Physics 333 – Fall 2021
Due October 29.

Homework 08 - REV B

Instructions:

Each problem should begin at the top of a new sheet of paper. The final answer (numerical or symbolic) should be boxed (or written in a different color). (For proofs, this isn't reasonable, so don't do it.) Each problem should have your name on the left and, below it, the *SPN*, circled. Problems should (usually) include a 3x3 inch sketch and begin with the general equations and the assumptions you make.

SPN 8–01. Do a sketch that includes a pinto-bean charge distribution, $\vec{r_T}$, $\vec{r_S}$, $\mathscr{r}$, and α. Then make a Taylor expansion of $1/\mathscr{r}$ and show that you end up with

$$1/\mathscr{r} \ = \frac{1}{r_T}\Sigma(\frac{r_S}{r_T})^n \ P_n(cos\alpha) \tag{1}$$

explicitly for the case n=0,1,2 and 3. You do NOT need to prove the case for ALL n.

HINTS:

1. I recommend defining $\Delta = \frac{r_S}{r_T}$ so that you do not have to write so much.
2. I also recommend defining $\epsilon = \Delta^2 - 2\Delta cos\alpha$ and writing the Taylor series in terms of ϵ.
3. Do the expansion of the powers of ϵ and then group all the terms in Δ, Δ^2 and Δ^3. This will give you the needed Legendre polynomials (P_n)
4. You do not have to use ϵ and Δ. Feel free to make up your own notation. Use Hebrew, Arabic or Russian letters or make up a letter or borrow one from Dr. Seuss's "On Beyond Zebra".

SPN 8–02 – An induced dipole. Rederive the potential of a conducting sphere of radius R placed at the origin in an external field $\vec{E} = E_0 \ \hat{z}$. You may follow the steps in Example 3.8. Justify in your own words every step in the math (and fill in intermediate steps if needed.)

SPN 8–03. *–Conceptual question, section 4.1*
Give an example of a molecule that requires a polarizability tensor. Why does it need one? (As opposed to a simple constant).

SPN 8–04: Atomic polarizability. Assume that the model of an atom as a uniform sphere of charge is correct. Table 4.1 gives the atomic polarizability of several atoms. What is the radius of a Hydrogen atom? A Cesium atom?

SPN 8–05: Polar molecules. A water molecule is sketched in Figure 4.4. The angle between the two O-H bonds is given, but the length of the bond is not. Take the published value that $d_{O-H} = 0.95$ Å. Calculate the dipole moment of water. (Note: Griffith's quotes a value which is somewhat different than what you will get.)

SPN 8–06. *–Conceptual question, section 4.2*
Is a bound charge density "real"? Discuss.

SPN 8–07: Force between charge and polarized atom. A free proton *(*particle A) is located 1 Å from a neutral hydrogen atom (particle B). Particle A thus induces polarization in the hydrogen atom. Calculate the attractive force between A and B. There are two ways to do this problem, described below. Do it both ways.

a: Use the published value of $\alpha_{Hydrogen}$ to calculate p_B. You know the field of a dipole, so use that to calculate the force on A.
b: Use the same value of p_B together with equation 4.5 for calculate the force on B. Of course $F_{AB} = F_{BA}$.

SPN 8–08. *–Conceptual question, section 4.3*
Since both "free" and "bound" charges affect the E-field, why should we even bother to talk about the difference between them?

SPN 8–09. *–Polarization*
A polarized dielectric cube of side a is centered at the origin. It has no free charge on it or inside. It has a Polarization vector inside that varies as $\vec{P} = (P_0 x^2 z)\,\hat{z}$.
$a = 4\ cm$ and $P_0 = 3 \times 10^{-3}$

a: Sketch the cube and the coordinate axes.
b: What units does P_0 need to have to make $\vec{P}$ have the proper units?
c: What is the bound surface charge density σ_B on each surface?
d: What is the total surface charge on each surface?
e: What is the bound volume charge density ρ_B inside the cube?
f: What is the total bound volume charge inside the cube?

SPN 8–10. *–Conceptual question, section 4.4*

(a) You are told by a somewhat confused chemist that the "permittivity thingy" of unobtanium is 2×10^{-11} SI units. By "thingy" do they mean *Permittivity* or *Relative permittivity* ... and how do you know?
(b) Whichever one it was in part a, what's the value of the other one?

SPN 8–11. Complete the following table. For each symbol, write the technical name (e.g. α="polarizability") as well as the correct units. This ought to help organize the material in your head better. It will make good exam review.

Symbol	Description	SI Units
ϵ_0		
ϵ		
ϵ_r		
χ_e		
α		
$\vec{p}$		
$\vec{P}$		
$\vec{E}$		
$\vec{D}$		