Lecture 17 02/21/2024

- Reflection/Transmission at Oblique incidence
  - Fresnel Equations
  - Brewster's Angle

Intensity at Normal Incidence

$$\tilde{E}_R = \tilde{E}_I \frac{v_T - v_I}{v_T + v_I} = \tilde{E}_I \frac{n_1 - n_2}{n_1 + n_2}$$

$$R \stackrel{\text{def}}{=} \frac{I_R}{I_I} = \frac{\epsilon_1 v_1 E_R^2}{\epsilon_1 v_1 E_I^2} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

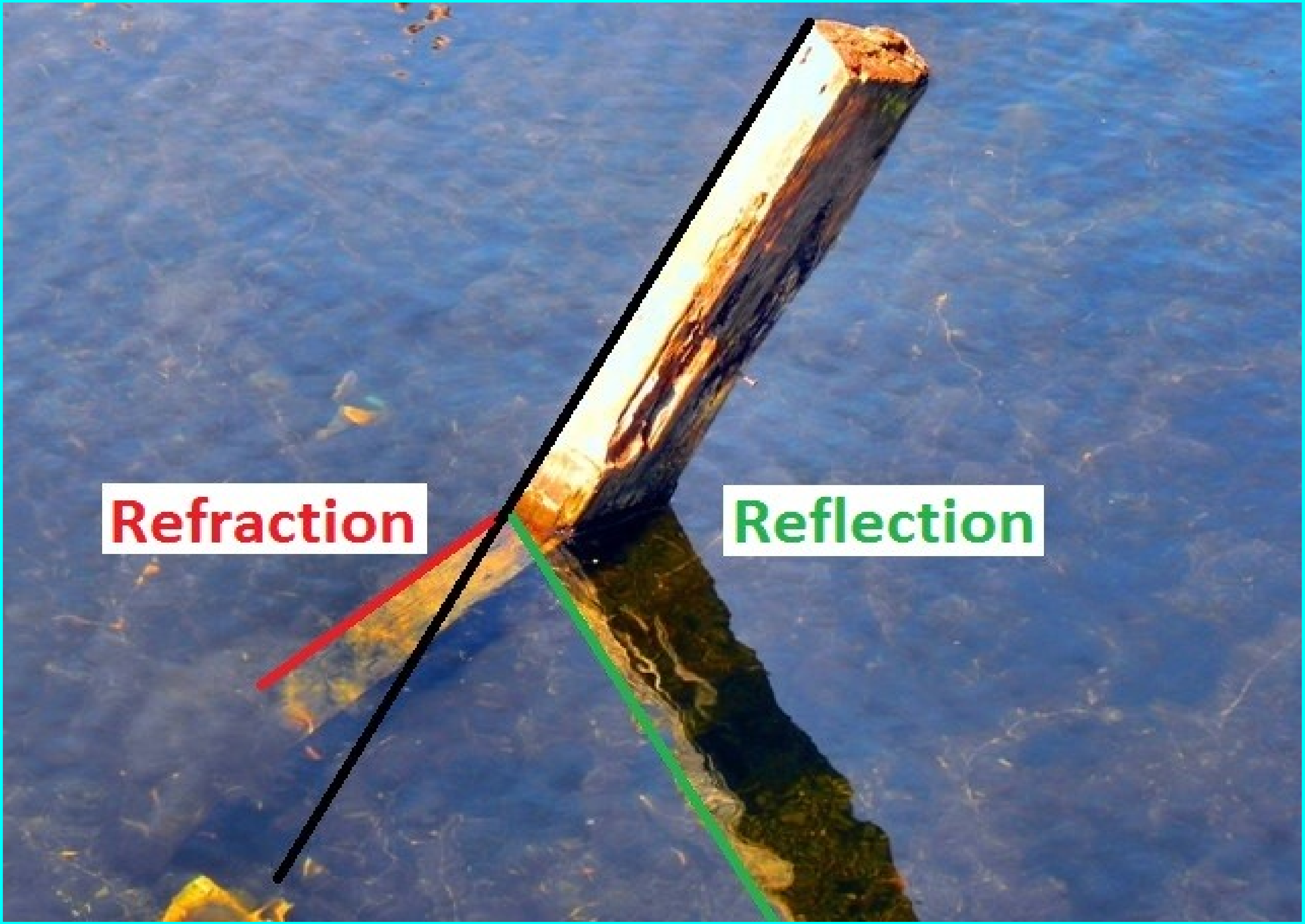
$$\tilde{E}_T = \tilde{E}_I \frac{2 v_T}{v_I + v_T} = \tilde{E}_I \frac{2 n_1}{n_1 + n_2}$$

$$T \stackrel{\text{def}}{=} \frac{I_T}{I_I} = \frac{\epsilon_2 v_2 E_T^2}{\epsilon_1 v_1 E_I^2} = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

