Physics 535 – Lecture 29
Physics of Lightning

Saha Equation

4/4/16

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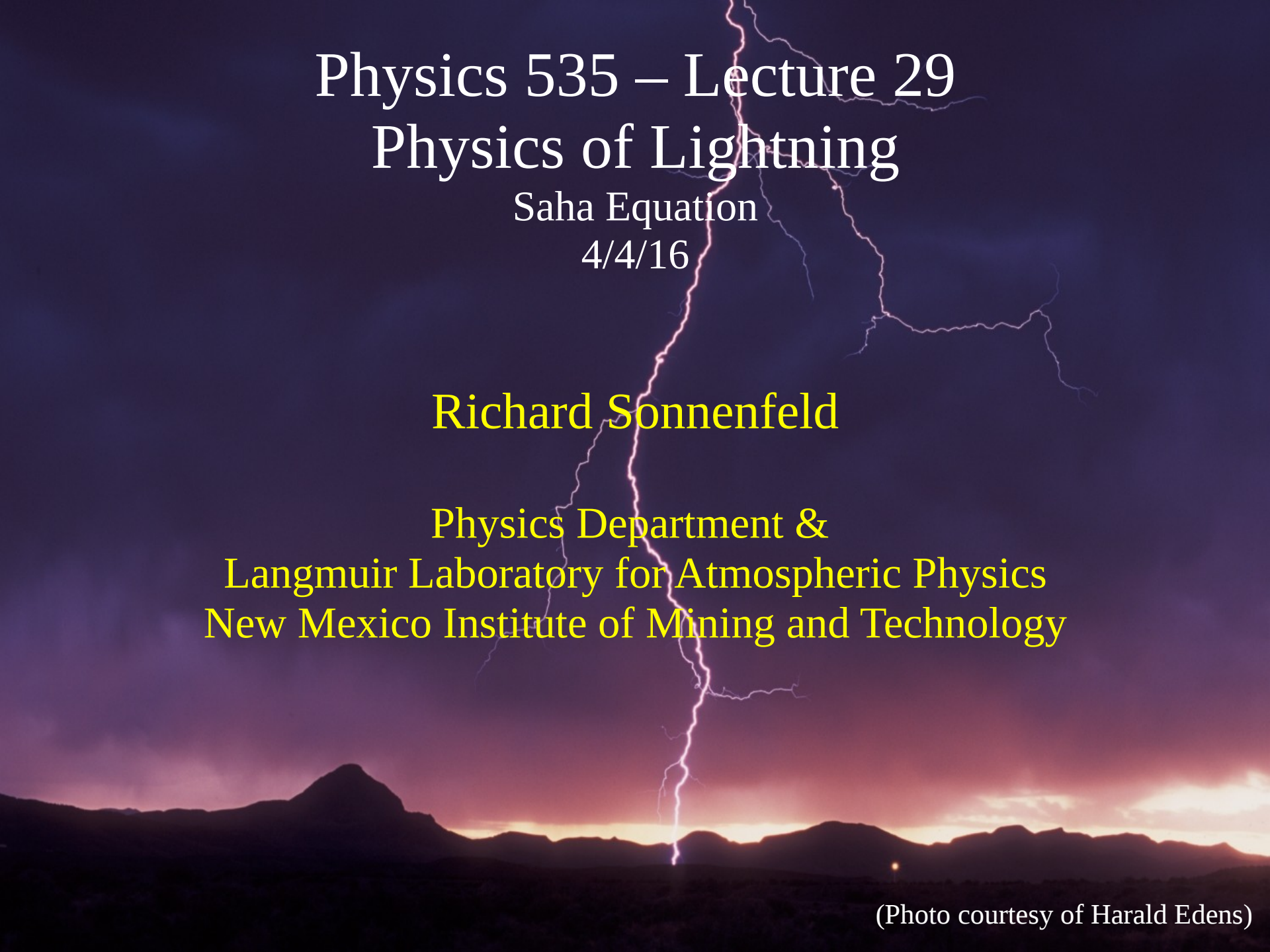
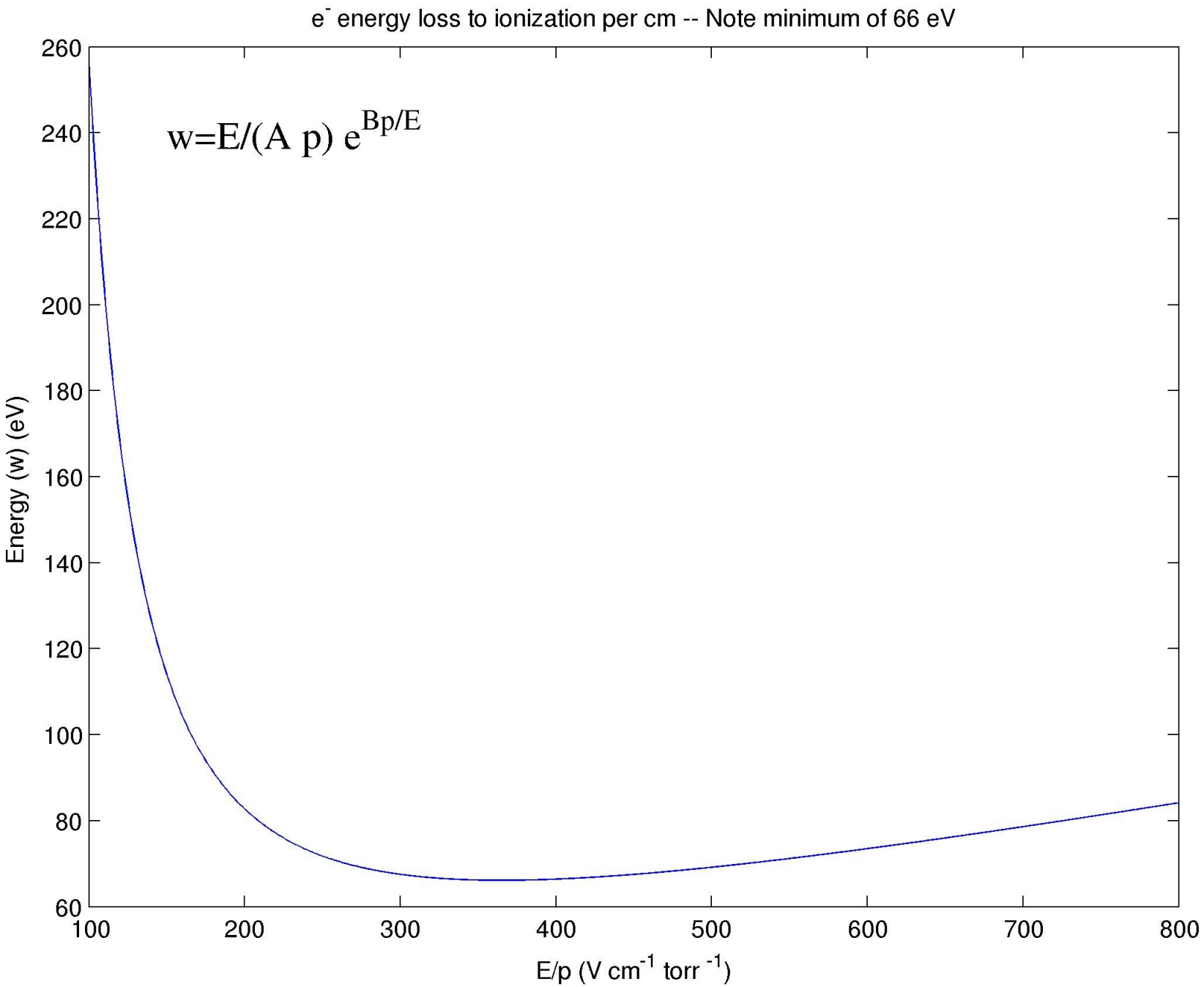


