√ $W_1 = 8\pi\lambda T_m^2 / I$ Power

√ $T_0 = T_m(1 - 2T_m/I)$

$(T_m = \text{sqrt}[(I/k)(W/8\pi\lambda)]$

√ $\sigma_m = b e^{-I/2T_m}$ Conductivity

$\sigma_0 = b e^{-I/2T_0}$

$r_0 = R e^{-(2\pi\lambda T_0 / W_1)}$ core radius

√ $E = W_1 / i$ E-field

√ $i = r_0 \sqrt{\pi \sigma W_1}$ current

$i^2 = \pi r_0^2 \sigma W_1$

$R = \rho \frac{l}{A} \rightarrow \frac{1}{R} = A\sigma$

$\frac{\text{Power}}{\text{length}} = \frac{i^2 R}{\text{length}}$

$i^2 = \frac{\text{Power}}{R}$

Given σ = average conductivity
r_0 = conducting radius
W_1 = power/unit length,
What is i?

$$P = i^2 R \qquad R = \rho \frac{l}{A} = \frac{l}{\sigma A} \qquad \frac{R}{\text{length}} = \frac{1}{\sigma A} \therefore \frac{1}{R} = \sigma A$$

$$i^2 = \frac{P}{R} = \frac{W_1}{R/\text{length}} = \sigma A W_1$$

$$i = \sqrt{\sigma \pi r_0^2 W_1} = r_0 \sqrt{\pi \sigma W_1}$$

Given T_m & λ, what is W_1?

2.42 $$T_m = \sqrt{\frac{I}{8\pi\lambda_m k} W_1}$$

$$T_m^2 = \frac{I}{8\pi\lambda_m k} W_1 \quad \therefore \quad W_1 = \frac{8\pi\lambda_m k T_m^2}{I}$$

Use λ_m = const $1.5 \frac{\text{Watts}}{\text{m} \cdot \text{K}}$

Start w 2.34 What is T_0? (Temp where conductivity goes to σ/e)

$$\ln\left[e = \frac{\sigma_m}{\sigma_0} = \frac{b e^{-I_{eff}/2kT_m}}{b e^{-I_{eff}/2kT_0}} = e^{\left(-\frac{I}{2kT_m} + \frac{I}{2kT_0}\right)} \right]$$

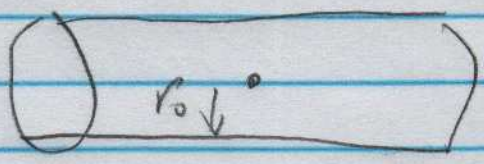
Assume =

2.41

$$1 = \frac{I}{2k}\left[\frac{1}{T_0} - \frac{1}{T_m}\right] = \frac{I}{2k}\left[\frac{T_m - T_0}{T_0 T_m}\right] \rightarrow T_m - T_0 = \frac{2k}{I} T_0 T_m$$

How much heat is removed?

By definition, it is W_1



$$P_{WR} = \lambda \frac{\partial T}{\partial r}(\text{Area})$$

$$\frac{P_{WR}}{L} = W_1 = \lambda \frac{T_m - T_0}{r_0}\, 2\pi r_0 \frac{L}{L}$$