*The intensity formulae are only correct if $\mu_I = \mu_R = \mu_{0.}$

$$R + T = 1$$

Reflection at surface of linear media for oblique incidence



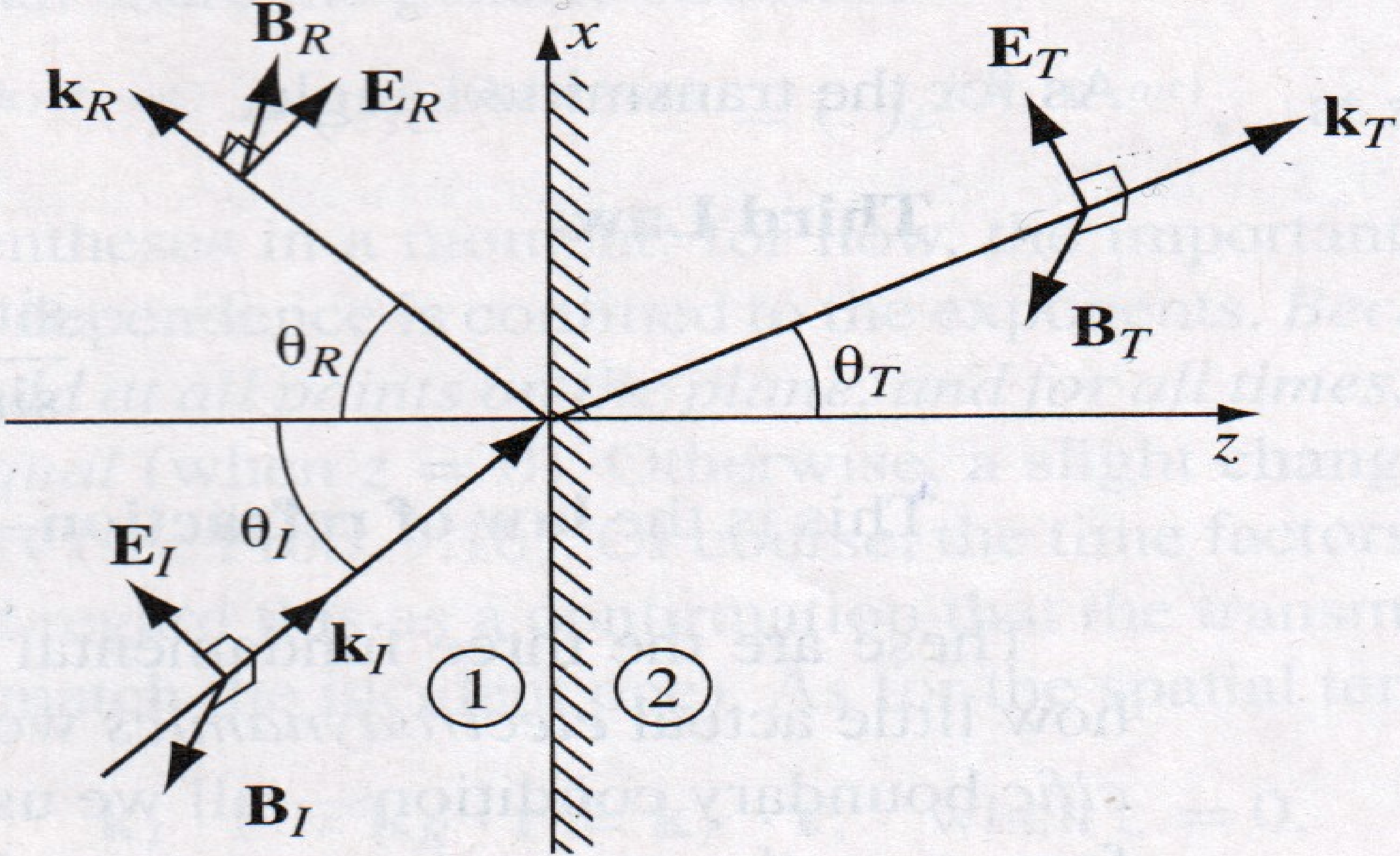
Continuity / Boundary Conditions – Linear Media, no free currents or charges

$$\vec{D}_1^{\perp} = \vec{D}_2^{\perp} \rightarrow \epsilon_1 \vec{E}_1^{\perp} = \epsilon_2 \vec{E}_2^{\perp}$$

$$\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$$

$$\vec{B}_1^{\perp} = \vec{B}_2^{\perp}$$

$$\vec{H}_1^{\parallel} = \vec{H}_2^{\parallel} \rightarrow \frac{\vec{B}_1^{\parallel}}{\mu_1} = \frac{\vec{B}_2^{\parallel}}{\mu_2}$$



$\mathbf{B}_R$
$\mathbf{k}_R$
$\mathbf{E}_R$
x
$\mathbf{E}_T$
$\mathbf{k}_T$
$\mathbf{B}_T$
θ_R
θ_T
z
θ_I
$\mathbf{E}_I$
$\mathbf{k}_I$
1
2
$\mathbf{B}_I$

