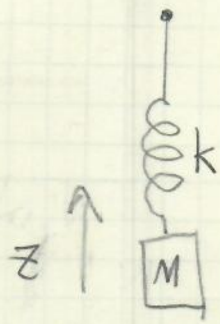
Oscillators to Waves

Lecture 5

$y = A\sin\omega t$

Can ignore gravity



$F_z = -kz - mg$

$m\ddot{z} + kz = -mg$

$z_{total}(t) = z_h + z_p$

Oscillator - One variable varying in time

Continuous Medium Varying in time

z_p when $\ddot{z} = 0$

$kz = -mg$

$z_p = -\frac{mg}{k}$ Small Offset

$m\ddot{z} + kz = 0$

$\ddot{z} = -\frac{k}{m}z$

$z = A\cos(\omega t + \phi)$

Let $z(0) = 0$

$= A\cos\omega t\cos\phi - A\sin\omega t\sin\phi$

$= \underbrace{(A\cos\phi)}_{C}\cos\omega t + \underbrace{(-A\sin\phi)}_{D}\sin\omega t$

$z = C\cos\omega t + D\sin\omega t$

Equivalent Results. 2nd Order Eqn, 2 ARB CONSTANTS

$z(0) = C\cos(0) + D\sin 0 = 0$

$C(1) + D(0) = 0 \rightarrow C = 0$

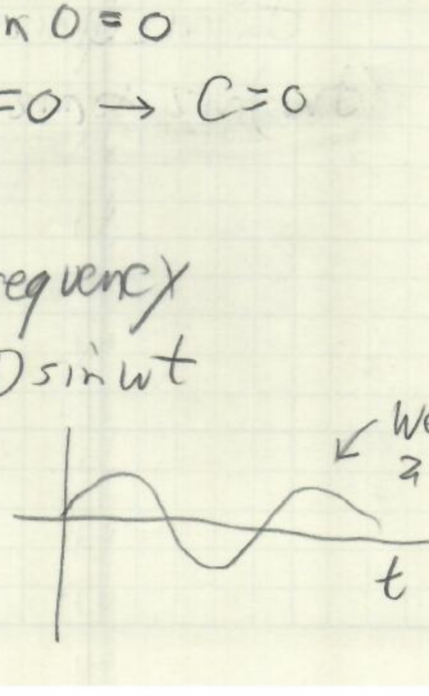
$z(t) = D\sin\omega t - \frac{mg}{k}$

Oscillator w/ char. frequency

$\ddot{z} = -D\omega^2\sin\omega t = -\frac{k}{m}D\sin\omega t$

$\omega^2 = \frac{k}{m}$

Get an oscillator

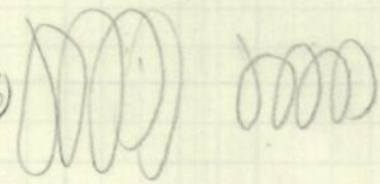


We call this a sine wave but this is not a wave.

Show an inductor – Adams Lab

100
10,000
1000000
100 MHz - 10 GHz

$I = I_0 \cos(\omega t)$



Assume current same everywhere in coil

When is this not valid? $L = (2920)(D)\pi$

$D = (3\text{ cm})$ -- $L \cong 9000\text{ cm}\,\pi \sim 28000\text{ cm} = 280\text{ m}$

$c = 3\times10^{8}\text{ m/s}$ $\quad cT = L \quad T = \frac{L}{c} = \frac{2.8\times10^{2}}{3\times10^{8}} \sim 10^{-6}$

Wave is a disturbance of a continuous medium that propagates w/ fixed shape at constant velocity

