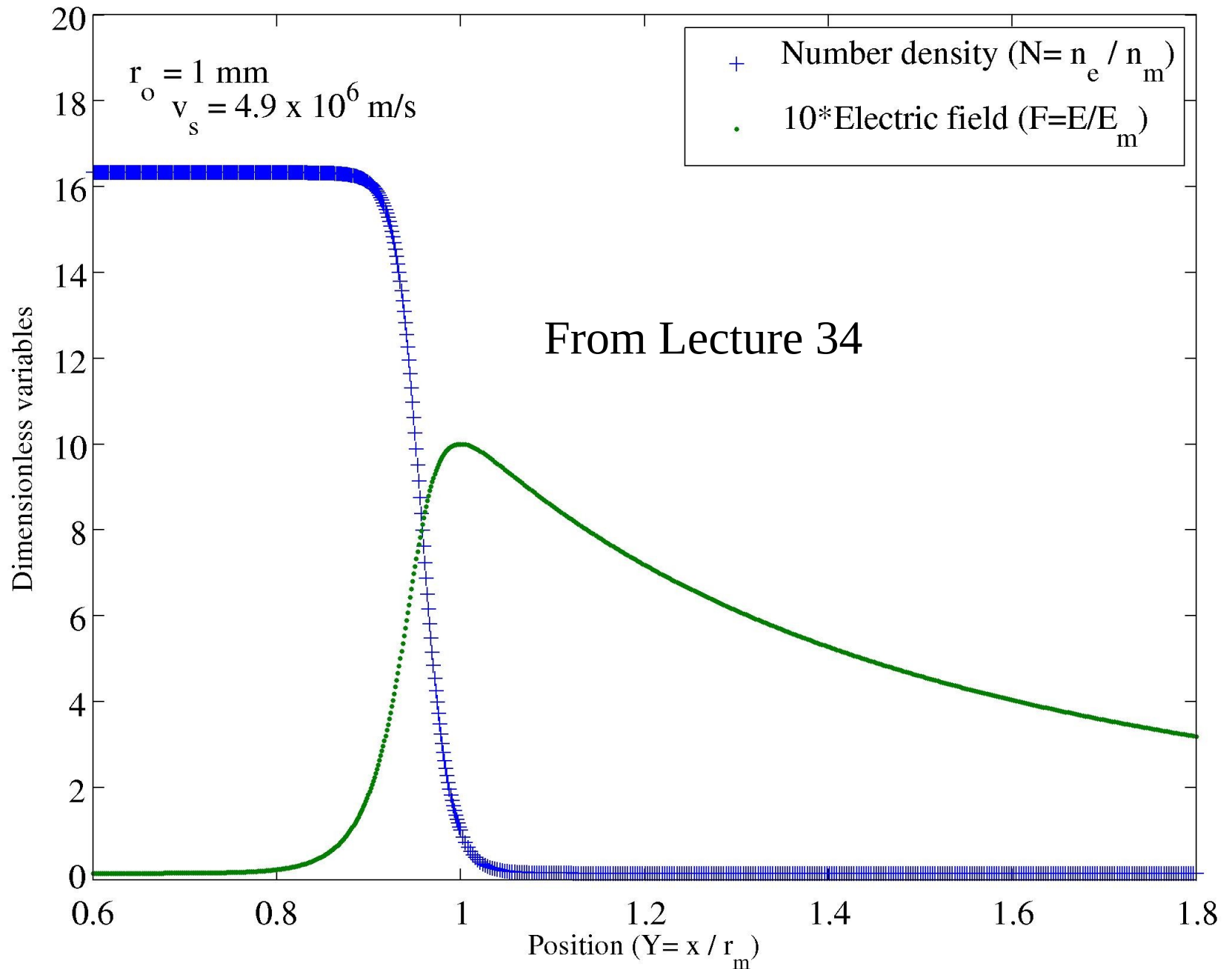
Physics 535 – Lecture 36  
Physics of Lightning  
Numerical Solution of Streamers (again)  
5/2/16

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(Photo courtesy of Harald Edens)