FIGURE 9.15

Reflection at surface of linear media for oblique incidence

[i] $\vec{D}_1^{\perp} = \vec{D}_2^{\perp}$

[ii] $\vec{B}_1^{\perp} = \vec{B}_2^{\perp}$

[i] $\epsilon_1 \vec{E}_1^{\perp} = \epsilon_2 \vec{E}_2^{\perp}$

[iv] $\vec{H}_1^{\parallel} = \vec{H}_2^{\parallel}$

[iii] $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

[iv] $\frac{\vec{B}_1^{\parallel}}{\mu_1} = \frac{\vec{B}_2^{\parallel}}{\mu_2}$

[i] $\epsilon_1 (E_{0I} + E_{0R})_z = \epsilon_2 (E_{0T})_z$

[ii] $(B_{0I} + B_{0R})_z = (B_{0T})_z$

[iii] $(E_{0I} + E_{0R})_{x,y} = (E_{0T})_{x,y}$

[iv] $\frac{1}{\mu_1} (B_{0I} + B_{0R})_{x,y} = \frac{1}{\mu_2} (B_{0T})_{x,y}$

[i] $\epsilon_1 \left(E_{0I} + E_{0R} \right)_z = \epsilon_2 \left(E_{0T} \right)_z$

[iii] $\left(E_{0I} + E_{0R} \right)_x = \left(E_{0T} \right)_x$

[iv] $\frac{1}{\mu_1} \left(B_{0I} + B_{0R} \right)_y = \frac{1}{\mu_2} \left(B_{0T} \right)_y$

