## Homework 11 - Rev A

SPN 11-01 - Field of a magnetic dipole
a Compare equation 5.88 and 3.103 . What do their similarities tell you?
b Imagine 5 loops of current on top of eachother and all laying in the $x-y$ plane. Each loop carries 80 Amperes and has a diameter of 1 cm . Calculate the magnetic field in the center of the loops ( $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0$ ). (You have previously derived this formula you do not need to rederive it.)
c Repeat the calculation for $\mathrm{z}=1 \mathrm{~cm}$ and again for $\mathrm{z}=10 \mathrm{~cm}$. Carry 5 sig. figs.
d What is the magnetic moment $\vec{m}$ of this configuration?
e Use the magnetic moment and equation 5.88 to repeat the calcu-


Problem 2: A constant B-field in a loop. lations from parts $b$ and $c$. At approximately what value of $z$ would you say the magnetic dipole approximation becomes valid?

SPN 11-02 Given a field of magnitude $B$ confined to a rectangular region as shown in the figure. What is the total flux through the hoop shown of inner radius $r_{1}$ and outer radius $r_{2}$.

SPN 11-03 A loop of wire is dragged into a constant magnetic field as shown. What is the magnitude, and direction of the induced EMF?

SPN 11-04. Below are three sketches of a 10 -cm-diameter loop in three different magnetic fields. The loop's resistance is $R=0.20 \Omega$. What is the induced current (direction and magnitude) in each case?

(a) $B$ increasing at $0.50 \mathrm{~T} / \mathrm{s}$


## (b) $B$ decreasing

 at $0.50 \mathrm{~T} / \mathrm{s}$
(c) $B$ decreasing at $0.50 \mathrm{~T} / \mathrm{s}$


Problems 3 and 4:

SPN 11-05. A coil of wire has an axis of rotation along $\hat{x}$. Initially, the area vector of the coil is oriented in the same direction as a uniform magnetic field (so that the flux through the coil is maximal). The coil has $N=1000$ windings, the uniform magnetic field is $\mathrm{B}=0.03 \mathrm{~T}$, and the coil diameter is $d=40 \mathrm{~cm}$. At $t=0$ the generator is turned on and instantly begins to spin at a frequency of $f=60 \mathrm{~Hz}$. Write an expression for the EMF of the generator as a function of time. Your expression should have the correct trig function, the correct value at $t=0$ and the correct maximum EMF.

SPN 11-06. Come up with a general formula for the mutual inductance $M_{\alpha} \beta$ of two coils of wire $\alpha$ and $\beta$ where coil $\beta$ fits entirely inside coil $\alpha$. Begin your derivation with the definition of mutual inductance. Sketch the Amperian and Faraday loops that you need for your derivation. Both coils may be assumed to be infinite solenoids. They have the following properties:

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diameter: \(d_{\alpha}, d_{\beta}\)
length: \(\ell_{\alpha}, \ell_{\beta}\)
number of windings: \(N_{\alpha}, N_{\beta}\)
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SPN 11-07. An actual measurement is more complex than in problem six. You are given two actual coils. A known voltage is applied to $\alpha$. What is the induced voltage $V_{\beta}$ ? What is the actual mutual inductance $M_{\alpha \beta}$, in Henries?

Voltage: $V_{\alpha}=44 \mathrm{~V}$
AC Resistance: $R_{\alpha}=500 \Omega, R_{\beta}=100 \Omega$
Applied frequency: 60 Hz
diameter: $d_{\alpha}=10 \mathrm{~cm}, d_{\beta}=2 \mathrm{~cm}$
length: $\ell_{\alpha}=\ell_{\beta}=0.1 \mathrm{~m}$
number of windings: $N_{\alpha}=3400, N_{\beta}=2900$

SPN 11-08. Come up with a general formula for the self inductance $L$ for a coil of length $\ell$, diameter $d$ and windings $N$. Also calculate the actual inductance if $L=10 \mathrm{~cm}, N=2900$ and $d=3 \mathrm{~cm}$.

SPN 11-09. Assume that the coil from problem 8 is made of resistanceless wire so that the entire voltage of the power supply goes to fighting back-emf $(\varepsilon)$. Assuming a voltage on the powersupply $V=V_{0} \cos (\omega t)$, $V_{0}=170 V$, and $f=300 \mathrm{~Hz}$. Come up with a numerical expression for the current $I(t)$ in the coil.

SPN 11-10a. Example 7.8 is weird enough that I have to ask you a numerical question to make sure you understand it. In Figure 7.26, assume that $B_{0}=1$ Tesla, $\mathrm{a}=1$ meter and $\mathrm{b}=2$ meters. Assume the B-field goes linearly from 1 Tesla to zero in 0.5 seconds (and remains zero afterwards). What is the magnitude and direction of the induced E-field at $\mathrm{r}=\mathrm{b}$ ? How about at $\mathrm{r}=\mathrm{a}$ ? How about $\mathrm{r}=\mathrm{a} / 2$ ? $\mathrm{r}=0$ ?
SPN 11-10b. Sketch $E_{\text {ind }}(r)$ vs. $r$ for, $0<r<5 a$. Also, if there is a charged ring of radius $b$ as described in Example 7.8, and $\lambda=3 \mathrm{milliC} / \mathrm{m}$, and the ring has a mass of $m=3 \times 10^{-4} \mathrm{~kg}$, how fast does it end up rotating? (The other conditions are the same as part 'a').

SPN 11-11. Do problem 7.13. Also, state which direction (clockwise or CCW) the induced current would flow.

SPN 11-12. Do problem 7.22. As the problem is written in Griffiths, the second integral is somewhat annoying. You can do it for $z=0$, which is much simpler. (We did this in class). Also, if you need formulae which we have already developed, you can quote the result from Griffiths rather than rederive it. Cite the equation number you found.

SPN 11-13. We had a lot of fun playing with the big rectangular magnet and the copper disk. The magnet had the North and South poles on two of the broad faces (not the ends). Draw two sketches showing the magnetic field lines at two different times as you bring the magnet toward the copper. Show the induced currents and show that $I \vec{\ell} \times \vec{B}$ produces the correct force. If the magnetic field lines came vertically out of the magnet would there be a repulsive force? (No!). Your sketch should reflect this.

