Physics 333 – Fall 2021 Due September 22 in class.

Homework 05

Instructions:

Where numerical calculations are requested, please provide numbers (no radicals, no factors of pi) to three sig. figs. Draw diagrams where appropriate (most of the time). Start every problem on a new page. You do not need to write your answers in the top right corner, but do indicate them clearly on the page (as in a box or with a colored highlighter.)

SPN 5–01. Four "point" charges (-0.001 C/each) are brought together from ∞ to form a square with sides of 0.5 meters.

(a) How much work does this take?

(b) If the charges are distributed uniformly over small spheres (negligible size compared to 0.5 meters) with mass 250 g each, and the charges are now released, what is their speed when the square has increased to 2 meter sides?

SPN 5–02. For EACH of the following four statements, write a brief explanation or sketch/label a picture/diagram that explains why it is true:

$$\nabla \times \vec{E} = 0 \text{ means } \oint \vec{E} \cdot d\vec{l} = 0 \tag{1}$$

$$\oint \vec{E} \cdot d\vec{l} = 0 \text{ means } \int_a^b \vec{E} \cdot d\vec{l} \text{ is independent of the chosen path.} \tag{2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} \text{ is independent of the chosen path means } \vec{E} = -\nabla V. \tag{3}$$

$$\vec{E} = -\nabla V \text{ means } \nabla \times \vec{E} = 0 \tag{4}$$

SPN 5–03. You are given a potential $V(x, y) = -Kxy$ (K a constant).

(a) Calculate the electric field ($\vec{E} = -\nabla V$).

(b) Sketch the electric field by putting appropriate vectors on a grid

(c) Calculate the curl $\nabla \times \vec{E}$ and state whether this results is consistent with your plot and why

(d) Repeat parts b and c if you are given $\vec{E} = Ky\hat{\mathbf{x}} - Kx\hat{\mathbf{y}}$.

SPN 5–04. Parallel-plate capacitors A parallel-plate capacitor is defined by two plates whose separation (z) is small compared to their other dimensions (x, y). The top plate has charge Q, and the bottom has equal and opposite charge $-Q$. This entirely confines the electric field to the region between the plates as well as making it purely in the z direction. Given this background:

(a) A parallel plate capacitor is made from square plates with side s and separation d. Use Gauss's law to determine the field between the plates given charges $Q/-Q$.

(b) Use $V(z) = -\int \vec{E} \cdot d\vec{l}$ to calculate the potential difference between the plates.

(c) Use result "b" and the definition Q=CV to arrive at a formula for the capacitance between the two plates. *Note: Q=CV defines capacitance. The parallel plate formula you just derived is NOT the definition of capacitance.*

(d) Use the definition $Q = CV$ and the relation $U = qV$ to begin with a discharged capacitor and arrive at the formula $U = \frac{1}{2}CV^2$ for total energy stored in a capacitor.

(e) Use results "a" and "d" to derive the formula for the energy stored in an electric field ($u = \frac{1}{2}\epsilon_0 E^2$). Why is it little u and not big U?

SPN 5–05. Capacitors can be made with rolled up aluminum foil plates separated by wax paper. Pick a reasonable thickness of aluminum foil and wax paper and "design" a 1000 pf capacitor. How long is it? What is the diameter of the rolled up cylinder? What did you have to do to the wax paper to stop the capacitor from being too darn large? (There is no single right answer to this problem. Simply explain your approach.)

SPN 5–06. (Related to 2–08) In a coaxial cable, a wire runs down the middle of a conducting tube made of metal foil or copper fabric. The tube is wrapped around a plastic "dielectric" spacer. Dielectrics are chapter 4. For now, let the spacer be air and let the wire stay centered in the tube by magic. (Or just ignore the dielectric!). Let the wire have a diameter of α and the tube have a diameter of β. The wire has a total charge Q and long length L while the tube has an equal and opposite charge -Q and the same length.

(a) What is the potential difference between the wire and the tube (You may simply quote the answer from problem 2–08 without rederivation.)
(b) This setup *is* a cylindrical capacitor. What is its capacitance?
(c) If $\alpha = 1\ mm$, $\beta = 10\ mm$, and $L = 6\ m$, what is the capacitance? (Give a number in Farads).
(d) If there were a dielectric, and not magic, separating the wire from the tube, it would reduce the electric field by a factor of three everywhere between α and β (without reducing the charge on either conductor. What is the capacitance now? You do not need to do any more math, but you do need a one sentence explanation in support of your new answer.

SPN 5–07. We've done cylindrical and planar capacitors now. The third "easy" capacitor to calculate is two concentric spheres (even if it's not practical). Consider oppositely charged concentric spheres of radii one and two meters. 1000 Volts is applied between them. What is their capacitance, and what are their charges?

SPN 5–08. Do Problem 2.39 in Griffiths. (– *Behavior of charged conductors*)

SPN 5–09. An electric potential varies along the x-axis as follows. What is the charge density $\rho(x)$ at x=$\frac{1}{2}$ m and x=$\frac{3}{4}$ m? (Numbers and units). (Big Hint: $\nabla^2 V = -\rho/\epsilon_0$)

$$V(x) = \begin{cases} 0 & (for \quad x < 0 \\ 2x^3 - 3x^2 + 7 & (for \ \ 0 \le x \le 1) \\ 0 & (for \quad x > 1 \end{cases}$$

SPN 5–10. A potential expressed in spherical coordinates is:

$$V(r, \theta, \phi) = \frac{kcos\theta}{r^2} \text{ Note: there is no } \phi \text{ dependence.} \tag{5}$$

(a) Express $\vec{E}$ in spherical coordinates.
(b) Assume $\phi = 0$. Rexpress $\vec{E}$ in cartesian coordinates (Hint: Look on inside back cover of book).
(c) Assume magnitude of $k = 1$. Assume $r = 10\ cm$. Calculate $\vec{E}$ at $\theta = 0, \theta = \pi/4, \theta = \pi/2$.
(d) Calculate ρ for this potential. If you do it right you will get a surprising result. Comment on the surprise and whether you think math or physics "broke".