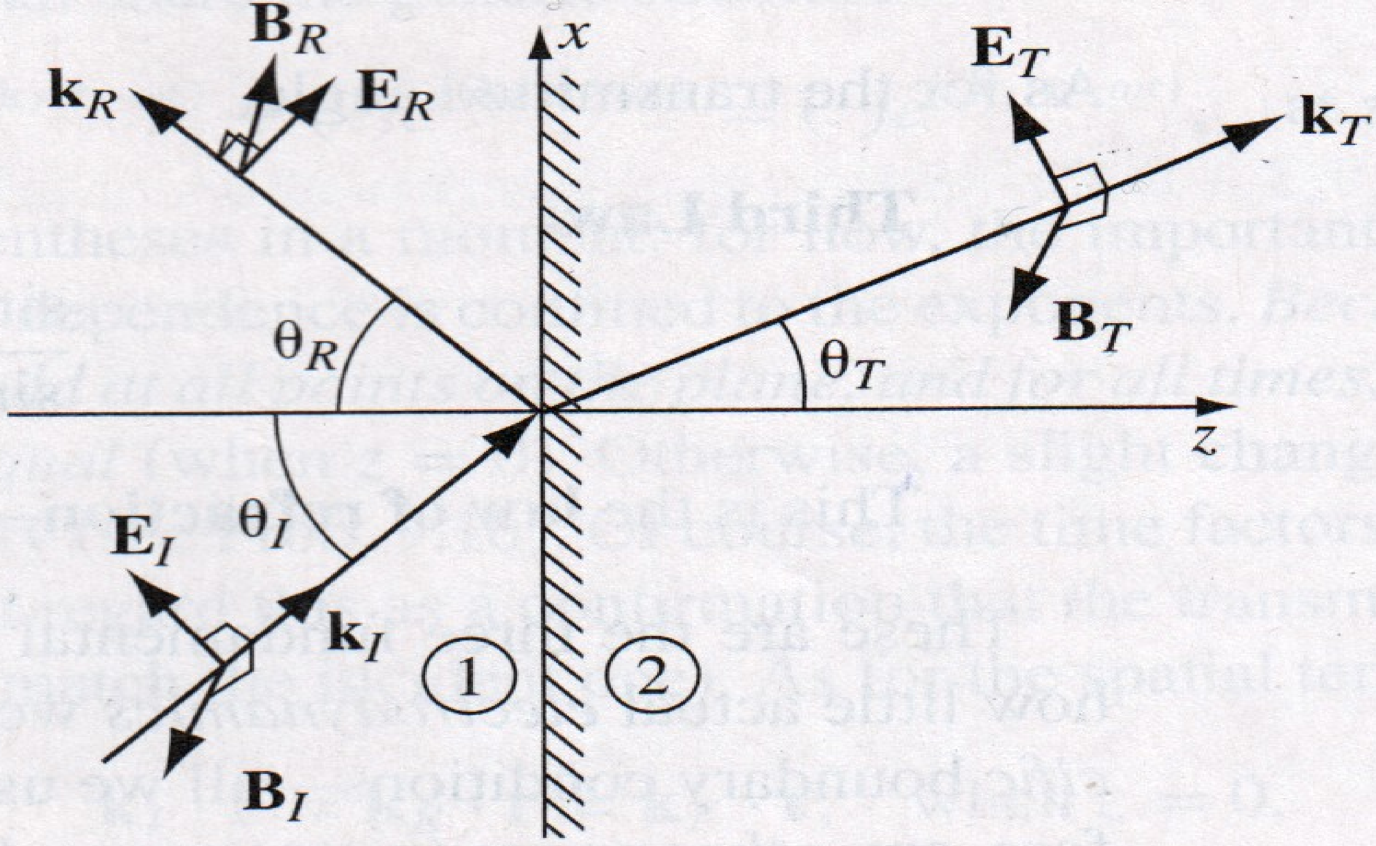


FIGURE 9.15

[iii] $\left(E_{0I}+E_{0R}\right)_x=\left(E_{0T}\right)_x$

$$\left(E_{0I}\cos\theta_I+E_{0R}\cos\theta_R\right)=\left(E_{0T}\right)\cos\theta_T$$

$$\left(E_{0I}+E_{0R}\right)=\left(E_{0T}\right)\alpha$$

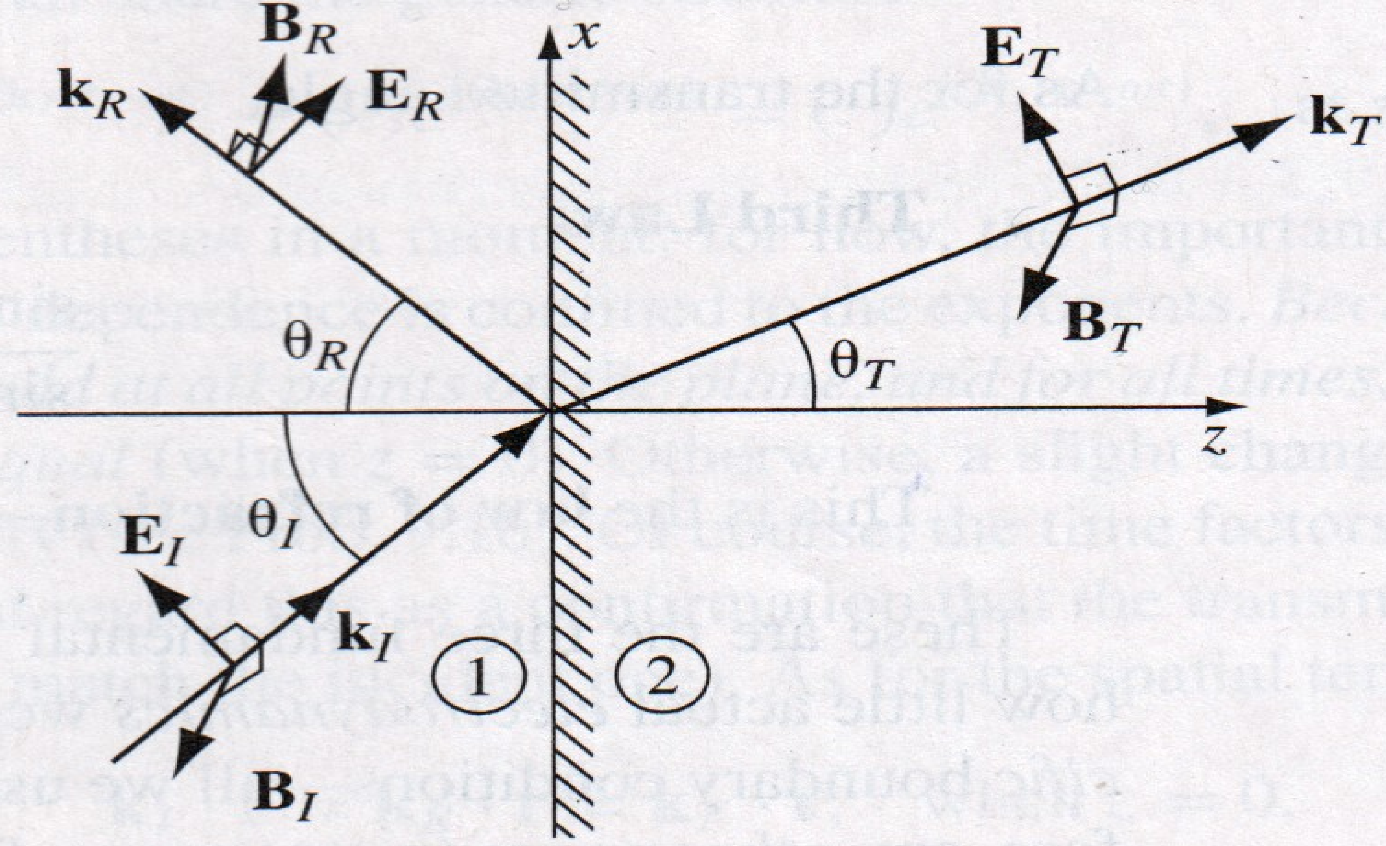


FIGURE 9.15

$$[\text{iv}] \quad \frac{1}{\mu_1}\left(B_{0I}+B_{0R}\right)_y=\frac{1}{\mu_2}\left(B_{0T}\right)_y$$

