Homework 01

Instructions:

- Each problem should begin at the top of a new sheet of paper.
- Each problem should be numbered and circled (or in a different color).
- Problems should (usually) include a 3x3 inch sketch and begin with the general equations and the assumptions you make. Some problems are purely numerical, but almost all will benefit from a labeled sketch to define the terms, origin, coordinates, or geometry you are using. If you omit a sketch where one is appropriate, points will be deducted.
- Final answers (numerical or symbolic) should be on a separate line from the other calculations and surrounded by a neatly drawn box (or written in a different color).
- Calculations should be well organized and neatly arranged. For example, if you arrive at intermediate answers, perhaps label and box these as well and then maybe indicate the final answer with a double box!
- For numerical answers, show numbers plugged into the equation before solving with a calculator.
- Numerical answers should include SI units.

SPN 1–01. The distance between two protons in a helium nucleus is roughly one “fermi” (femtometer). How large is the electrical repulsion between them? Compare this repulsion with their gravitational attraction. Why does the nucleus stay together? (a sentence, not a calculation)

SPN 1–02. Two charged conducting spheres with a diameter of 8 cm each hang as indicated in the figure. The angle \(\theta = 7^\circ\). The gap \(d = 0.5\ m\). What is the charge on each ball? What approximation did you have to make to solve the problem?

SPN 1–03. An equilateral triangle of side length \(S = 2\ m\) meters has its base resting on x-axis and centered at origin. Point A is to left of origin and Point B is equal distance to right. The Apex at C is on the y-axis. The test point, Q, is centered at (-1,1). The charges on A, B, C, and Q are +1 milliC, 2 mC, -1 mC, and 30 \(\mu\)C respectively. What is the vector force on Q? What is the Electric field vector at point Q?

SPN 1–04. A sphere of radius \(R\) has uniform charge density \(\rho\) and total charge \(Q\). Use Gauss’s Law to arrive at \(E(r)\) for \(r < R\) and \(r > R\). Draw a sketch that indicates the Gaussian surface you are using and draw a couple of representative \(d\bar{A}\)s on it. Give your answer in two forms; in terms of \(\rho\) and in terms of \(Q\).

SPN 1–05. Gauss’s law applies to the gravitational field \(\vec{g}\) (for spherical or point masses) just as well as it applies to the electric field \(\vec{E}\) (for spherical or point charges). Beginning with Newton’s Law of gravitation and Coulomb’s Law, work by analogy to arrive at Gauss’s law for gravitation. Specifically, if \(\int \vec{E} \cdot d\bar{A} = Q_{enc}/\epsilon_0\), complete the equation \(\int \vec{g} \cdot d\bar{A} = ?\)

SPN 1–06. If you lived in Southern Argentina and could dig straight through the Earth, you would end up in China. Let’s imagine someone in Argentina digs a straight shaft through the Earth to China. The shaft is narrow enough that it does not disturb the local value of \(\vec{g}\). Also, air resistance may be ignored (not a valid assumption, but then neither is digging a clean hole straight through the earth).

(a) Write down the differential equation of motion for the person who jumped into the shaft.
(b) How long after the person enters the shaft do they resurface in China. (You have enough information to get a numerical answer, in minutes.)

Hint: The differential equation you got in part A, if you did it correctly, should be very familiar. Hope you have an ”aha” moment.
SPN 1–07. Consider a high-voltage direct current power line that consists of two parallel conductors suspended five meters apart. The lines are oppositely charged. If the electric field strength midway between the wires is 4000 N/C, how much excess positive charge resides on 100 m of the positive conductor? This problem can be done by direct integration or Gauss’s law. Do it with Gauss’s law. Show the Gaussian surface you use for one wire and a few representative dÃs.

SPN 1–08. Find the angle between the following two vectors: \( \vec{A} = \hat{x} + 2\hat{y} + 3\hat{z} \) and \( \vec{B} = 3\hat{x} - \hat{y} \). \(- Use a dot-product to find angle between vectors\)

SPN 1–09. Given two vectors \( \vec{A} \) and \( \vec{B} \), the cross product \( \vec{A} \times \vec{B} \) is normal to both vectors. Using this relation, find the unit vector normal to the vectors \( \vec{A} \) and \( \vec{B} \) from the previous problem.

SPN 1–10. Do Griffiths Problem 1.19. \(- geometric interpretation of curls\)

SPN 1–11. Show that the curl of a gradient is zero in cartesian coordinates.

SPN 1–12. Show that the curl of a gradient is zero in cylindrical coordinates.

SPN 1–13. Find the divergence AND the curl of the vector field \( \vec{V} = (x^2 + yz)\hat{x} + (y^2 + zx)\hat{y} + z\hat{z} \).

SPN 1–14. Do Problem 1.34. \(- test Stoke’s theorem\)

SPN 1–15. Do Problem 1.44. \(- delta-function practice\)

SPN 1–16. Test the divergence theorem for the function of problem 13. Take as your volume the cube shown in figure 1.30 of Griffiths, with sides of length 2. \(- test the divergence theorem\)