Physics 535 – Lecture 34
Physics of Lightning

Numerical Solution of Streamers

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(Photo courtesy of Harald Edens)

Streamers (Ch. 3.1-3.2)

Gas is at ambient temperature but electrons are at > 1 eV. Ionization is by electron impact. Electrical conductivity is low except at the streamer tip. E-field is very high. “Equilibrium” in streamer frame only, contingent on streamer growing at 0.01c.

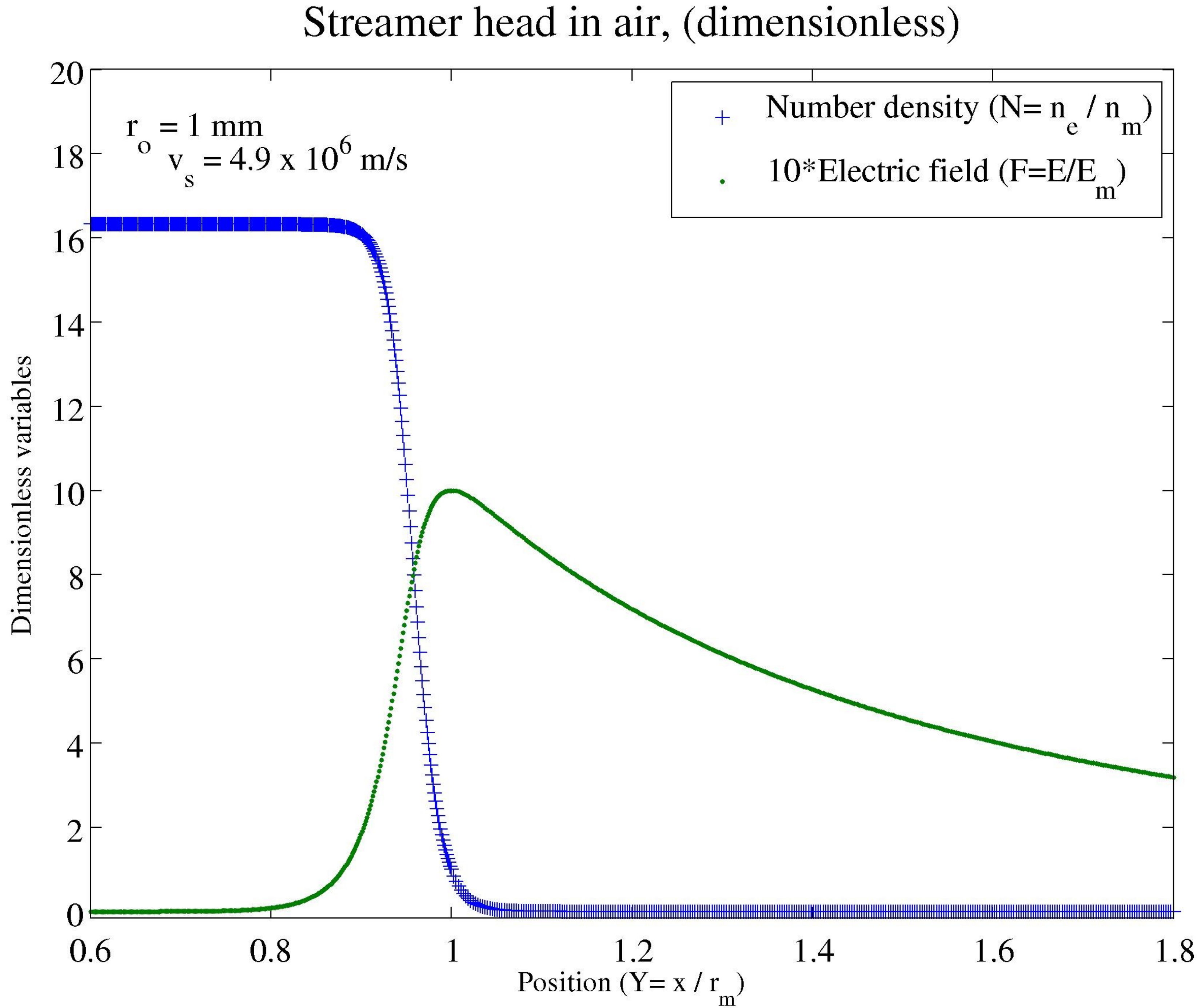

Figure3_2

R. Sonnenfeld -- As of: 14-Apr-2016

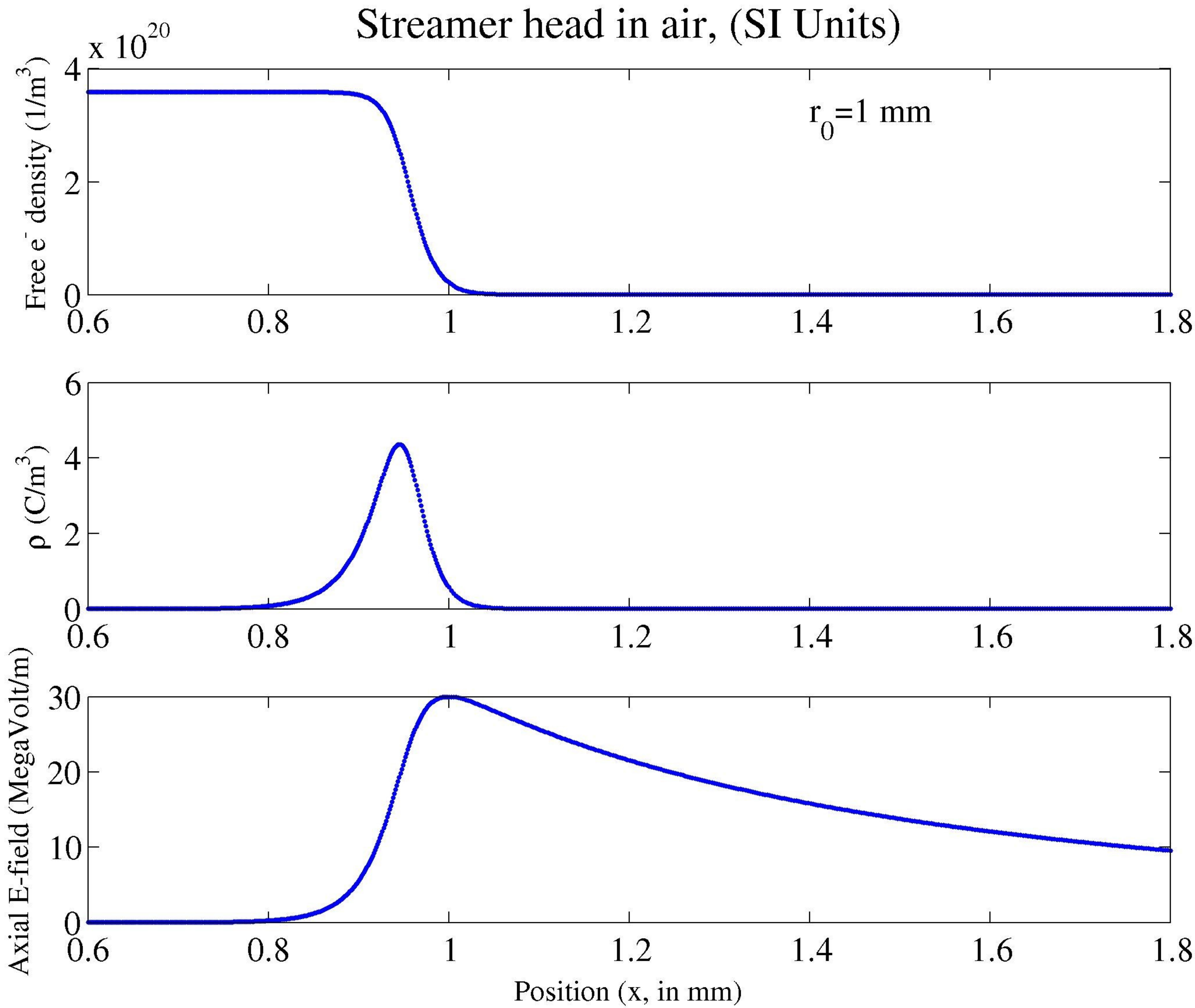

Figure3_2

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Streamer Derivation I

$$[3.3]\ \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow \frac{2E}{x} + \frac{dE}{dx} = \frac{e}{\epsilon_0}\left(n_+ - n_e\right)$$

$$[3.6]\ j_t = e\, n_e\, v_e + e\left(n_+ - n_e\right) v_s = 0 \rightarrow \left(n_+ - n_e\right) = \frac{-n_e\, v_e}{v_s}$$

$$[3.9]\ v_s = \frac{e\,\mu_e\, n_m\, r_m}{2\,\epsilon_0} \ldots + \text{algebra} \ldots$$

$$\frac{dF}{dY} + 2\frac{F}{Y} = -2\,NF$$

Streamer Derivation II

$$[3.5]\ -\frac{d}{dx} n_e (v_s - v_e) = \nu_i n_e \rightarrow n_e \frac{dv_e}{dx} + v_e \frac{dn_e}{dx} - v_s \frac{dn_e}{dx} = \nu_i n_e$$

$$[2.1]\ v_e = -\mu_e E$$