$$\frac{1}{\mu_1 v_1}\left(E_{0I}-E_{0R}\right)=\frac{1}{\mu_2 v_2}\left(E_{0T}\right)$$

$$\left(E_{0I}-E_{0R}\right)=\left(E_{0T}\right)\beta$$

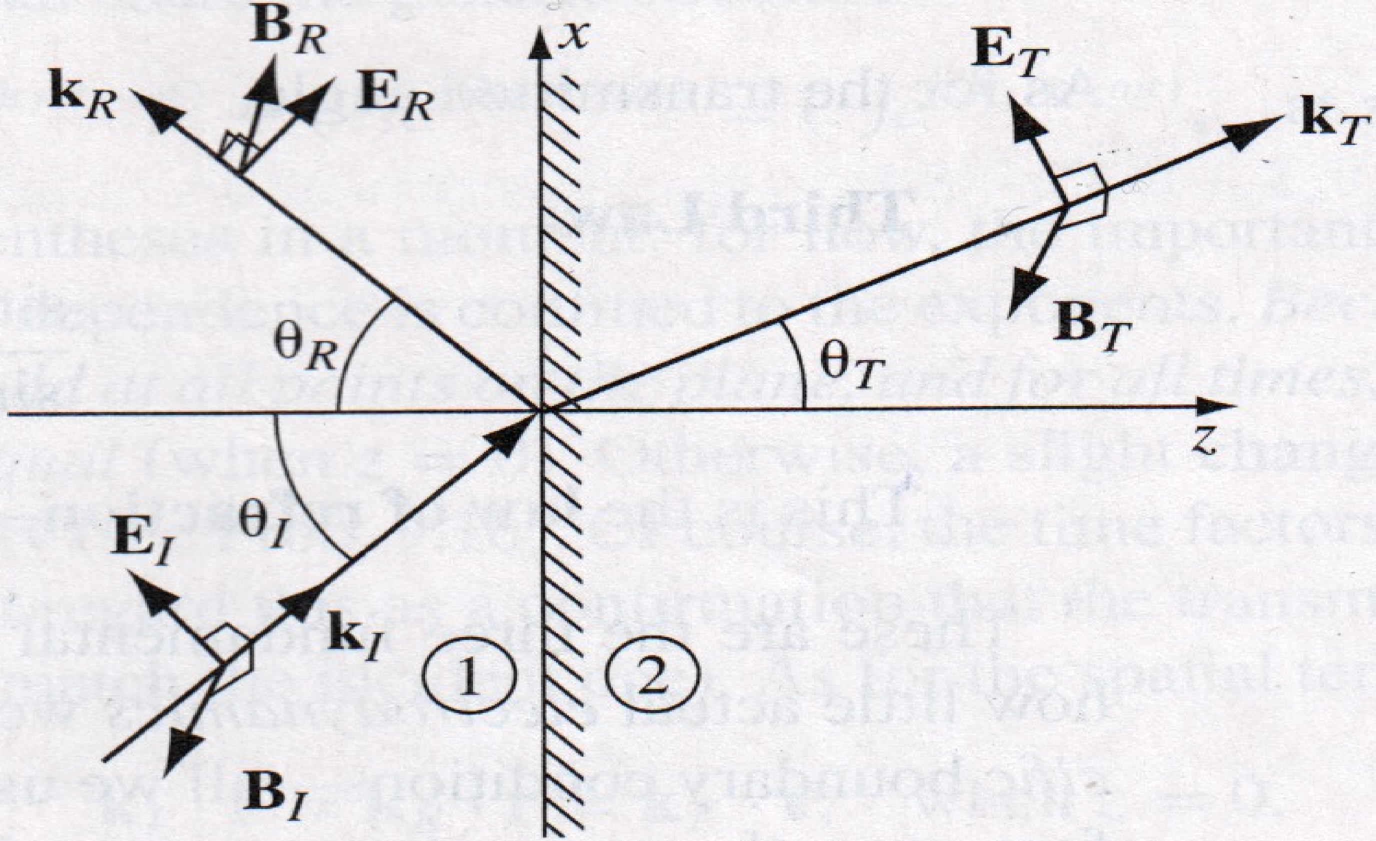


FIGURE 9.15

$$\left(E_{0I}+E_{0R}\right)=\left(E_{0T}\right)\alpha \qquad \frac{\cos\theta_T}{\cos\theta_I}=\alpha$$

$$\left(E_{0I}-E_{0R}\right)=\left(E_{0T}\right)\beta \qquad \frac{n_2\mu_1}{n_1\mu_2}=\beta$$

- Polarization dependent reflection, Fresnel equations.

