Lecture 34-1

3.3 $\nabla\cdot E = \frac{\rho}{\epsilon_0} \rightarrow \frac{1}{r^2}\frac{d}{dr} r^2 E = \frac{e}{\epsilon_0}(n_+ - n_e)$ [Spherical Coords]

$$\frac{1}{r^2} 2rE + \frac{r^2}{r^2}\frac{dE}{dr} = \frac{2E}{r} + \frac{dE}{dr} = \frac{e}{\epsilon_0}(n_+ - n_e)$$

3.6 $J_t = e n_e V_e + e(n_+ - n_e) V_s = 0$

Ahead of streamer $J_t = 0$

$$-e n_e V_e = e(n_+ - n_e) V_s \rightarrow (n_+ - n_e) = -\frac{n_e V_e}{V_s}$$

Combine 3.3 and 3.6

(a) $\frac{dE}{dx} + \frac{2E}{x} = \frac{-e}{\epsilon_0}\left(\frac{n_e V_e}{V_s}\right)$

3.5 $-\frac{d}{dx} n_e (V_s - V_e) = \nu_i n_e$ Use product rule

$$n_e \frac{dV_e}{dx} + V_e\frac{dn_e}{dx} - n_e \cancelto{0}{\frac{dV_s}{dx}} - V_s \frac{dn_e}{dx} = \nu_i n_e$$

Streamer velocity is assumed constant in space

$\frac{dV_s}{dx} \approx 0$ Also $V_e = -\mu_e E$ (or allow μ_e to be negative)

$$n_e\left(-\mu_e \frac{dE}{dx}\right) - (\mu_e E)\frac{dn_e}{dx} - V_s\frac{dn_e}{dx} = \nu_i n_e$$

Also $\nu_i = \nu_m (E/E_m)^k$ ← come back to this derivation

(c) $n_e\left(-\mu_e \frac{dE}{dx}\right) - (\mu_e E)\frac{dn_e}{dx} - V_s \frac{dn_e}{dx} = n_e \nu_m \left|\frac{E}{E_m}\right|^k$

3.9 $v_s = \frac{e\mu_e n_m r_m}{2\epsilon_0} \rightarrow \frac{1}{v_s} = \frac{2\epsilon_0}{e\mu_e n_m r_m}$ plug into (a)

(b) $\frac{dE}{dx} + \frac{2E}{x} = \frac{-\cancel{e}}{\cancel{\epsilon_0}}(n_e V_e)\frac{2\cancel{\epsilon_0}}{\cancel{e}\mu_e n_m r_m} = \frac{-2 n_e V_e}{\mu_e n_m r_m}$

Multiply (b) by $\frac{r_m}{E_m}$

$$r_m \frac{d}{dx}\left(\frac{E}{E_m}\right) + \frac{2r_m}{x}\frac{E}{E_m} = \frac{-2 n_e V_e\, r_m}{\mu_e n_m\, r_m\, E_m}$$

$$-\mu_e E = V_e \quad \therefore \quad \frac{-V_e}{\mu_e} = E$$

Also, define Y as $\frac{x}{r_m}$ $\rightarrow$ $\frac{d}{dx} = \frac{d}{dY}\frac{dY}{dx} = \frac{1}{r_m}\frac{d}{dY}$

Also define F as E/E_m

" " N as n_e/n_m

$$\frac{r_m}{r_m}\frac{d}{dY}\left(\frac{E}{E_m}\right) + \frac{2}{(x/r_m)}\left(\frac{E}{E_m}\right) = +\frac{n_e}{n_m}\frac{E}{E_m}$$

$$\boxed{[1] \quad \frac{dF}{dY} + \frac{2F}{Y} = +2NF}$$

Now develop 3.5 into (c)

Multiply (c) by $\frac{r_m}{E_m n_m}$

$$\frac{n_e}{n_m}\left(-\mu_e \frac{dE}{dx}\frac{r_m}{E_m}\right) - \mu_e \frac{E}{E_m}\frac{dn_e}{dx}\frac{r_m}{n_m} - \frac{V_s r_m}{E_m n_m}\frac{dn_e}{dx} = \frac{\nu_i r_m}{E_m}\frac{n_e}{n_m}$$

$$\frac{d}{dx} = \frac{1}{r_m}\frac{d}{dY}$$

(d) $$\left[N\left(-\mu_e \frac{dF}{dY}\right) - \mu_e F \frac{dN}{dY} - \frac{V_s}{E_m}\frac{dN}{dY} = \frac{\nu_i r_m}{E_m} N\right]\frac{1}{\mu_e}$$

RHS of (d): $\frac{\nu_i r_m}{\mu_e E_m} N = \nu_m \left(\frac{E}{E_m}\right)^k \frac{r_m N}{\mu_e E_m}$

$$= N F^k \left(\frac{\nu_m r_m}{\mu_e E_m}\right) \leftarrow \text{"A"}$$

(d) $$-N\frac{dF}{dy} - F\frac{dN}{dy} - \frac{V_s}{\mu_e E_m}\frac{dN}{dy} = ANF^k$$

$$-\left(F + B\right)\frac{dN}{dy} = N\frac{dF}{dy} + ANF^k$$

$$A = \frac{\nu_m r_m}{\mu_e E_m}$$

$$B = \frac{V_s}{\mu_e E_m}$$

$N = n_e / n_m$

$F = E / E_m$

n_+