Lecture 24: 04/16/2024

- Announcements
  Last assignment will be next week
  Faraday's law and inductance (Ch 13/14)
  Omit Ch 16

- Last Time
  The infinite formula sheet
  Circular motion of a charged particle
  Motor review
  Ampere's Law

- Today
  Maxwell equations
  Ampere's Law

Drowning in Equations – Circular motion update!

An electron is accelerated through a 1000V potential in a 1 mT magnetic field. What is the radius of its circular orbit?

$U_i + K_i = U_f + K_f \rightarrow qV + 0 = 0 + \frac{1}{2}mv_f^2$

$U = qV$

"Too many v's"

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$ Lorentz force Law

$$U = qV$$ Potential energy from potential

$F_c = \frac{mv^2}{R}$ $F_B = qvB$

$qvB = \frac{mv^2}{R}$

Maxwell's Equations

Gauss

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$ To calculate E for symettrical charges.

$$\oiint \vec{B}\cdot d\vec{A} = 0$$ Cannot have North magnet w/o a South pole.

Ampere's

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$ To calculate B for symettrical currents.

Faraday's

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$ Magnetic induction! Generators! Light!

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$ Total flux through closed surface prop. to Q.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$ Line integral of B prop. to current thru loop.

Electric fields are calculated with Coulomb's law or Gauss's Law

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q\,\hat{r}}{r^2} \quad \text{Coulomb}$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0} \quad \text{Gauss's Law}$$

Magnetic Fields are calculated with the Biot-Savart Law or Ampere's Law

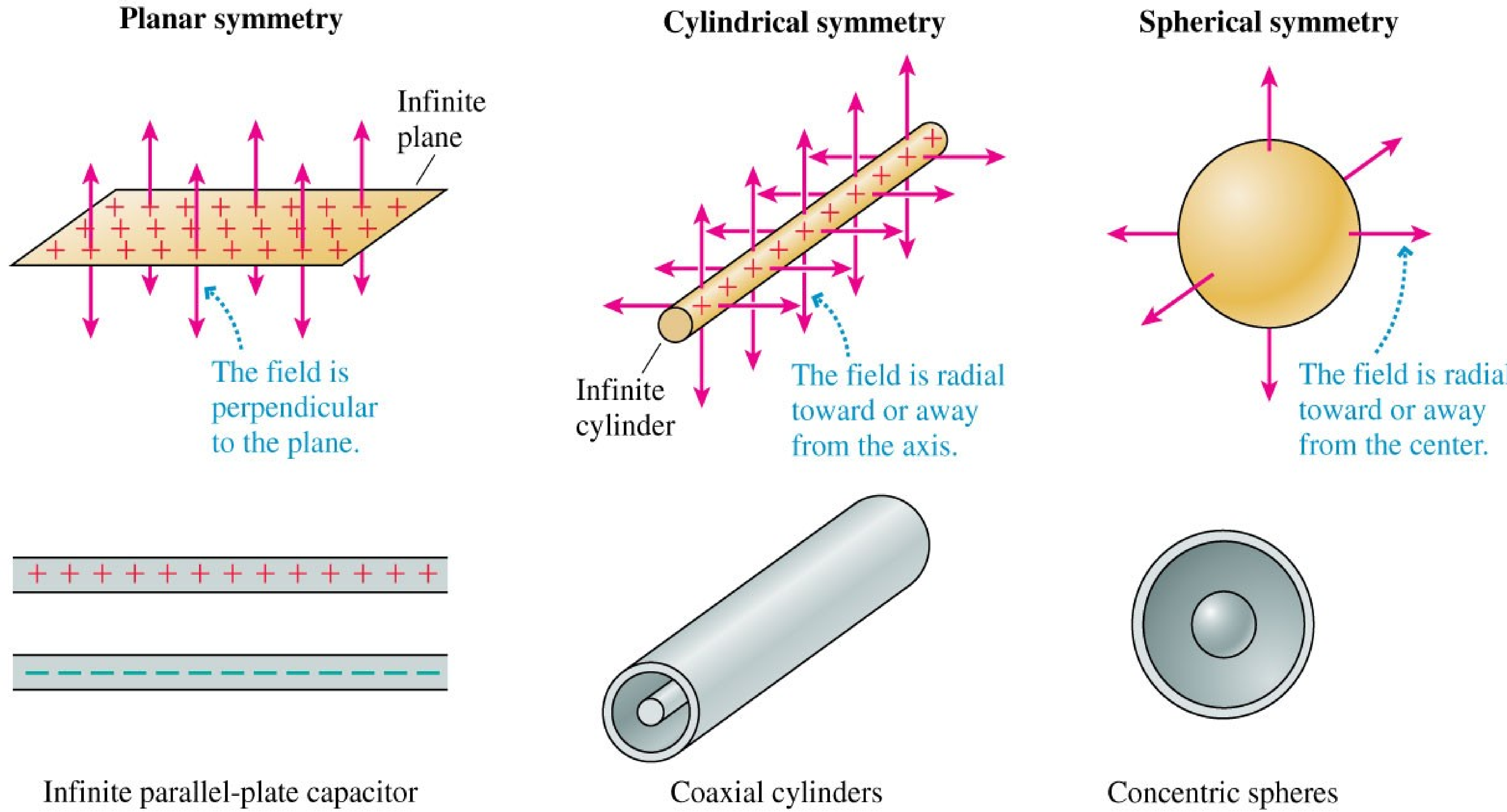
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{Biot Savart}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} \quad \text{Ampere's Law.}$$

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

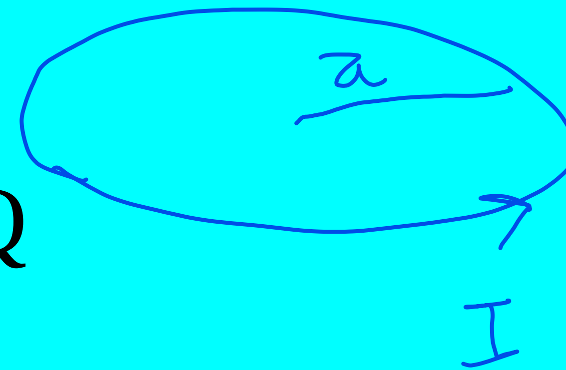
Gauss to calculate E for symettrical charges.