# Streamer head in air, (dimensionless)



# Streamer head in air, (SI Units)

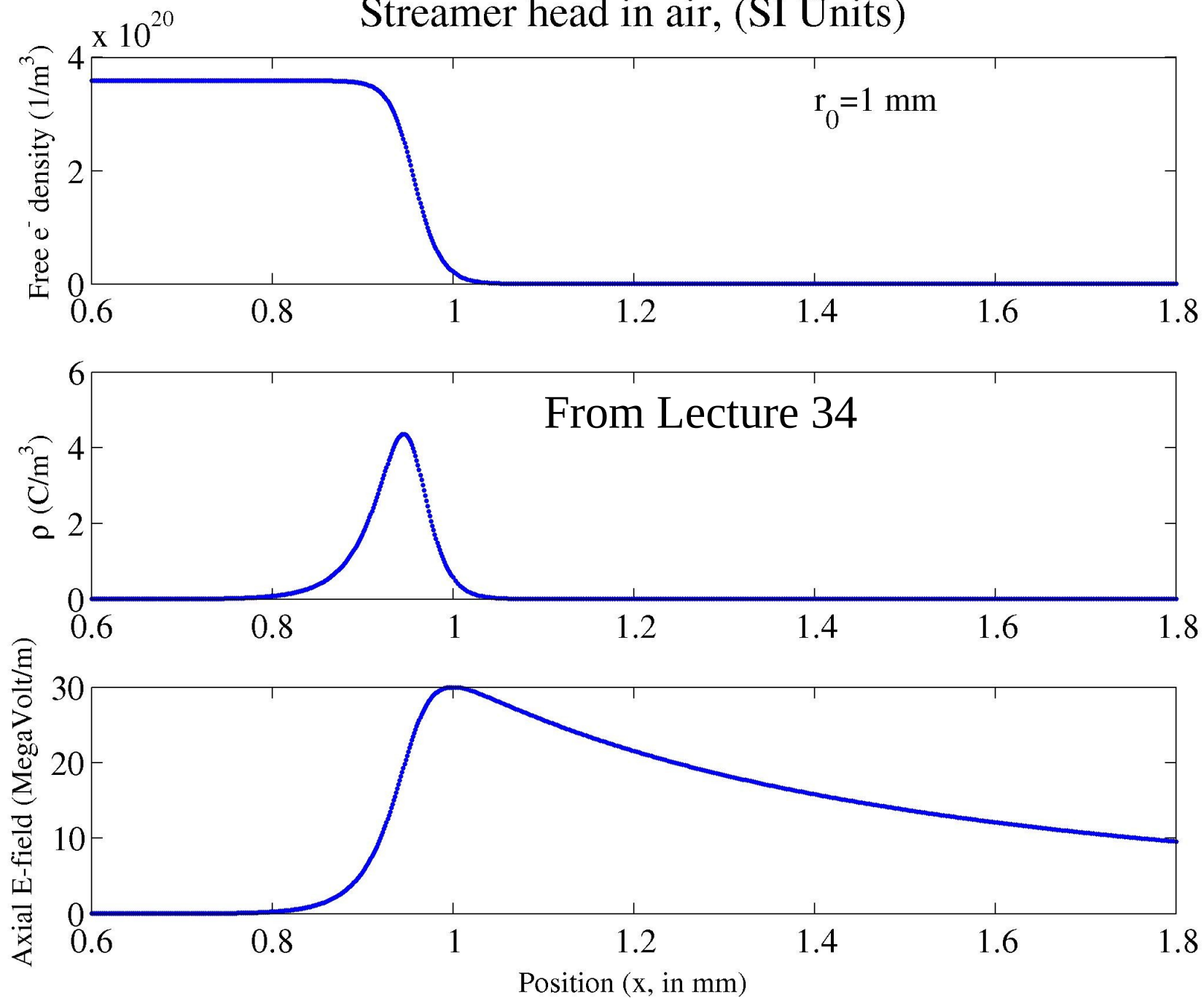
