Physics 333 – Fall 2021

This is due October 6, 2021.

Homework 06

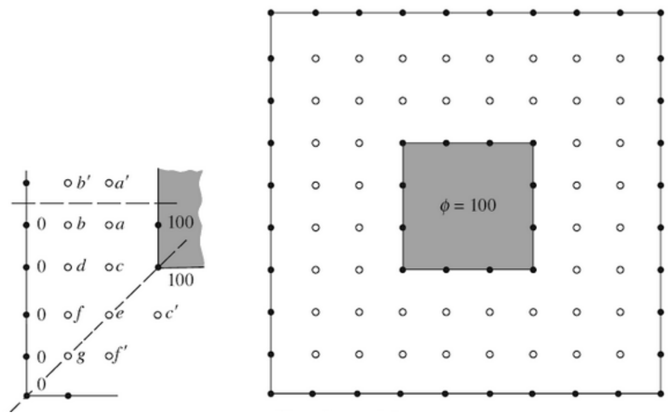
Instructions:

Each problem should begin at the top of a new sheet of paper. The final answer (numerical or symbolic) should be circled or clearly indicated in a different color. (You don't need to copy to top right of page.)

Each problem should have your name on the left and, below it, the *SPN*, circled. Problems should (usually) include a 3x3 inch sketch and begin with the general equations and the assumptions you make. For numerical answers, show numbers plugged into the equation before solving with a calculator. Numerical answers should include SI units.

SPN 5–01. Let the x-y plane be a grounded conductor. Charges $Q_1 = -1\ \mu C$ and $Q_2 = +3\ \mu C$ are stacked on the z-axis such that $\vec{r}_1 = 1\,\hat{z}\ cm$ and $\vec{r}_2 = 3\,\hat{z}\ cm$. Draw a sketch of this situation and then calculate the net force on Q_2. (–*Multiple image charges*)

SPN 5–02. Last week you calculated the capacitance of coaxial cylinders with radii "a" and "b". Begin with the expression you derived. (No need to rederive it). Calculate the form of the capacitance in the limit that $b = a + d$ and $d << a$. (Yes it goes to infinity ... but BEFORE that). Show that the formula derived is consistent with what you expect.



Problem 3: Relaxation method. Keep boundary potentials fixed.

SPN 5–03. I keep talking about the "relaxation method"; the numerical method most often used to solve general purpose Laplace equation problems. We can do a simple version of this by hand to see that it works. Next week I will give you some code to let you do it by computer.

A square grid represents two square conductors separated by an air gap. The outer conductor is fixed at 0 volts and the inner at 100 volts. By symettry there are only 7 unique nodes in the interior. These are labeled a–g. We pick an initial guess for each node. I recommend setting $a, c, e = 50$ and $b, d, f, g = 25$ initially. Refer to these values as $a_1, b_1, c_1, ...g_1$. In the relaxation method, you generate the next value for each node by taking an average of its four nearest neighbors. Thus, $c_2 = (1/4) * (100 + a_1 + d_1 + e_1)$. When you have calculated $a_2 ... g_2$ in this way, you repeat the process, generating $c_3 = (1/4) * (100 + a_2 + d_2 + e_2)$. Stop on convergence, defined as no value changes by more than one unit out of 100. At that point, draw the grid and estimate (and sketch) the eqiupotentials for $\phi = 25$ and $\phi = 50$. Submit on a piece of paper a table showing the a_n, b_n... through every iteration. For extra credit, you can figure out how to do this with a computer yourself. Run 10 iterations and use computer to plot.

SPN 5–04. Use plotting software to reproduce figure 3.19. Sample python code that does the plotting work can be found here: `http://kestrel.nmt.edu/~rsonnenf/phys333/python_code/`. This is also linked off the front page of Canvas for the course.

SPN 5–05. Use plotting software to reproduce figure 3.18. Use any plotting software you like. Sample python and matlab code that does the plotting work can be found here: `http://kestrel.nmt.edu/~rsonnenf/phys333/python_code/`. Of course you have to insert the correct potential.

SPN 5–06. Sticking with example 3.3, determine (and plot) the charge density on the strip at x=0, assuming the potential on the strip is a constant V_0. (– *Surface charge for example 3.3*)

SPN 5–07. A rectangular pipe, running parallel to the x-axis (from $-\infty \rightarrow \infty$) has three grounded metal sides, at y=0, y=a and z=0. The fourth side, at z=b, is maintained at potential $V_0(y)$. (– *Potential in a rectangular pipe*)

a. Solve for V(y,z) inside the pipe for the general form of $V_0(y)$.

b. Assume $V_0(y)$ is a constant 13 V. What is the specific solution now?

SPN 5–08. Check some things about the separable solution to Laplace's equation in spherical coordinates.

a. Show by direct differentiation that $R(r) = Ar^{\ell}$ and $R(r) = \frac{B}{r^{\ell+1}}$ satisfy equation 3.58 (– *Legendre Polynomials properties*)

b. Show by direct differentiation that $P_1(cos\theta)$ and $P_3(cos\theta)$ satisfy 3.60. Finally, show that P_1 dotted with P_2=0 and P_2 dotted with itself goes to $\frac{2}{2\ell+1}$. (Integrating the product of two Legendre polynomials over the defined interval is a generalized form of a dot product)

SPN 5–09. Do Problem 3.19 in Griffiths. (– *Potential and charge on a sphere*)