Section 7.2.4 Shows $u_B = \frac{B^2}{2\mu}$ also $u_E = \frac{1}{2}\epsilon E^2$

So $u = \frac{1}{2}\left(\epsilon E^2 + \frac{B^2}{\mu}\right)$

Given $\vec{E}$ + $\vec{B}$ let charges move. What is work done?

$dW = \vec{F}\cdot d\vec{l} = \vec{F}\cdot\vec{v}\,dt = q\,(\vec{v}\times\vec{B} + \vec{E})\cdot\vec{v}\,dt$

$(\vec{v}\times\vec{B})\cdot\vec{v} = 0$

$dW = q\vec{E}\cdot\vec{v}\,dt \qquad q = \rho\,d\tau \quad \rho\vec{v} = \vec{J}$

Prove Mag Force does no work. Use translation stage

Section 8.3

$dW = \vec{F}\cdot q\vec{v}\,dt = \vec{E}\cdot\rho\vec{v}\,dt\,d\tau = \vec{E}\cdot\vec{J}\,dt\,d\tau$

$\frac{dW}{dt} = \int \vec{E}\cdot\vec{J}\,d\tau \qquad \nabla\times\vec{B} = \mu J + \mu\epsilon\frac{\partial E}{\partial t}$

$\vec{E}\cdot\vec{J} = \vec{E}\cdot\left(\frac{\nabla\times\vec{B}}{\mu} - \epsilon\frac{\partial\vec{E}}{\partial t}\right) \qquad \vec{J} = -\epsilon\frac{\partial E}{\partial t} + \frac{\nabla\times\vec{B}}{\mu}$

Also $\nabla\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{B})$

So $\frac{\vec{E}\cdot(\nabla\times\vec{B})}{\mu} = \frac{\vec{B}\cdot(\nabla\times\vec{E})}{\mu} - \frac{\nabla\cdot(\vec{E}\times\vec{B})}{\mu}$

$= \frac{1}{\mu}\vec{B}\cdot(\nabla\times\vec{E}) - \frac{\nabla\cdot\vec{E}\times\vec{B}}{\mu} - \epsilon\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$

$= \frac{\vec{B}}{\mu}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) - \epsilon\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} - \frac{1}{\mu}\nabla\cdot(\vec{E}\times\vec{B})$

$\vec{E}\cdot\vec{J} = -\frac{1}{\mu}\frac{\partial}{\partial t}B^2 - \epsilon\frac{\partial}{\partial t}E^2 - \frac{1}{\mu}\nabla\cdot(\vec{E}\times\vec{B})$

$$-\frac{dW}{dt} = -\int \vec{E}\cdot\vec{J}\,d\tau$$

$$-\frac{dW}{dt} = \frac{\partial}{\partial t}\int u\,d\tau + \int \frac{1}{\mu}\nabla\cdot(\vec{E}\times\vec{B})\,d\tau$$

$$= \frac{\partial}{\partial t}\int u\,d\tau + \oint \frac{\vec{E}\times\vec{B}}{\mu}\cdot da$$

$$-\frac{dW}{dt} = \frac{\partial}{\partial t}\int u\,d\tau + \int \vec{S}\cdot da$$

charges working on field

$\frac{dW}{dt}$ field working on charges (changing their KE)

$$\text{or} \quad \int \vec{S}\cdot d\vec{a} = -\frac{\partial}{\partial t}\int u\,d\tau = -\frac{\partial U}{\partial t}$$

Analogy $\int \vec{J}\cdot d\vec{a} = -\frac{\partial}{\partial t} Q$

Also $\nabla\cdot\vec{S} = -\frac{\partial u}{\partial t}$

$\nabla\cdot\vec{J} = -\partial\rho/\partial t$

Work out units of $\vec{S}$ for HW. Plug in E, B, μ

$$\frac{\partial U}{\partial t} = -\int \vec{S}\cdot d\vec{a}$$

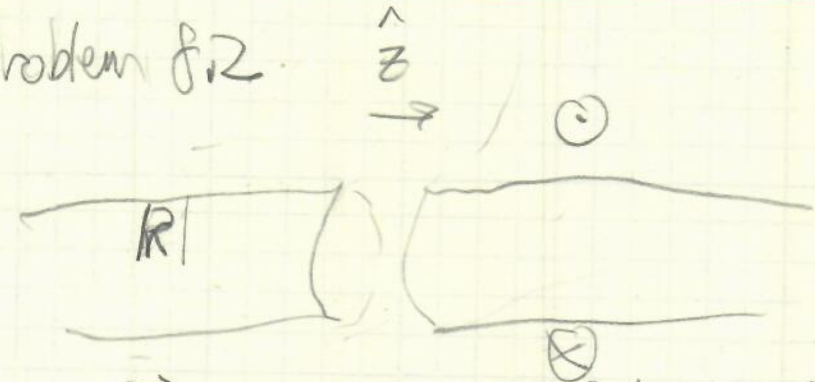
A) W/m^2

B) J

C) J/m^3

D) W

Problem 8.2



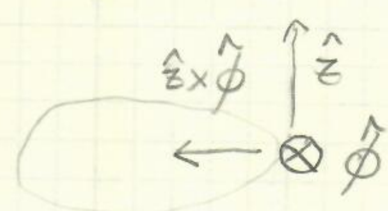
$$\frac{d\vec{E}}{dt} = \frac{dQ}{dt}\frac{1}{\epsilon_0}\hat{z} = \frac{1}{A}\frac{dQ}{dt}\frac{1}{\epsilon_0}\hat{z} = \frac{I}{A\epsilon_0}\hat{z}$$

$$E(t) = It/A\epsilon_0\,\hat{z}$$

$$\int \vec{B}\cdot d\vec{l} = \mu I_{enclosed} = \mu\frac{\pi r^2}{\pi R^2}I$$

$$2\pi r B = \mu\frac{r^2 I}{R^2}$$

$$\vec{B} = \frac{\mu}{2\pi}\frac{r I}{R^2}\hat{\phi}$$



$$\vec{S} = -\hat{r}\,\frac{\mu r I}{2\pi R^2}\frac{1}{\mu}\frac{It}{A\epsilon_0} = \frac{I^2 r t}{2\pi R^2 \pi R^2 \epsilon_0}$$

$$u = \frac{1}{2}\epsilon_0\frac{I^2 t^2}{A^2\epsilon_0^2} + \frac{1}{2\mu_0}\frac{\mu_0^2 r^2 I^2}{4\pi^2 R^4}$$

$$\frac{du}{dt} = \frac{\epsilon_0 I^2 t}{A^2\epsilon_0^2}$$

$$\nabla\cdot\vec{S} = \frac{1}{r}\frac{\partial}{\partial r} r S_r$$
cylindrical

$$= \frac{1}{r}\frac{\partial}{\partial r}\frac{I^2 r^2 t}{2\pi^2 R^4\epsilon_0} = \frac{I^2 t}{\pi^2 R^4 \epsilon_0} \quad ✓$$