$$E_{0R} = E_{0I}\frac{\alpha - \beta}{\alpha + \beta}$$

$$\frac{\cos\theta_T}{\cos\theta_I} = \alpha = \frac{\sqrt{1 - (n_1/n_2)^2 \sin^2\theta_I}}{\cos\theta_I}$$

$$E_{0T} = E_{0I}\frac{2}{\alpha + \beta}$$

$$\frac{n_2\mu_1}{n_1\mu_2} = \beta$$

Reflection Coefficient at Normal Incidence

$$\tilde{E}_{R-Glass} = \tilde{E}_I \frac{n_1 - n_2}{n_1 + n_2} = \tilde{E}_I \frac{1.5 - 1.0}{2.5} = 0.20\,\tilde{E}_I$$

$$R \stackrel{def}{=} \frac{I_R}{I_I} = \frac{\epsilon_1 v_1 E_R^2}{\epsilon_1 v_1 E_I^2} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

Reflection Coefficient at Oblique Incidence

$$E_{0R} = E_{0I} \frac{\alpha - \beta}{\alpha + \beta} \qquad \alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1 - (n_1/n_2)^2 \sin^2\theta_I}}{\cos\theta_I}$$

$$\beta = \frac{n_2 \mu_1}{n_1 \mu_2}$$

$$R \stackrel{def}{=} \frac{I_R}{I_I} = \frac{\epsilon_1 v_1 E_R^2 \cos\theta_R}{\epsilon_1 v_1 E_I^2 \cos\theta_I} = \frac{(\alpha - \beta)^2}{(\alpha + \beta)^2}$$

Transmission Coefficient at Normal Incidence

$$\tilde{E}_{R-Glass} = \tilde{E}_I \frac{2 n_1}{n_1 + n_2}$$

$$T \stackrel{\text{def}}{=} \frac{I_T}{I_I} = \frac{\epsilon_2 v_2 E_T^2}{\epsilon_1 v_1 E_I^2} = \frac{(4 n_1 n_2)}{(n_1 + n_2)^2}$$

Transmission Coefficient at Oblique Incidence

$$E_{0T} = E_{0I} \frac{2}{\alpha + \beta} \qquad \alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1 - (n_1/n_2)^2 \sin^2\theta_I}}{\cos\theta_I}$$

$$\beta = \frac{n_2 \mu_1}{n_1 \mu_2}$$

$$T \stackrel{\text{def}}{=} \frac{I_T}{I_I} = \frac{\epsilon_2 v_2 E_T^2 \cos\theta_T}{\epsilon_1 v_1 E_I^2 \cos\theta_I} = \alpha\beta \times \frac{4}{(\alpha + \beta)^2}$$