(Photo courtesy of Harald Edens)


e- energy loss to ionization per cm -- Note minimum of 66 eV
w=E/(A p) e^Bp/E
Energy (w) (eV)
E/p (V cm-1 torr -1)
260
240
220
200
180
160
140
120
100
80
60
100
200
300
400
500
600
700
800

Saha Equation

Begins with principles of statistical mechanics.

Probability of occupation of a state E_n is Boltzman factor $e^{-E_n/k_{BT}}$

\# number of possible states $g\frac{d^3x\,d^3p}{h^3}$

Reaction is something like

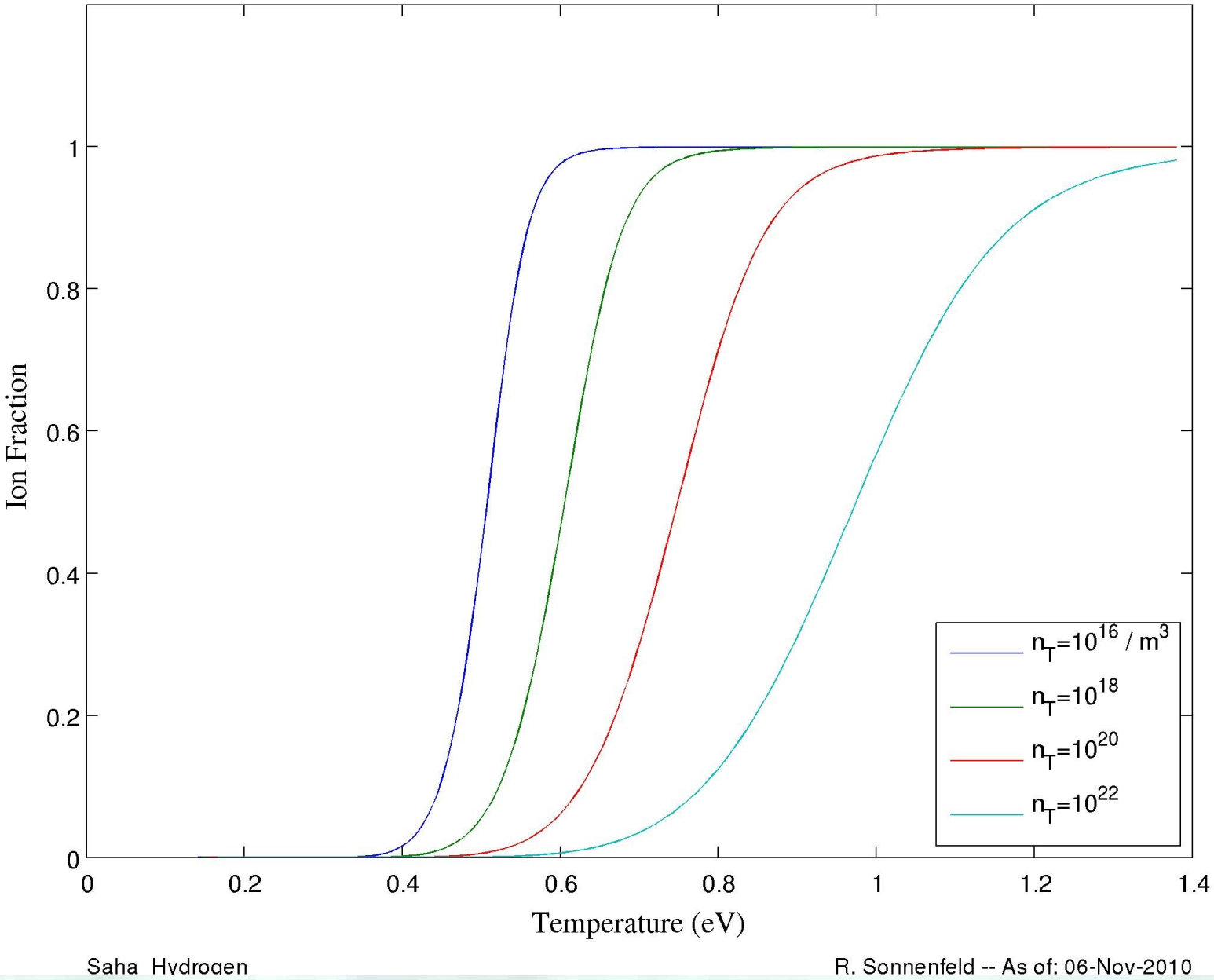
$$N_2 \rightarrow N_2^+ + e^-$$

State "A", neutral molecule, $E_A = 0$

State "B", ion+electron, $E_B = I + \frac{p^2}{2m}$

Reproducing Lecture notes (pg 14) of John Howard at Australian National Univ

Plot of Saha equation for singly ionized Hydrogen



Saha Equation

$$z(T) = \frac{n_e n_+}{n_n} = \frac{g_+}{g_n}\frac{2}{h^3} e^{-I/kT} (2 m_e \pi k T)^{3/2}$$

$$\frac{\chi^2}{1-\chi} = \frac{z}{n_{Total}} \qquad \chi = \frac{n_e}{n_{Total}}$$

