Reflection & Transmission

$f_I(z,t) = A_I \cos(k_1 z - \omega t)$

$f_R(z,t) = A_R \cos(-k_1 z - \omega t)$

$f_T(z,t) = A_T \cos(k_2 z - \omega t)$

Clicker

Which Properties of a wave Don't change at the knot?

ω

T

f

k

$f_I + f_R \big|_{z=0} = f_T \big|_{z=0}$

Discontinuity Not OK

$\frac{\partial (f_I + f_R)}{\partial t}$ $\frac{\partial f}{\partial z}\Big|_{z=0}$

$F_{net} = T\left[\frac{\partial f_T}{\partial z} - \frac{\partial (f_I + f_R)}{\partial z}\right]$ if $\neq 0$ $a = \frac{F}{m} \to \infty$ m ← negligible

$f_I + f_R = f_T$

$A_I \cos(-\omega t) + A_R \cos(-\omega t) = A_T \cos(-\omega t)$

$A_I + A_R = A_T$

$\frac{\partial f_I}{\partial z}\Big|_{z=0} = -k_1 A_I \sin(-\omega t)$ $\frac{\partial f_R}{\partial z}\Big|_{z=0} = k_1 A_R \sin(-\omega t)$

$\frac{\partial f_T}{\partial z}\Big|_{z=0} = -k_2 A_T (\sin(-\omega t))$

$$-k_1 A_I + k_1 A_R = -k_2 A_T$$

$$k_1 (A_I - A_R) = k_2 A_T$$

$$A_I - A_R = \frac{k_2}{k_1} A_T$$

$$A_I + A_R = A_T$$

$$2A_I = A_T(1 + k_2/k_1)$$

$$\frac{2A_I}{1 + k_2/k_1} = A_T$$

$$\frac{2k_1 A_I}{k_1 + k_2} = A_T \qquad \omega/k = V_1$$

$$\text{so } k_1 = \omega/V_1$$

$$\frac{2\,\omega/V_1\, A_I}{\omega/V_1 + \omega/V_2} \quad \text{.... shortcut} \quad \frac{k_2}{k_1} = \frac{\omega/V_2}{\omega/V_1} = \frac{V_1}{V_2}$$

$$\frac{2A_I}{1 + V_1/V_2} = \frac{2V_2 A_I}{V_1 + V_2} \rightarrow \frac{A_T}{A_I} = \frac{2V_2}{V_1 + V_2}$$

$$A_R = A_T - A_I \text{ -- Easier}$$

$$A_I - A_R = \frac{V_1}{V_2} A_T$$

$$-\ \frac{V_1}{V_2}(A_I + A_R) = A_T \frac{V_1}{V_2}$$

$$A_I\left(1 - \frac{V_1}{V_2}\right) - A_R\left(1 + \frac{V_1}{V_2}\right) = 0$$

$$\frac{A_R}{A_I} = \frac{1 - \frac{V_1}{V_2}}{1 + V_1/V_2} = \frac{V_2 - V_1}{V_2 + V_1}$$

If $V_2 > V_1$, A_R & A_I have same sign

What if $V_2 = 0$

$A_R / A_I = -1$

$A_T / A_I = 0$

$V_2 = \infty$

$A_R = A_I$

$A_T = 2 A_I$