Polarization
- Bound Charges
- Gauss's law with bound charges
- Conducting sphere in external E-field

Electric Fields in Matter

Neutral atoms may be polarized

$$\vec{p} = \alpha \vec{E}$$

Alpha is called "polarizability"

$$\vec{P} = N\vec{p} = N\alpha \vec{E}$$

Polarization is dipole moment/volume

To keep things clear

$$\vec{P} \overset{\text{def}}{=} \epsilon_0 \chi_E \vec{E}$$
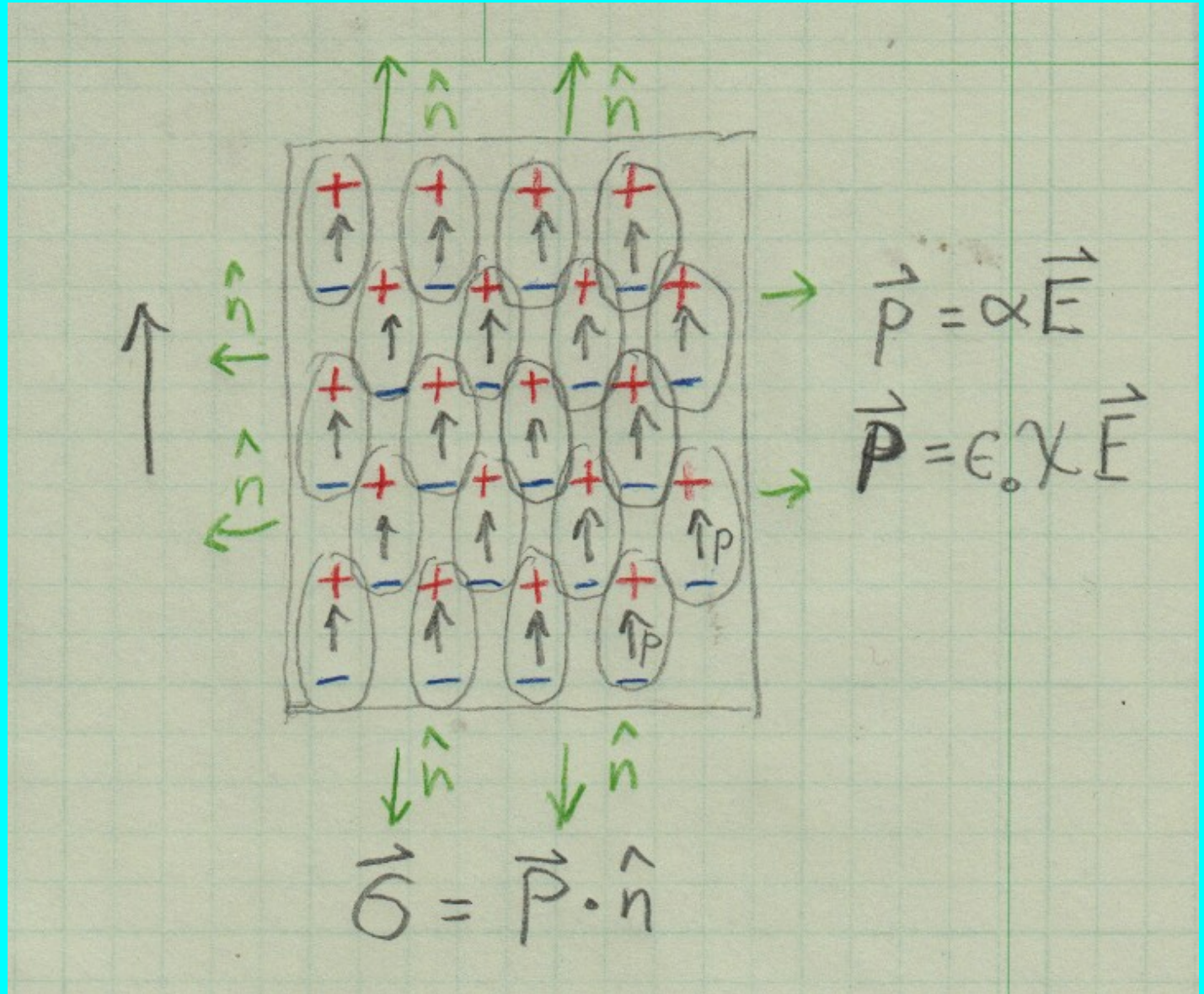
Chi is called "electric susceptibility"

In case you aren't confused

$$\epsilon \overset{\text{def}}{=} \epsilon_0 (1 + \chi_E)$$

Epsilon is called "permittivity"

$$\epsilon_r \overset{\text{def}}{=} (1 + \chi_E)$$

Epsilon_r is called "relative permittivity" or "dielectric constant"
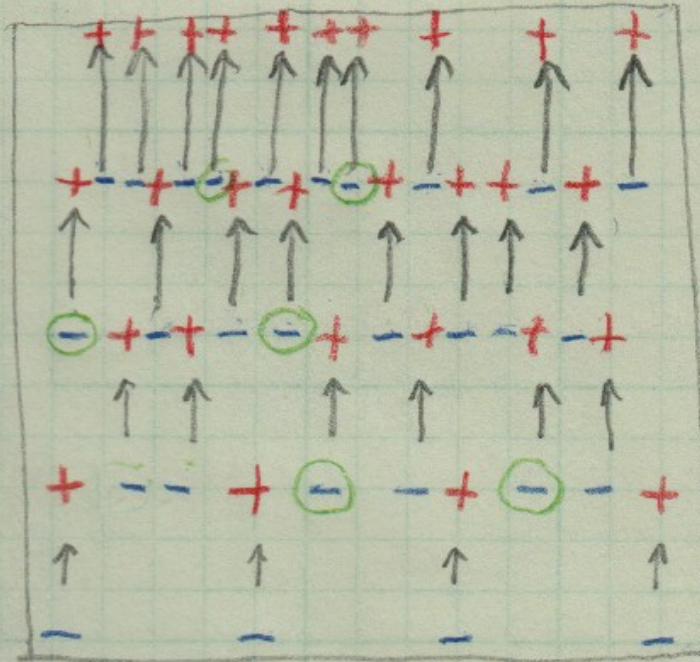
Bound surface charges

