$$\tilde{\vec{E}} = \tilde{\vec{E}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\tilde{\vec{B}} = \tilde{\vec{B}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Satisfy $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\tilde{E}$ or $\tilde{B} = 0$

Let's try individual maxwell

$\nabla\cdot\vec{B} = 0 \quad \nabla\cdot\vec{E} = 0$

$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$ Let $\vec{E}_0 = \tilde{E}_0\,\hat{x}$

$= i\frac{\partial}{\partial x}(k_x x + k_y y + k_z z)\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$= 0 = i k_x e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

Means $k_x = 0$ if $\hat{n} = \hat{x}$

What if don't know $\hat{n}$

$\tilde{\vec{E}} = \tilde{\vec{E}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\nabla\cdot\vec{E} = \frac{\partial}{\partial x} E_{0x} e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \frac{\partial}{\partial y} E_{0y} e^{i(\;)}$

$+ \partial/\partial z\, E_{0z} e^{i(\;)}$

$= i k_x E_{0x} e^{i(\;)} + i k_y E_{0x} e^{i(\;)} + i k_z \tilde{E}_{0z} e^{i(\;)}$

$= i\vec{k}\cdot\vec{E} = 0$ Says prop direction $\perp$ to $\vec{E}$

$\nabla\vec{E} = \nabla E_x + \nabla E_y + \nabla E_z$

$= \hat{i}\frac{\partial E_x}{\partial x} + \hat{j}\frac{\partial E_x}{\partial y} + \hat{k}\frac{\partial E_{\cancel{x}}}{\partial z}$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

But $\partial/\partial x \, \vec{E} = i k_x \vec{E}$

$$\partial/\partial y \, \vec{E} = \frac{\partial}{\partial y} \tilde{E}_x e^{i(\,)} \hat{x} + \frac{\partial}{\partial y} \tilde{E}_y e^{i(\,)} \hat{y} + \frac{\partial}{\partial x} \hat{E}_z e^{i(\,)} \hat{z}$$

$$= i k_y \vec{\hat{E}}$$

So

$$\nabla \times \vec{E} = \begin{vmatrix} x & \hat{y} & \hat{z} \\ i k_x & i k_y & i k_z \\ E_x & E_y & E_z \end{vmatrix} = i \vec{k} \times \vec{E}$$

Lecture 10

So Faraday:

$$\cancel{i} \vec{k} \times \vec{E} = \cancel{i} \omega \vec{B} \qquad \left|\frac{\omega}{k}\right| = v$$

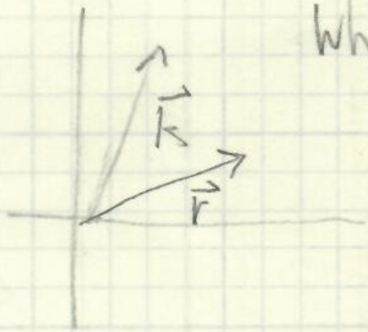
$$\vec{B} = \frac{\vec{k}}{\omega} \times \vec{E} = \frac{\hat{k}}{c} \times \vec{E}$$

Also

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$i k \times \vec{B} = \frac{1}{c^2} i \omega \vec{E}$$

$$\hat{k} \times \vec{B} = \frac{1}{c^2} \frac{\omega}{k} \vec{E} \rightarrow \hat{k} \times \vec{B} = \frac{1}{c} \vec{E}$$



What is $\nabla(\vec{k}\cdot\vec{r})$?

$\hat{x}\frac{\partial}{\partial x}\vec{k}\cdot\vec{r} = \hat{x}\,k_x +$

$\hat{y}\frac{\partial}{\partial y}\vec{k}\cdot\vec{r} = \hat{y}\,k_y +$

$\hat{z}\frac{\partial}{\partial z}\vec{k}\cdot\vec{r} = \hat{z}\,k_z$

$= \vec{k}$ $\therefore$ $\vec{k}$ is direction of prop

Because where argument of $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ changes quickest

We have shown in general that

$\nabla\cdot\vec{E} = i\vec{k}\cdot\vec{E}$ likewise

$\nabla\cdot\vec{B} = i\vec{k}\cdot\vec{B}$

Given $\nabla\cdot E = 0 = \nabla\cdot B$ then $\vec{k}\cdot\vec{E} = 0 = \vec{k}\cdot\vec{B}$

Definitely transverse

Lets try $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\delta)} = \tilde{\vec{E}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$= \vec{B}_0 e^{i(\quad)} = \tilde{\vec{B}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$-\frac{\partial B}{\partial t} = i\omega\vec{B}$

$\nabla\times\vec{E} =$