Physics 3034 – Spring 2024

Upload by January 25.

Homework 01

For numerical answers, show numbers plugged into the equation before solving with a calculator. Numerical answers should include SI units.

SPN 1–01 The original form of Ampere's law (before Maxwell) is

$$\nabla \times \vec{B} = \mu \vec{J} \tag{1}$$

[**a**] Take the divergence of this equation and explain (briefly) the problem.
[**b**] Repeat the previous step including the Maxwell Current density and show that the problem is fixed.

SPN 1–02 – Energy in a magnetic field.
The energy stored in an inductor is $U = \frac{1}{2}LI^2$. This can be used to derive the energy density in a magnetic field.

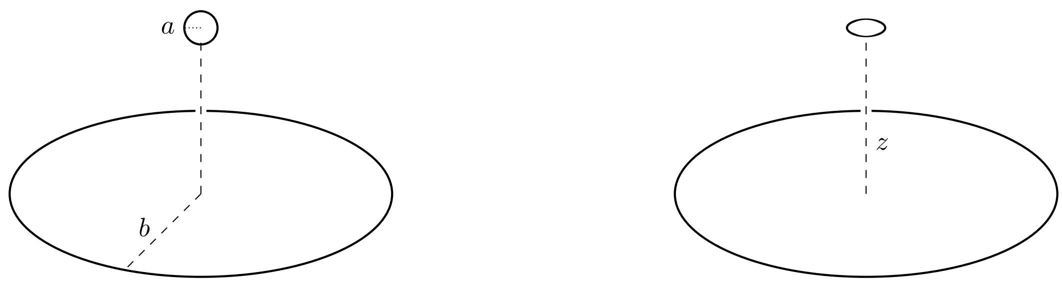
[**a**] Beginning with Ampere's law, derive an expression for the magnetic field in a long solenoid of radius s, length ℓ, with total windings N and current I. (You have done this before, but it is worthy review.)
[**b**] Using the result from part a, and the definition of inductance $\Phi = LI$, derive an expression for the self inductance of this solenoid.
[**c**] Use the result from parts a and b to derive the energy density in a magnetic field.
[**d**] A neutron star can have a magnetic field of a Teragauss. How much energy is stored in a cubic centimeter of neutron star magnetic field?
[**e**] Express the answer to d in kilowatt-hours.

SPN 1–03 – Application of displacement current.
Do Griffiths 7.34.

SPN 1–04 – Application of displacement current.
Do Griffiths 7.40.

SPN 1–05 Displacement current vs. conduction current in a wire.
Imagine a copper wire of diameter $d = 1$ mm that is carrying a current as follows $I = I_0 cos(\omega t)$, where $I_0 = 100$ Amperes.

[**a**] Let $f = 1\ kHz$ and derive an expression for $\vec{E}(t)$ inside the wire.
[**b**] In this case, compare the maximum displacement current to the normal current in the wire.
[**c**] Assuming that nothing changes but the frequency, repeat this calculation for $f = 1\ THz$
[**d**] Assume next that the current is oscillating at the frequency of green light ($\lambda = 500\ nm$). What is the frequency of green light?
[**e**] Compare the conventional and the displacement current in this case.
[**f**] At what frequency and wavelength of light would the conventional and displacement currents become equal?



Problem 6: Small wire loop (radius a) centered above large loop (radius b). Their centers are separated by a vertical distance z.

SPN 1–06 Mutual inductance Small wire loops "A" (radius a) are centered above large loops "B" (radius b). The large loops lie in the $x-y$ plane. The small loop on the right also lies in $x-y$ plane. However, the small loop on the left lies in the $x-z$ plane.

[The figure is intended to accurately convey the relative orientations of the loops. As drawn $a < b$. Feel free to assume in your answers that $a << b$.]

(a) Write down the definition of mutual inductance.
(b) For which configuration is the mutual inductance larger? It is not necessary to calculate the mutual inductances to answer this, you can justify your answer with a sketch and a sentence.
(c) For the case on the right, calculate the mutual inductance M_{AB}.