$$[\text{Bonus}]\ \nu_i = \nu_n \left(\frac{E}{E_m}\right)^k \ldots + \text{algebra} \ldots$$

$$dN = \frac{-dY}{(F+B)}\left[N \frac{dF}{dY} + A N |F|^k\right]$$

Dimensionless Equations

$$\frac{dF}{dY} + 2\frac{F}{Y} = -2NF$$

$$F \stackrel{\text{def}}{=} \frac{E}{E_m}$$

$$-(F+B)\frac{dN}{dY} = N\frac{dF}{dY} + AN|F|^k$$

$$N \stackrel{\text{def}}{=} \frac{n_e}{n_m}$$

$$A = \frac{\nu_m r_m}{\mu_e E_m} \qquad B = \frac{v_s}{\mu_e E_m}$$

$$Y \stackrel{\text{def}}{=} \frac{x}{r_m}$$

$$k \simeq 2.5$$

Solve for dF and dN

$$[1]\ dF = \left(2FN - 2\frac{F}{Y}\right) dY$$

$$F \stackrel{\text{def}}{=} \frac{E}{E_m}$$

$$[2]\ dN = \frac{-dY}{(F+B)}\left[N\frac{dF}{dY} + A N |F|^k\right]$$

$$N \stackrel{\text{def}}{=} \frac{n_e}{n_m}$$

Boundary conditions:

$$Y \stackrel{\text{def}}{=} \frac{x}{r_m}$$

At Y=1, F and N are both=1

Numerical Solution method

$$[1] \quad dF = \left(2FN - 2\frac{F}{Y}\right) dY$$

$$[2] \quad dN = \frac{-dY}{(F+B)}\left[N\frac{dF}{dY} + A\,N\left|F\right|^{k}\right]$$

Create an array of 500 Y values (with 1<Y<2)
Y(1)=1, F(1)=1 and N(1) is also 1.
Use eqn [1] to calculate dF, and thus F(2)
Use eqn [2] to calculate dN and thus N(2).
dF/dY(1) is also obvious.
Continue generating new F's and N's.
Create another array of Y's (0<Y<1). Repeat the process going backwards.

Practice Plot, A=10, B=1