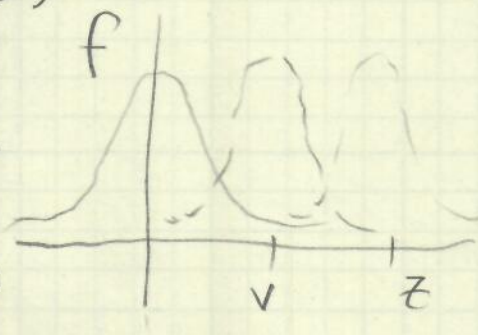
Arb function $f(z,t)$

special case $f(z-vt)$

$f_1(z) = Ae^{-bz^2}$

Max at $z=0$

$f_1(z,t) = Ae^{-b(z-vt)^2}$



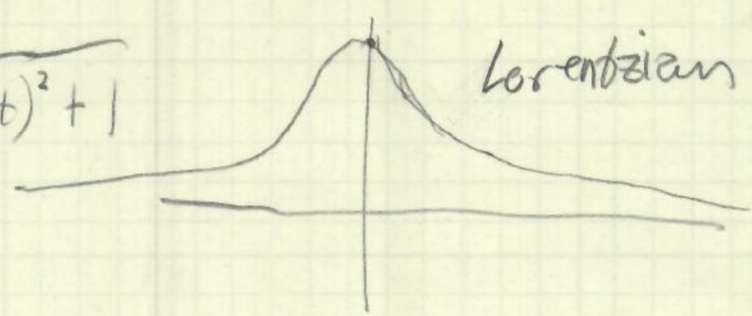
Gaussian

$t=1$ Max at $z-V=0$

$z=V$

$t=2$ Max at $z-2V=0$

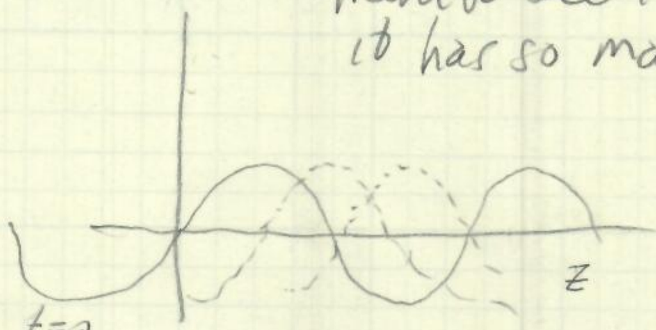
$$f_3(z,t) = \frac{A}{b(z-vt)^2+1}$$



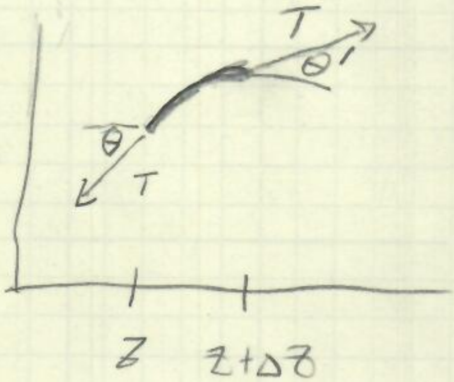
Lorentzian

$f(z,t) = A\sin(b(z - vt))$

Hard to see it move because it has so many peaks



Do a guitar string — Demos



$$\Delta F = T(\sin\theta' - \sin\theta)$$
$$\cong T(\tan\theta' - \tan\theta)$$
$$= T\left(\frac{df}{dz}\Big|_{z+\Delta z} - \frac{df}{dz}\Big|_{z}\right)$$

$$\lim_{\Delta z \to 0} \frac{\left(\frac{df}{dz}\Big|_{z+\Delta z} - \frac{df}{dz}\Big|_{z}\right)}{\Delta z}$$

$= \frac{d^2f}{dz^2}$ so $F_{net} = T\Delta z \frac{d^2f}{dz^2}$ It's partial because f is fn of t also

Restart Here for Lecture 6

So $F_{net} = ma = m\frac{\partial^2 f}{\partial t^2}$

So $\frac{m\,\partial^2 f}{\partial t^2} = T\Delta z \frac{\partial^2 f}{\partial z^2}$

But $m = \mu\Delta z \rightarrow \mu\Delta z \frac{\partial^2 f}{\partial t^2} = T\Delta z \frac{\partial^2 f}{\partial z^2}$

$$T\frac{\partial^2 f}{\partial z^2} - \mu\frac{\partial^2 f}{\partial t^2} = 0$$

$$\frac{\partial^2 f}{\partial z^2} - \frac{\mu}{T}\frac{\partial^2 f}{\partial t^2} = 0$$

$$\frac{\partial^2 f}{\partial z^2} - \frac{1}{V^2}\frac{\partial^2 f}{\partial t^2} = 0 \rightarrow V = \sqrt{\frac{T}{\mu}}$$

Proof that speed is V.

Assume a function of form $f(z,t) = g(b(z-vt))$

Plug & Chug $\quad u = b(z-vt)$

$$\frac{\partial f}{\partial z} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial z} = \frac{\partial g}{\partial u} b$$

$$\frac{\partial f}{\partial t} = \frac{\partial g}{\partial u}\frac{\partial u}{\cancel{\partial z}} = \frac{\partial g}{\partial u}(-bv)$$

$$\frac{\partial^2 f}{\partial z^2} = b\frac{\partial^2 g}{\partial u^2}\frac{\partial u}{\partial z} = b^2\frac{\partial^2 g}{\partial u^2}$$

$$\frac{\partial^2 f}{\partial t^2} = -bv\frac{\partial}{\partial t}\frac{\partial g}{\partial u} = -bv\frac{\partial^2 g}{\partial u^2}\frac{\partial u}{\partial t} = b^2v^2\frac{\partial^2 g}{\partial u^2}$$

So $\frac{\partial^2 f}{\partial z^2} = b^2\frac{\partial^2 g}{\partial u^2}$ or $\frac{\partial^2 g}{\partial u^2} = \frac{1}{b^2}\frac{\partial^2 f}{\partial z^2}$

$\frac{\partial^2 f}{\partial t^2} = b^2v^2\frac{\partial^2 g}{\partial u^2}$ or $\frac{\partial^2 g}{\partial u^2} = \frac{1}{b^2v^2}\frac{\partial^2 f}{\partial t^2}$