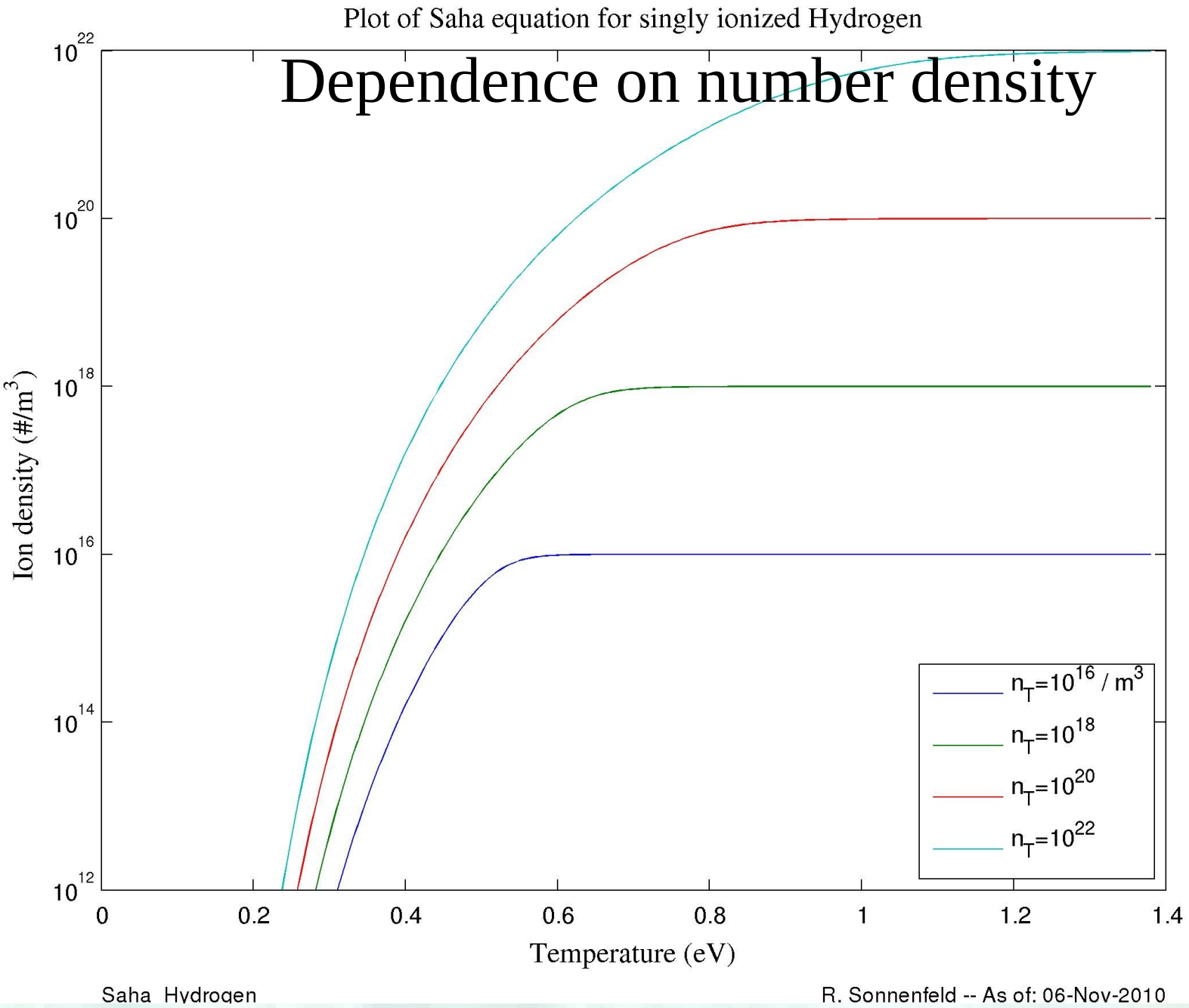
n_e = free e^- density

n_+ = positive ion density (assume $n_e = n_+$)

