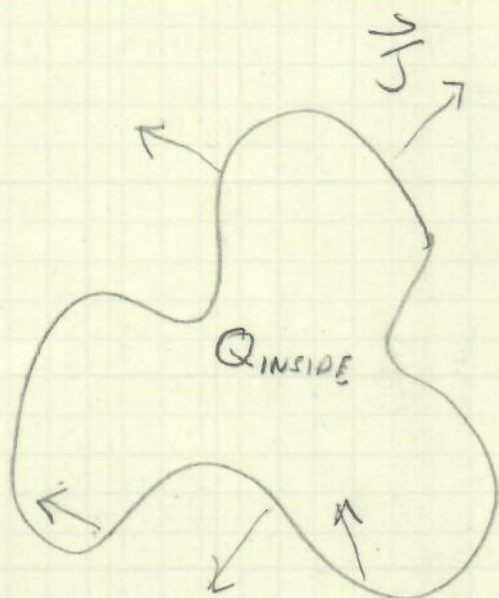


1/19/2024



$$I = -\frac{dQ}{dt}$$

$$\int \vec{J} \cdot d\vec{a} = -\frac{dQ}{dt}$$

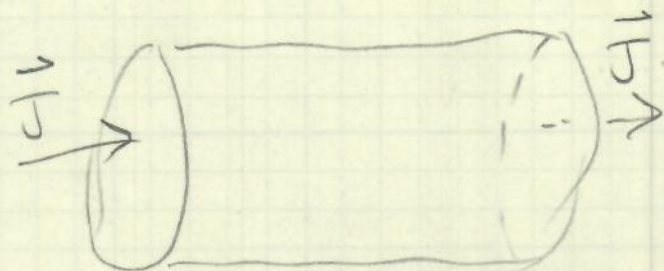
Egn 5.29 p. 222

Self Evident

$$\int \vec{J} \cdot d\vec{a} = \int_V \nabla \cdot \vec{J} d\tau = -\frac{d}{dt} \int_V \rho d\tau$$

For arb V

$$\therefore \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{Continuity Equation}$$

Wire $\int \vec{J} \cdot d\vec{a} = 0$
In equilibrium

$$\nabla \cdot \vec{E} = \rho/\epsilon \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \mu \vec{J} \quad \text{w/o Maxwell term}$$

$$\nabla \cdot \nabla \times \vec{A} = 0 \quad \text{for any vector fn } \vec{A}$$

$$\text{So } \nabla \cdot \nabla \times \vec{E} = \underbrace{-\frac{\partial}{\partial t}}_0 \underbrace{\nabla \cdot \vec{B}}_0$$

0, identically

$$\nabla \cdot \nabla \times \vec{B} = \underbrace{0}_0 = \underbrace{\mu \nabla \cdot \vec{J}}_{\text{Not 0}} = \mu \frac{\partial \rho}{\partial t}$$

But $\mu \frac{\partial}{\partial t} \nabla \cdot \vec{E} = \mu \frac{\partial}{\partial t} \rho / \epsilon$ By Gauss

So $\mu \epsilon \nabla \cdot \frac{\partial \vec{E}}{\partial t} = \mu \frac{\partial \rho}{\partial t}$

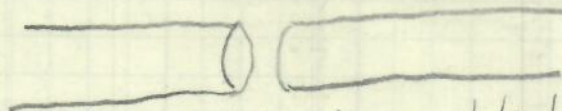
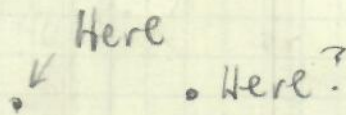
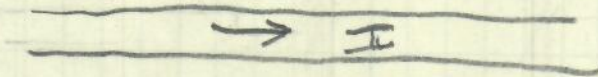
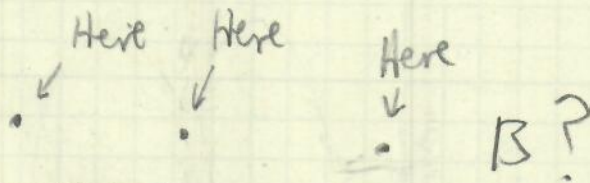
So change Ampere's:

$\nabla \times \vec{B} = \mu \vec{J} \rightarrow \nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$

Now $\nabla \cdot \nabla \times \vec{B} = 0$

$\nabla \cdot (\mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t})$
 $= -\mu \frac{\partial \rho}{\partial t} + \mu \epsilon \frac{\partial}{\partial t} \frac{\rho}{\epsilon} = 0$