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 f}{\partial t^2} \text{ or } \frac{\partial^2 f}{\partial z^2} - \frac{1}{V^2}\frac{\partial^2 f}{\partial t^2} = 0$$

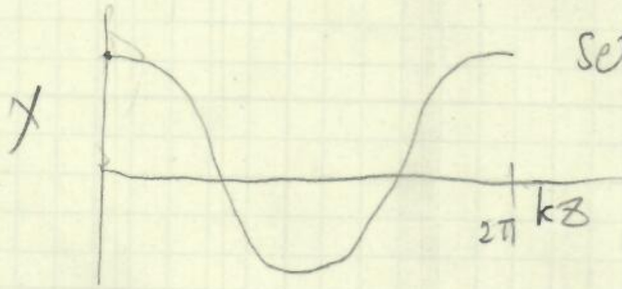
Shows wave eqn is what you get from any fn of form $g(b(z-vt))$

$g(z+vt)$ also works

Now try $g(b(z-vt)) = A\cos(b(z-vt))$

Change b to k

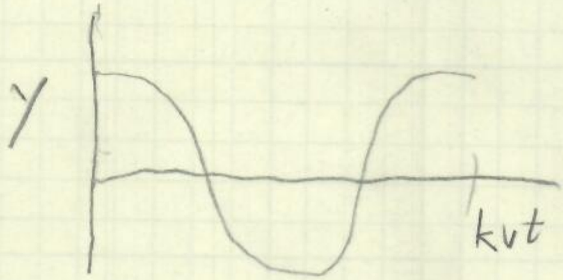
$y(z,t) = A\cos(kz - kvt)$



Set $t=0$

When $kz = 2\pi$ this is one cycle. We would say that $z=\lambda$ at this point

$k\lambda = 2\pi \quad \lambda = \frac{2\pi}{k} \quad k = \frac{2\pi}{\lambda}$



When $kvt = 2\pi \quad t = T$

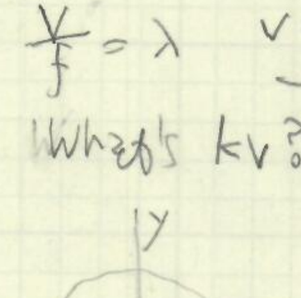
Thus $kvT = 2\pi$

$T = \frac{2\pi}{kv} = \frac{2\pi\lambda}{2\pi v}$

$vT = \lambda$

$\frac{v}{f} = \lambda \quad \underrightarrow{v = f\lambda} \quad k = \frac{2\pi}{\lambda}$

What's kv? $= \frac{2\pi v}{\lambda} = 2\pi f \rightarrow$ we call this ω



$\theta = \omega t \quad x = Re\, Ae^{i\omega t} = A\cos\omega t$

$y = Im\, Ae^{i\omega t} = A\sin\omega t$

As wheel spins with frequency ω

x & y oscillate at that frequency

$y = A\sin(7x - 3t + \pi/4)$

$f = \left(\frac{\omega}{2\pi}\right) \rightarrow \frac{3}{2\pi}$ (Not $-\frac{3}{2\pi}$) ? Freqs are negative not

$\lambda = \frac{2\pi}{k} = \frac{2}{7}\pi$

$y = -A\sin(-7x + 3t + \pi/4)$ ← Same Wave

Not useful.

Griffiths likes this form

$y = A\cos(kx - \omega t + \delta)$

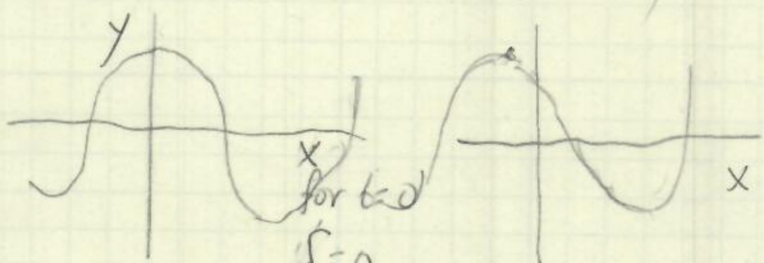
Wave to right

$A\cos(kx + \omega t + \delta)$

Wave to left

δ is "delay"

$y = A\cos(kx - \omega t + \delta)$



Which is not true?

$f = \frac{1}{T}$

$v = \omega/k$

$f = 2\pi\omega$

$\lambda = v/f$

$T = 2\pi/\omega$

$kx + \delta = 0$ peak

If $\delta = \frac{\pi}{10}k$

$kx + \frac{\pi}{10}k = 0$

Peak when $x = -\frac{\pi}{10}$

$L = A\cos(kx + \omega t - \delta)$

Wave to left delay means shift to right

Not worry about δ

Get sign of direction here

$y_{right} = A\cos(kx - \omega t)$

$y_{left} = A\cos(kx + \omega t) = A\cos(-kx - \omega t)$