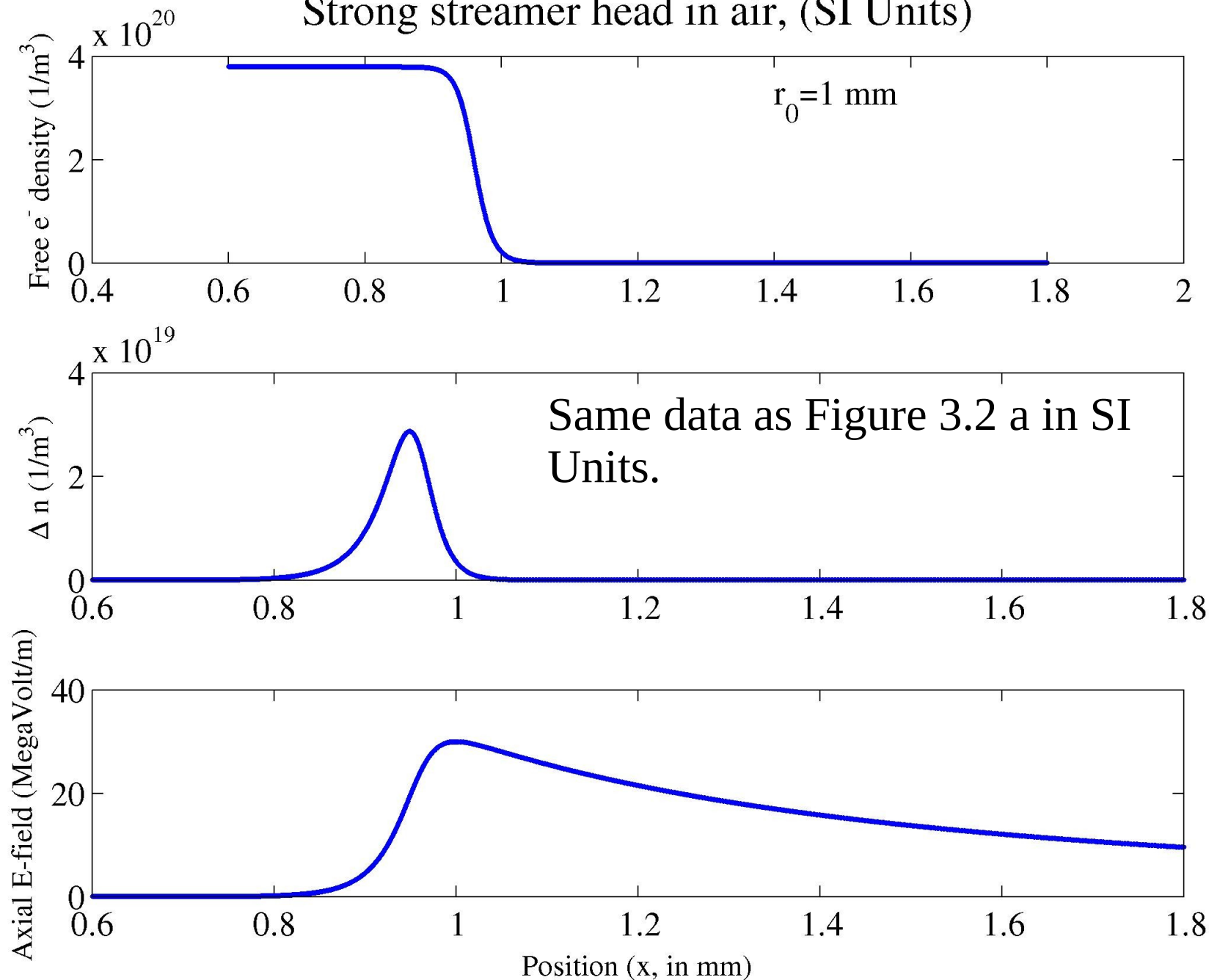
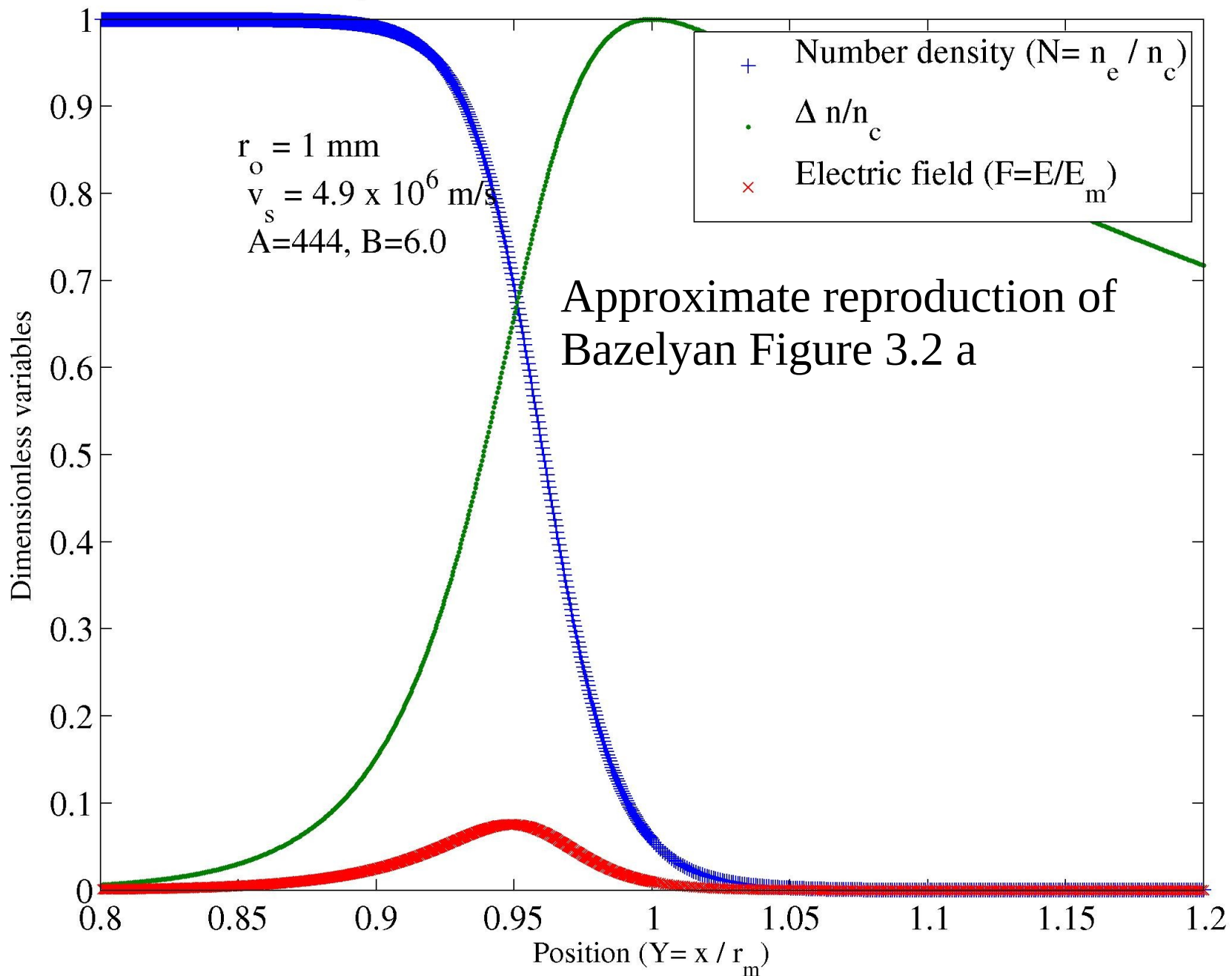


