Energy of an EM wave

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

$$= \frac{1}{2}\epsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta) + \frac{B_0^2}{2\mu_0}\cos^2(\quad)$$

$$B_0 = \frac{E_0}{c}$$

$$= \left(\frac{1}{2}\epsilon_0 E_0^2 + \frac{1}{2\mu_0}\frac{E_0^2}{c^2}\right)\cos^2(\quad)$$

$$\frac{1}{c^2} = \epsilon_0\mu_0$$

$$= \left(\frac{1}{2}\epsilon_0 E_0^2 + \frac{\epsilon_0\mu_0}{2\mu_0}E_0^2\right)\cos^2(\quad)$$

$$u = \epsilon_0 E_0^2 \cos^2(\quad) \rightarrow \langle u \rangle = \frac{1}{2}\epsilon_0 E^2$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\vec{E}\times\left(\frac{\vec{k}}{\omega}\times\vec{E}\right)$$

$$\vec{A}\times\vec{B}\times\vec{C} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{S} = \frac{\vec{k}}{\omega\mu_0}E^2 - \vec{E}\left(\frac{1}{\mu_0}\vec{E}\cdot\vec{k}\right)$$

$$\searrow 0$$

$$= \frac{k\hat{k}}{\omega\mu_0}E^2 = \frac{\hat{k}}{c\mu_0}E^2 \qquad \frac{1}{c^2} = \epsilon_0\mu_0$$

$$\frac{1}{\mu_0 c} = \epsilon_0 c$$

$$\vec{S} = \epsilon_0 c E^2 \hat{k}\cos^2(\vec{k}\cdot\vec{r} - \quad)$$

$$\langle\vec{S}\rangle = \frac{1}{2}\epsilon_0 c E^2 \hat{k}$$

What about momentum?

$$\vec{g} = \frac{1}{c^2}\vec{S}$$

$$\langle \vec{g} \rangle = \frac{1}{2}\epsilon_0 E^2 / c$$

Average Power/area $\equiv I =$ Intensity

$$I = \langle \vec{S} \rangle \cdot \hat{k} = \frac{1}{2}\epsilon_0 c E^2$$

In general $I = \frac{1}{2}\epsilon v E^2$

I is scalar

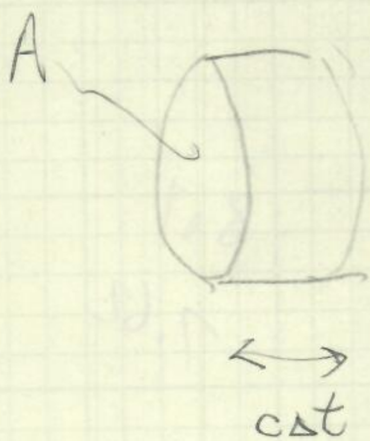
Try units $\vec{S} = \frac{W}{m^2}$ $\quad \vec{g} = \frac{W}{m^2}\frac{1}{m^2/s^2}$

$$\vec{g} = \frac{W \cdot s^2}{m^4} = \frac{J s^2}{s \cdot m^4} = \frac{J \cdot s}{m^4} = \frac{N \cdot s}{m^3}$$

$\vec{g}$ = Momentum Density

$\vec{p} = m\vec{v} = kg\,\frac{m}{s}$ $\qquad N = \frac{kg\, m}{s^2}$

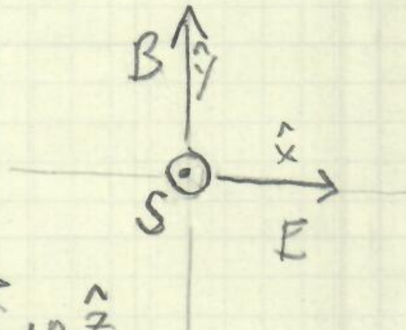
$\vec{p} = \frac{N s^2}{\cancel{m}}\frac{\cancel{m}}{\cancel{s}} = N \cdot s$ $\qquad kg = N s^2/m$

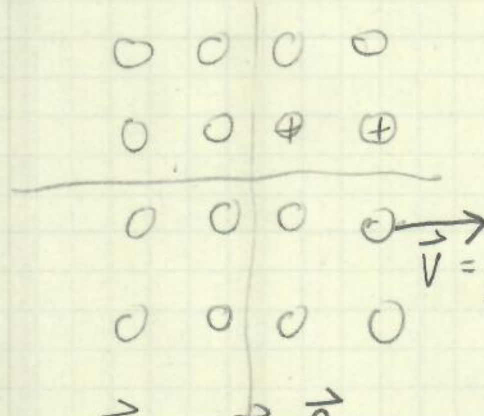


$$\Delta\vec{p} = \vec{g}\,\tau = \vec{g} A c \Delta t$$

$$\frac{\Delta \vec{p}}{\Delta t} = \vec{F} = \frac{1}{2}\epsilon_0 \frac{E^2}{c} c A$$

$$\text{Pressure} = \frac{F}{A} = \frac{1}{2}\epsilon_0 E^2 = \frac{I}{c}$$





$\vec{S}$ in $\hat{z}$

$\vec{V} = \frac{q\vec{E}}{m}\Delta t$

$\vec{F} = q\vec{V}\times\vec{B}$

Breakthrough Starshot

$\frac{q^2}{m} E \Delta t \ \hat{x}\times\hat{y}$

in $\hat{z}$ direction

3.5 kW in 0.6 mm $\varnothing$ spot

$I = 1.24\ MW/cm^2 = 10^6\ W/cm^2 = 10^{10}\ W/m^2$

Force on 1 cm^2

$Press = \frac{I}{c}$ $\quad Force = \frac{10^{10}\ W/m^2}{3\times10^8\ m/s} \cdot 10^{-4}\ m^2$

$= \frac{10^6}{3\times10^8} = \frac{1}{300} = 3.3$ milliN $\approx 330\ \mu g$

10 g satellite

$a = F/m = \frac{3.3\times10^{-3}\ N}{10^{-2}\ kg} = .33\ m/s^2$

Sunlight $10^3\ W/m^2$. If $(10m)^2 = 10^6\ cm^2$