$$\sigma_B = \vec{P} \cdot \hat{n}$$

Bound bulk charges

$$\rho_B = -\nabla \cdot \vec{P}$$

$$\vec{P} = (z_0 + 2z) \cdot \quad -\nabla \cdot \vec{P} = -2$$

$$\rho_b = -\nabla \cdot \vec{P}$$

Bound charge example

Cube of side 2L centered at origin with polarization
$$\vec{P} = kyx\,\hat{x}$$

What is the bound surface and volume charge distribution?

Gauss' Law with free (controlled) and bound (induced) charges

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \rho_{Free} + \rho_{Bound}$$

$$\rho_{Bound} = -\nabla \cdot \vec{P}$$
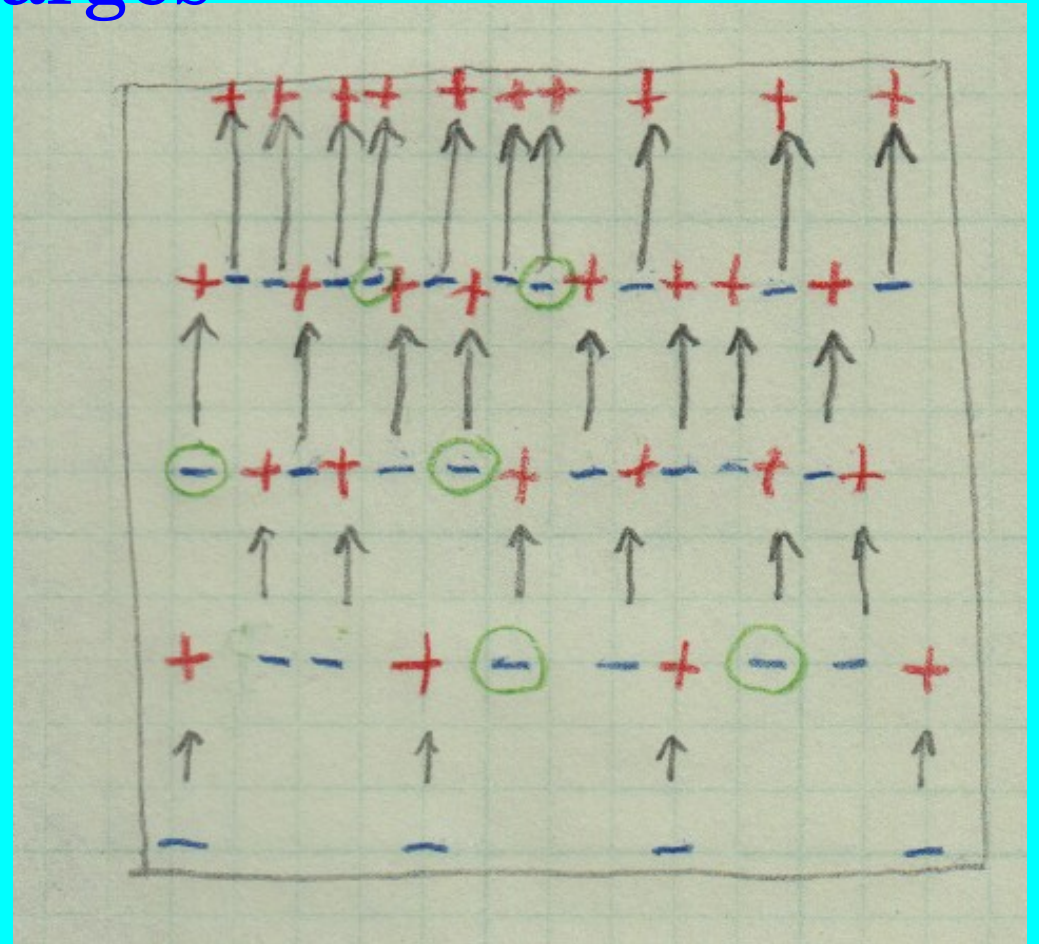
$$\epsilon_0 \nabla \cdot \vec{E} = \rho_F - \nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_F - \nabla \cdot \vec{P}$$

$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_F$$

$$(\epsilon_0 \vec{E} + \vec{P}) \overset{\text{def}}{=} \vec{D}$$

$$\nabla \cdot \vec{D} \overset{\text{def}}{=} \rho_F$$

Using Gauss' law in a dielectric

A wire (1 mm diameter) has linear charge density (lambda) 35 nC/m.

It is surrounded by a dielectric insulator out to 2 cm.

What are D and E at 1 cm?
At 3 cm?

Bound charge example

Cube of side 2L centered at origin with polarization

$$\vec{P} = kyx \, \hat{x}$$

Where and what are the free and bound charges?

Bound charge example

Cube of side 2L centered at origin with polarization

$$\vec{P} = kyx\,\hat{x}$$

What is the bound volume charge in the cube middle?

$(A)\quad kL^2$

$(B)\quad 4\,kL^2$

$(C)\quad 2\,kL$

$(D)\quad kL$

$(E)\quad 0$