$$W_1 = \lambda (2\pi)(T_m - T_0)$$

$$\frac{\text{Watt}}{m} = \left[\frac{W}{m \cdot K}\right][K]$$

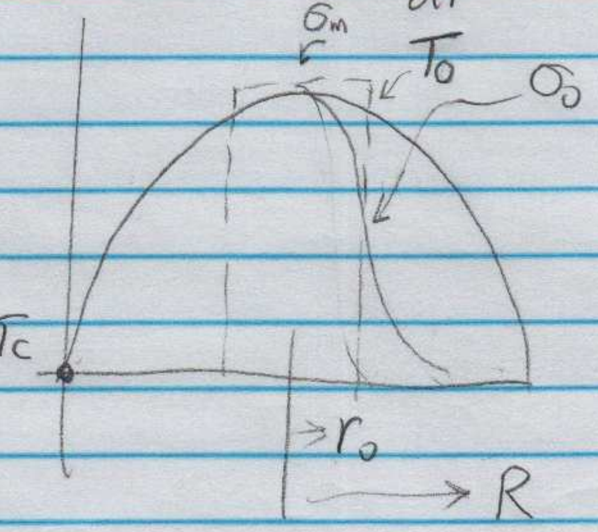
More rigorous

$$2.36 \quad -\frac{1}{r}\frac{d}{dr} r J + \sigma E^2 = 0$$

$$J = -\lambda \frac{dT}{dr} \qquad \frac{dT}{dr} = 0 \text{ at } r = 0$$

$$T = T_c \text{ at } r = R$$

$$T_c \sim 0$$



$$i = E \int_0^R \sigma (2\pi r)\, dr = E \sigma_m \pi r_0^2$$

σ_m is good choice since most current flows in that region

$(I = JA \quad J = \sigma E)$

$$\frac{1}{r}\frac{d}{dr}\left(-\lambda\frac{dT}{dr}\right) = \sigma_m E^2$$

$$\frac{d}{dr}\left(-\lambda\frac{dT}{dr}\right) = r\sigma_m E^2$$

$$-\lambda\frac{dT}{dr} = r_0\sigma_m E^2$$

$$\int_{T_0}^{T_m} dT = -\frac{\sigma_m E^2}{\lambda}\int_0^{r_0} r_0\, dr$$

$$T_m - T_0 = -\frac{\sigma_m E^2}{\lambda} r_0^2$$

$$\lambda(T_m - T_0) = -\sigma_m E^2 r_0^2$$

$$W_1 = i\frac{V}{\ell} = iE \rightarrow i = \sigma E A$$
$$= \sigma_m E \pi r_0^2$$

$$\underline{\pi\lambda(T_m - T_0) = W_1}$$

I don't get the 4π. Use it

2.40 $\quad 4\pi\lambda_m(T_m - T_0) = W_1 = 4\pi\lambda_m\left(2\frac{kT_m^2}{I}\right)$

2.42 $\quad W_1 = \frac{8\pi\lambda_m k T_m^2}{I}$

solve for T_0

$$4\pi\lambda_m(T_m - T_0) = 4\pi\lambda_m\left(2\frac{kT_m^2}{I}\right) \therefore$$

$$T_0 = T_m - \frac{2kT_m^2}{I}$$

If convert I to a temp
Then no "k"

For $r_0 < r < R \quad G = 0$

2.36 $\rightarrow$ $$-\frac{1}{r}\left(\frac{d}{dr}(-r\lambda)\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}(r\lambda)\frac{dT}{dr} = 0 \rightarrow$$

$$r\lambda\frac{dT}{dr} = \text{const}$$

$$\int_0^T \lambda \, dT' = \text{const}\int_{r_0}^{R}\frac{dr}{r}$$

$$\Theta \equiv \int_0^T \lambda \, dT' = \text{const} \ln\left(\frac{R}{r_0}\right)$$

Bazelyan says $\text{const} = \frac{W_1}{2\pi}$ (If $\lambda = \text{const}$)

Why?

$$\lambda \frac{d}{dr} r \frac{dT}{dr}$$

Heat flow is $\frac{dT}{dr}$ Total heat is $2\pi r L \frac{dT}{dr}$

Per length

$$2\pi r \lambda \frac{dT}{dr} = W_1 \quad \text{QED}$$

So $$\Theta = \frac{W_1}{2\pi}\ln R/r_0$$

If $\lambda = \text{const}$ w/ temp then $\int_0^{T_0} \lambda \, dT = \lambda T$

$$\lambda T = \frac{W_1}{2\pi}\ln(R/r_0) \rightarrow R\exp\frac{-2\pi\lambda T_0}{W_1} = r_0$$