Reflected electric field of glass vs. incidence angle

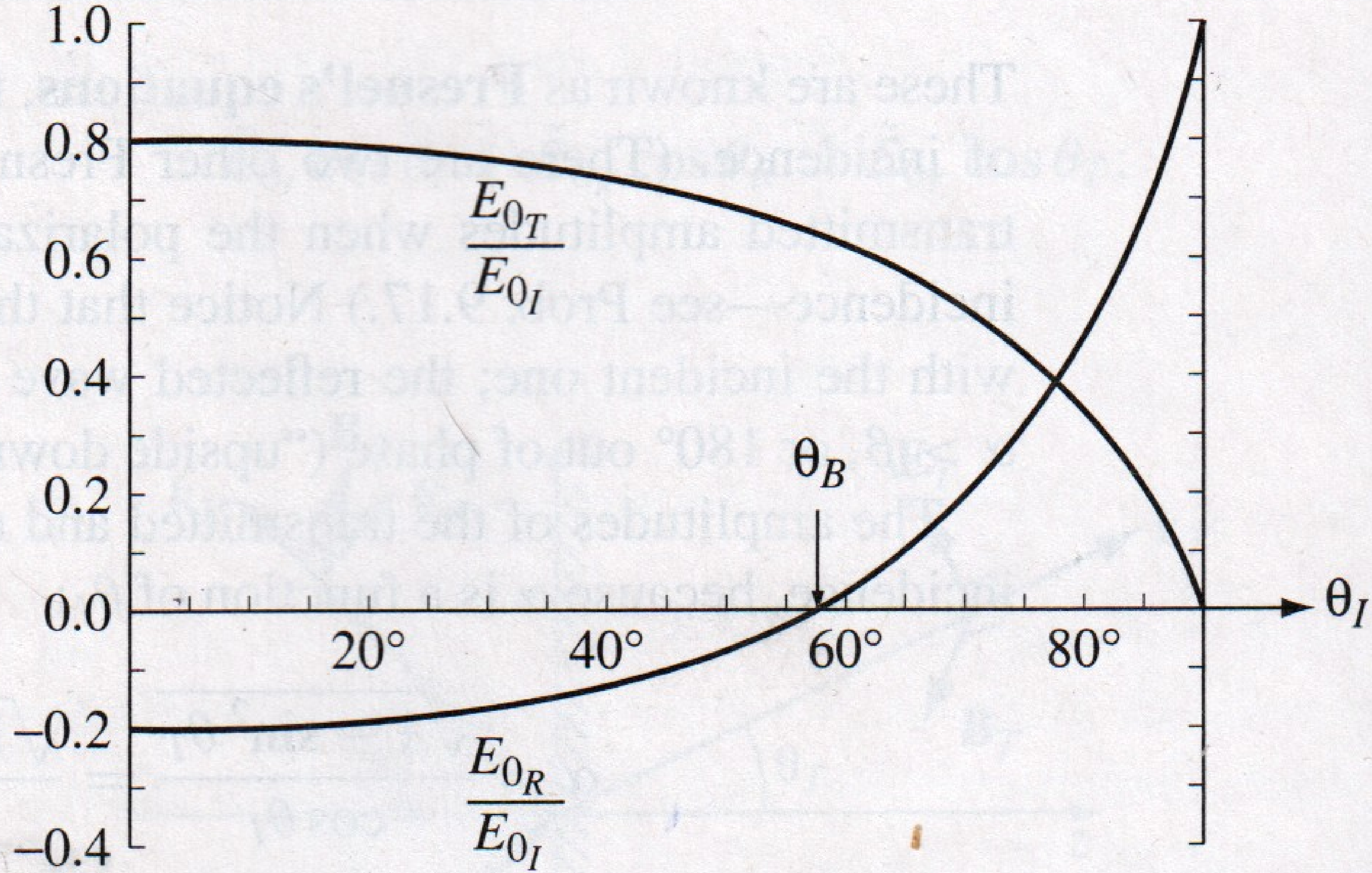


FIGURE 9.16


The index of refraction of glass is about 1.5
What is the transmission coefficient at the Air/Glass interface at normal incidence?