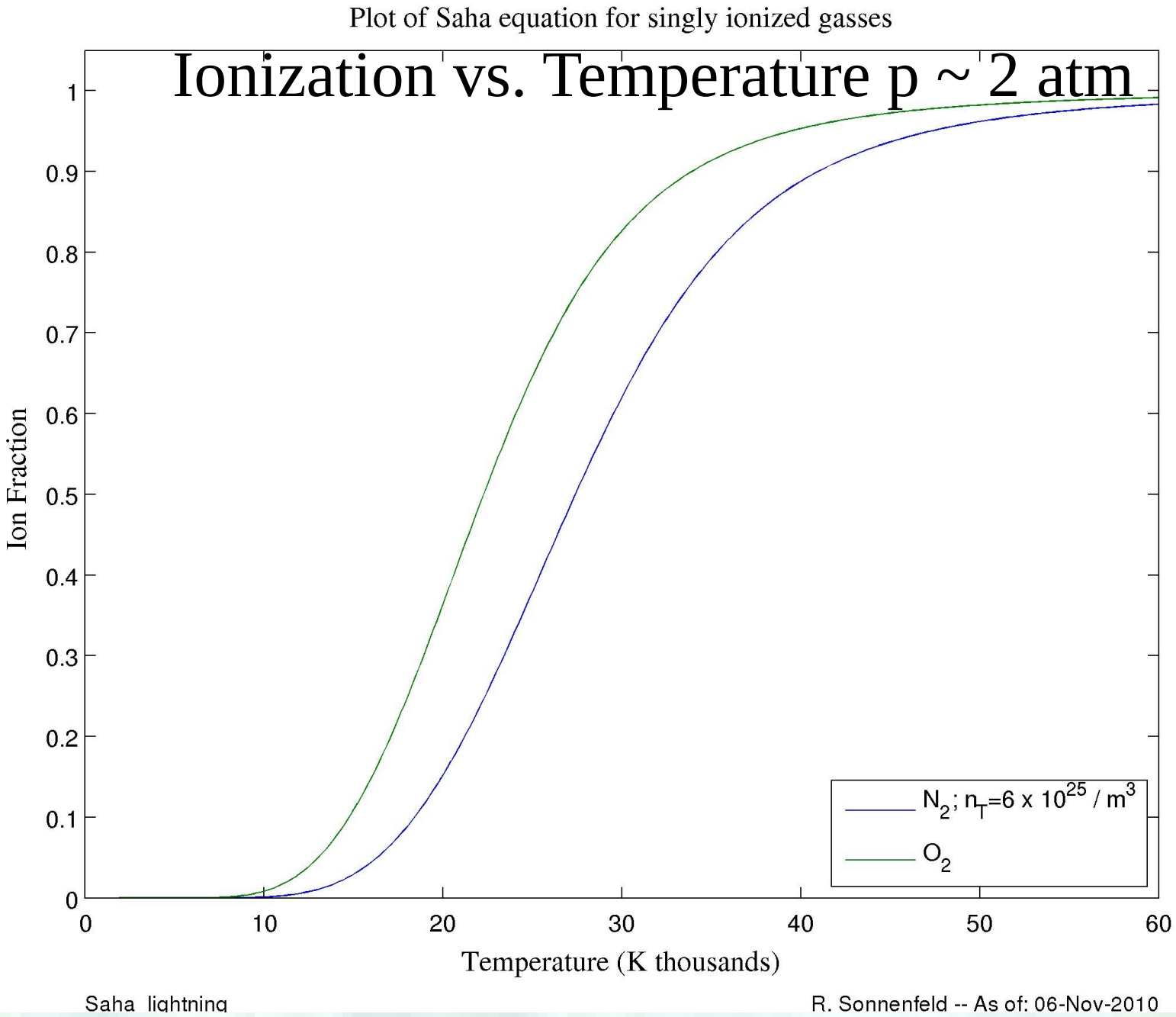
n_n = neutral molecule density

n_T = total particle density (not including n_e)

g = degeneracy factor


Plot of Saha equation for singly ionized Hydrogen
Dependence on number density
Ion density (#/m^3)
10^22
10^20
10^18
10^16
10^14
10^12
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Temperature (eV)
n_T=10^16 / m^3
n_T=10^18
n_T=10^20
n_T=10^22
Saha Hydrogen
R. Sonnenfeld -- As of: 06-Nov-2010

Ionization vs. Temperature p ~ 2 atm



Ionization vs. Temperature (eV)