Physics 333 – Fall 2021
Due October 20.

Homework 07 – Rev B

Instructions:

Each problem should begin at the top of a new sheet of paper. The final answer (numerical or symbolic) should be copied into a box (or written in a different color). Each problem should have your name on the left and, below it, the *SPN*, circled.

Problems should (usually) include a 3x3 inch sketch and begin with the general equations and the assumptions you make.

SPN 7–01. As promised I have provided code (*relax_hw.py*), which may be found here: `http://kestrel.nmt.edu/~rsonnenf/phys333/python_code/relax_hw.py`. This code implements the relaxation method to calculate the potential $V(x, y)$ inside a square tube of side 1 and infinite length. The four conducting sides of the tube are equipotentials, but they are separated from each other by insulators so that each can be set to a different potential. For the code provided, V(0,y)=1 Volt, while V(1,y)=V(x,0)=V(x,1)=0 Volts. The code has two easily adjustable quantities, *maxiterate* (set to 10,000) and *tolerance* (set to 1.0e-0). Every time you run it, it produces a new file with a *root* filename that you can set and that automatically includes the variables *iterate* and *tolerance* in the file name.

(a) Put your name in the title slot. Try adjusting the tolerance and see how it affects the result. What tolerance and how many iterations do you need before the calculations seem to your eye to be fully converged? (Hint, use a photo or image viewer program and rapidly switch between files with different tolerances. You can see quite small changes in solutions this way).

(b) Submit four figures representative of the "path to convergence". The first should show that you are obviously not converged. The next should be "close" to converged. The last two should be convincingly similar and support your selection of tolerance for full convergence. (Note: You could spend hours trying to get the tolerance accurate to many decimal places. That is not the intent! Question "a" is really just trying to get you to a tolerance of approximately the right order of magnitude.)

(c) Explain the functions of the variables *tolerance*, *iterate* and *maxiterate*.

(d) Change the code so that the boundary conditions are now V(0,y)=1 Volt, V(x,0)=0.5 Volt, V(1,y)=V(x,1)=0 Volts. Submit four figures showing path to convergence for these new boundary conditions.

SPN 7–02. The goal of this problem is assure you have mastered the separation of variables general method. You are welcome to follow along in the book as you do your solution. This problem only differs from section 3.3.1 in that V is a function of x, y, *and* z.

(a) Begin with Laplace's equation for V(x,y,z) and go through the separation process to yield ODEs. Briefly justify each step of the process. You may STOP when you get to the equivalent of equation 3.26 for this slightly more general problem.

(b) What is the relationship between the constants that appeared as you separated part "a"?

(c) Can you tell which of these constants are positive? If so, how? If not, what information would you need to decide?

SPN 7–03. Do Problem 3.29. (– *Moments of a discrete charge distribution*)

SPN 7–04. (– *Moments of a continuous charge distribution*)
A sphere of radius R is centered at the origin. It's charge density is expressed in spherical coordinates as

$$\rho(r, \theta) = a\frac{R}{r^2}(R - 2r)cos\theta.$$

where a is an arbitrary constant. Express the approximate potential along the z-axis where $z >> R$.

SPN 7–05. (– *Moments of a continuous charge distribution*)
This problem is exactly like 7–04 except for the $sin\theta$. However this change greatly changes the result! A sphere of radius R is centered at the origin. It's charge density is expressed in spherical coordinates as

$$\rho(r, \theta) = a\frac{R}{r^2}(R - 2r)sin\theta.$$

where a is an arbitrary constant. Express the approximate potential along the z-axis where $z >> R$.

SPN 7–06. Equal and opposite charges $\pm Q$ are on either end of the vector $\vec{L} = L\ \hat{z}$ m. Express your answers to $a - c$ in cartesian coordinates. For part d, you might find the integral easier to do in polar coordinates.

(a) What is the dipole moment $\vec{p}$ of this system? OK ... that took about 10 seconds. Yay!
(b) Assume for the rest of this problem that you are at a distance b large compared to L, so that only the dipole term of the potential matters. A point charge q is located at $r_1 = b\hat{x}$. What is the force on this charge?
(c) What is the force on the same charge q if it is located at $r_2 = b\hat{z}$?
(d) What work must you do to move q from r_1 to r_2?
(e) Express this work in Joules given that $Q = 3\ muC$, $L = 2$ m $b = 10$ m, and $q = 1\ mC$.