(A) 4%

(B) 20%

(C) 80%

(D) 96%

Reflection coefficient of glass vs. incidence angle

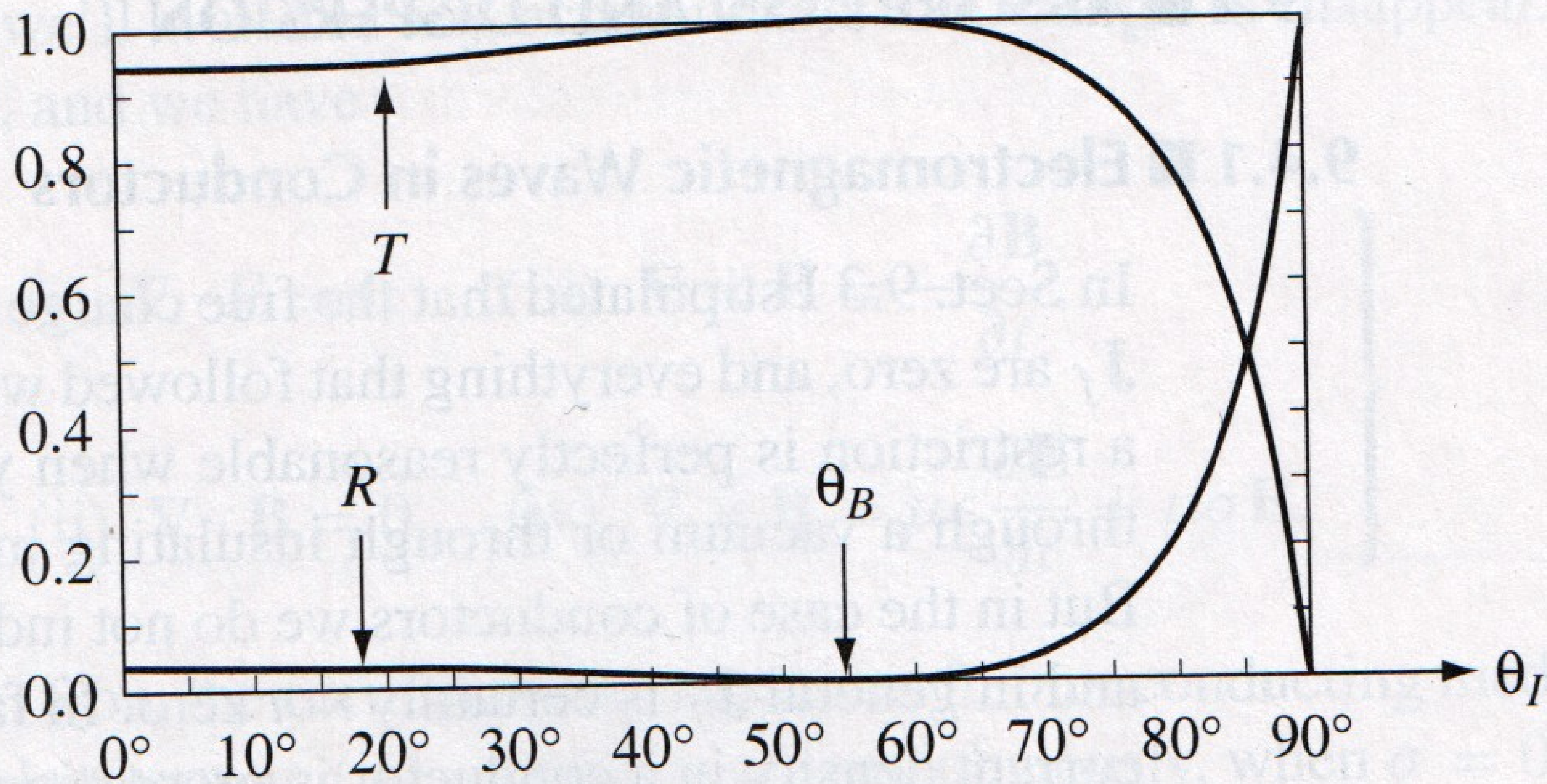
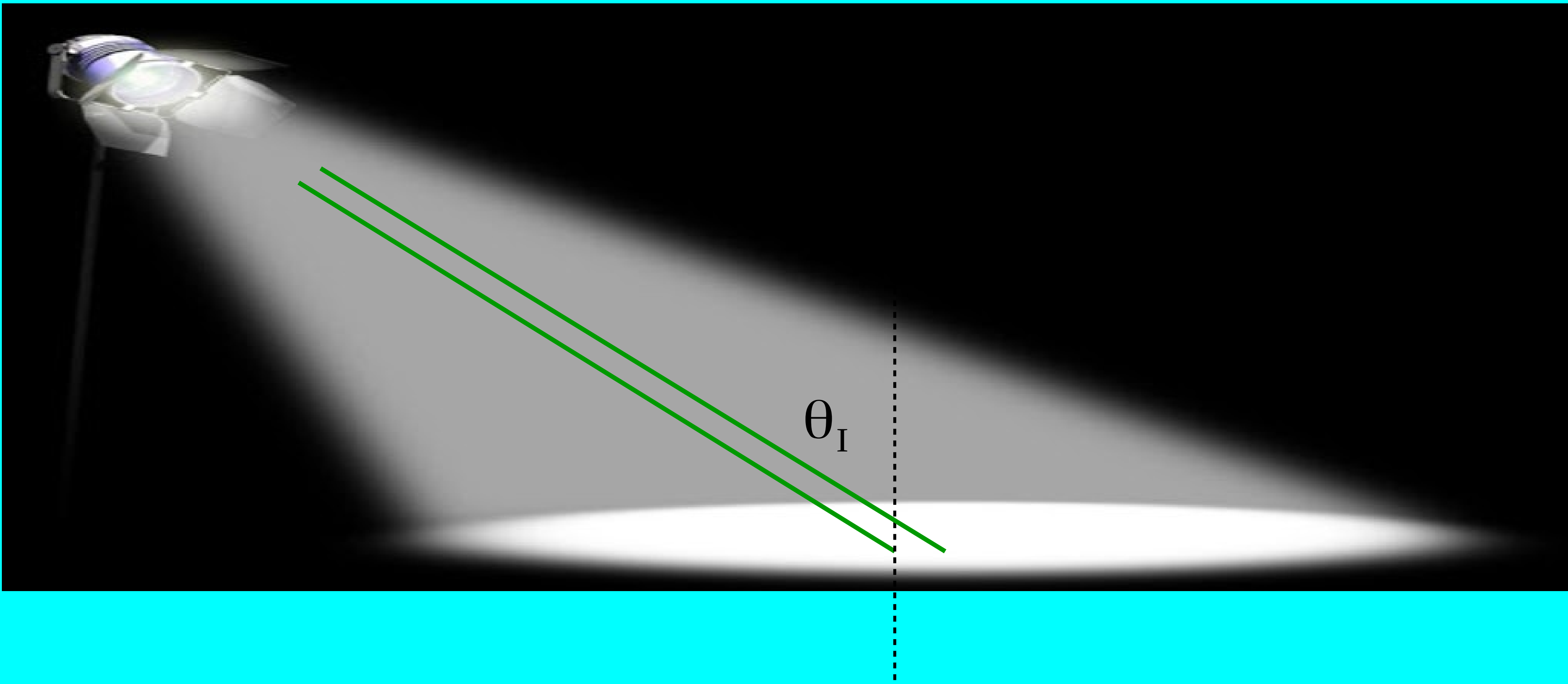


FIGURE 9.17

Why Cosine theta?



- Calculating Brewster's Angle

$$\tan\theta_B = \frac{n_2}{n_1}$$

The index of refraction of water is about 1.33
What is Brewster's Angle for the Air/Water interface?

(A) $\theta_B = 42^o$

(B) $\theta_B = 53^o$

(C) $\theta_B = 56^o$

(D) $\theta_B = 68^o$