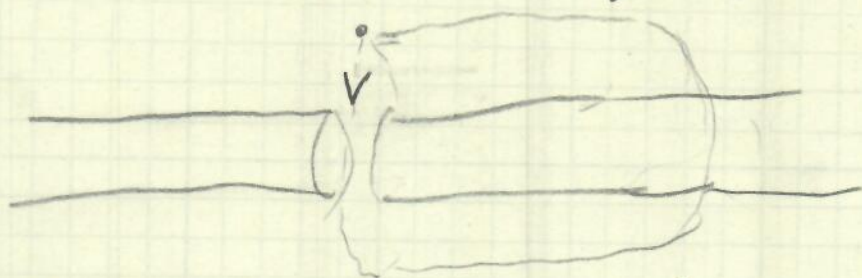
Also



It might be less. We broke symmetry
But still does not depend on theta & should
be only a fn of r

Amperian Bag

2-3



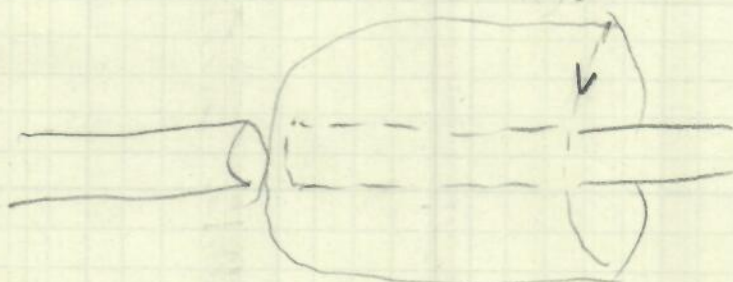
Same current is enclosed

$$\int \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{--- No Difference}$$

ALSO



The Bag can go in the gap, but there is no current in the gap!

$$E = \frac{\sigma}{\epsilon_0} \quad \sigma = \frac{Q}{A} \quad \frac{dQ}{dt} = I = A \frac{d\sigma}{dt}$$

$$A \frac{d\sigma}{dt} = A \epsilon_0 \frac{dE}{dt}$$

$$\int \vec{B} \cdot d\vec{\ell} = \mu_0 I + \underbrace{\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}}_{I_D}$$

What's the Maxwell Displacement current?

$$\mu_0 \mathbf{J}?$$

$$\mu_0 \epsilon_0 \mathbf{E}$$

$$\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\epsilon_0 \frac{\partial \Phi_{\mathbf{E}}}{\partial t}$$

$$\vec{A}$$

None of these

$$\vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{J}}{r} d\tau$$

Skip
this

Students - This is from 7.2.4 of new book (5th ed)
same as 7.2.4 of 4th ed

B-field energy -- Next Time? $u = \frac{B^2}{2\mu}$ $u = \frac{1}{2} \epsilon E^2$

$$W = qV \rightarrow \frac{dW}{db} = \frac{dq}{dt} V = IV = \text{Power}$$

$$\text{Power} = \frac{dW}{db} = -\mathcal{E}I = L \frac{dI}{dt} I \leftarrow \Phi = LI$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

$$\int \frac{dW}{db} dt = W \quad \int LI \frac{dI}{dt} dt = \int \frac{d}{dt} \frac{1}{2} LI^2 dt$$

$$= \frac{1}{2} LI^2$$

Analogous to $\frac{1}{2} CV^2$

Write $\frac{1}{2} LI^2$ in terms of B in general

$$\Phi = \int \vec{B} \cdot d\vec{a} = \int \nabla \times \vec{A} \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{\ell} \quad \text{Stokes} = LI$$

$$\text{So } \frac{1}{2} LI^2 = \frac{1}{2} I LI = \frac{1}{2} I \oint \vec{A} \cdot d\vec{\ell}$$

$$\text{But } I = \int \vec{J} \cdot d\vec{a}$$

$$\frac{1}{2} LI^2 = \frac{1}{2} \oint \vec{A} \cdot d\vec{\ell} \int \vec{J} \cdot d\vec{a} = \frac{1}{2} \int_V \vec{A} \cdot \vec{J} d\tau$$

$$\frac{1}{2} LI^2 = \frac{1}{2} \int_V \vec{A} \cdot \vec{J} d\tau \quad \text{Given } \nabla \times \vec{B} = \mu_0 \vec{J}$$

$$= \frac{1}{2\mu_0} \int_V \vec{A} \cdot (\nabla \times \vec{B}) d\tau$$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B}) \quad \text{Product Rule}$$

$$\begin{aligned} \vec{A} \cdot \nabla \times \vec{B} &= \vec{B} \cdot (\nabla \times \vec{A}) - \nabla \cdot (\vec{A} \times \vec{B}) \\ &= \vec{B} \cdot \vec{B} - \nabla \cdot (\vec{A} \times \vec{B}) \end{aligned}$$

$$W = \frac{1}{2\mu_0} \left[\int_V B^2 d\tau - \int_V \nabla \cdot (\vec{A} \times \vec{B}) d\tau \right]$$

$$= \frac{1}{2\mu_0} \int_V B^2 d\tau - \oint_S (\vec{A} \times \vec{B}) \cdot d\vec{a}$$

Take surface to ∞ $A, B \rightarrow 0$ Because far from currents