Figure3 2

# Strong streamer head in air, (SI Units)



# Strong Streamer head in air, (dimensionless)



# Streamer Derivation I

$$[3.3] \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow \frac{2E}{x} + \frac{dE}{dx} = \frac{e}{\epsilon_0} (n_+ - n_e)$$

$$[3.6] \quad j_t = e n_e v_e + e (n_+ - n_e) v_s = 0 \rightarrow (n_+ - n_e) = \frac{-n_e v_e}{v_s}$$

$$[3.9] \quad v_s = \frac{e \mu_e n_m r_m}{2 \epsilon_0} \dots + \text{algebra} \dots$$

$$\frac{dF}{dY} + 2 \frac{F}{Y} = -2 NF$$

# Streamer Derivation II

$$[3.5] \quad -\frac{d}{dx} n_e (v_s - v_e) = v_i n_e \rightarrow n_e \frac{dv_e}{dx} + v_e \frac{dn_e}{dx} - v_s \frac{dn_e}{dx} = v_i n_e$$

$$[2.1] \quad v_e = -\mu_e E$$

$$[\text{Bonus}] \quad v_i = v_n \left( \frac{E}{E_m} \right)^k \dots + \text{algebra} \dots$$

$$dN = \frac{-dY}{(F+B)} \left[ N \frac{dF}{dY} + A N |F|^k \right]$$

# Dimensionless Equations

$$\frac{dF}{dY} + 2 \frac{F}{Y} = -2NF$$

$$F \stackrel{\text{def}}{=} \frac{E}{E_m}$$

$$-(F+B) \frac{dN}{dY} = N \frac{dF}{dY} + A N |F|^k$$

$$N \stackrel{\text{def}}{=} \frac{n_e}{n_m}$$

$$A = \frac{v_m \Gamma_m}{\mu_e E_m}$$

$$B = \frac{v_s}{\mu_e E_m}$$

$$Y \stackrel{\text{def}}{=} \frac{X}{\Gamma_m}$$

$$k \simeq 2.5$$



Solve for dF and dN

$$[1] \quad dF = \left( 2FN - 2\frac{F}{Y} \right) dY$$

$$F \stackrel{\text{def}}{=} \frac{E}{E_m}$$

$$[2] \quad dN = \frac{-dY}{(F+B)} \left[ N \frac{dF}{dY} + A N |F|^k \right]$$

$$N \stackrel{\text{def}}{=} \frac{n_e}{n_m}$$

Boundary conditions:

$$Y \stackrel{\text{def}}{=} \frac{X}{r_m}$$

At  $Y=1$ ,  $F$  and  $N$  are both=1

# Numerical Solution method

$$[1] \quad dF = \left( 2FN - 2\frac{F}{Y} \right) dY$$

$$[2] \quad dN = \frac{-dY}{(F+B)} \left[ N \frac{dF}{dY} + A N |F|^k \right]$$

Create an array of 500 Y values (with  $1 < Y < 2$ )

$Y(1)=1$ ,  $F(1)=1$  and  $N(1)$  is also 1.

Use eqn [1] to calculate  $dF$ , and thus  $F(2)$

Use eqn [2] to calculate  $dN$  and thus  $N(2)$ .

$dF/dY(1)$  is also obvious.

Continue generating new F's and N's.

Create another array of Y's ( $0 < Y < 1$ ). Repeat the process going backwards.

# Solution method – details

$$Y_{SI} = Y * r_m;$$

$$N_{SI} = N * n_m;$$

$$E_{SI} = F * E_m;$$

$$Nc_{SI} = N_{SI}(1);$$

$$v_e = -\mu_e * E_{SI};$$

$$\Delta n = -N_{SI} * v_e / v_s;$$

# Constants (Strong Wave)

$$E_m = 3.0E7 \text{ (V/m)}$$

$$v_s = 4.9E6 \text{ (m/s)}$$

$$r_m = 1.0E-3 \text{ (m)}$$

$$\mu_e = 2.7E-2 \text{ (m}^2\text{/V s)}$$

$$\nu_m = 3.6E11 \text{ (1/s)}$$

# Constants (Weak Wave)

$$E_m = 8.0E6 \text{ (V/m)}$$

$$v_s = 5.8E3 \text{ (m/s)}$$

$$r_m = 1.0E-3 \text{ (m)}$$

$$\mu_e = 2.7E-2 \text{ ?? (m}^2\text{/V s)}$$

$$\nu_m = 3.6E11 \text{ ???? (1/s)}$$