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k} \qquad \vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r} \qquad \vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}$$



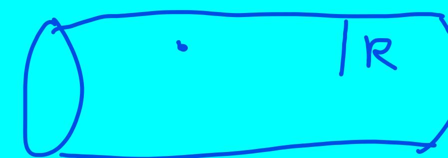
Equations of Magnetism

$\vec{F} = Q\,\vec{v} \times \vec{B}$ Force on charge Q



$\vec{F} = I\,\vec{L} \times \vec{B}$ Force on current I

$\vec{B} = \dfrac{\mu_0 I}{2\pi r}\hat{\phi}$ Field of Infinite wire



$\vec{B} = \dfrac{\mu_0 I r}{2R^2}\hat{\phi}$ Field inside a wire of radius R

$\vec{B} = \mu_0 n I \hat{z}$ Field of an infinite coil (solenoid)

$\vec{B} = \dfrac{\mu_0 I}{2a}\hat{z}$ Field in center of wire loop

Equations from Ampere's Law

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$ Field of Infinite wire

$$\vec{B} = \frac{\mu_0 I r}{2R^2}\hat{\phi}$$ Field inside a wire of radius R

$$\vec{B} = \mu_0 n I \hat{z}$$ Field of an infinite coil (solenoid)

Advanced Math Alert!! (Optional)

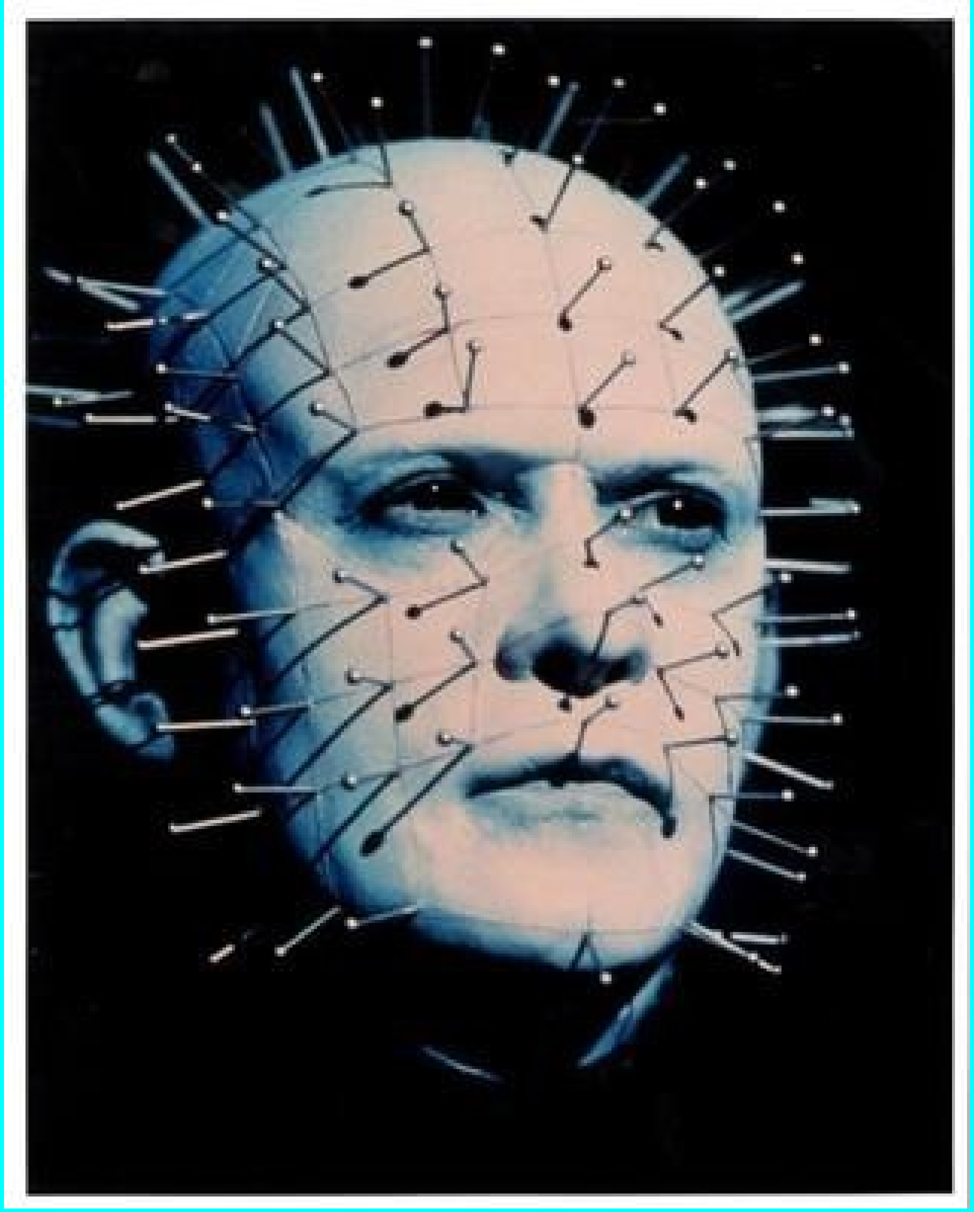
$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

What's a surface integral really?

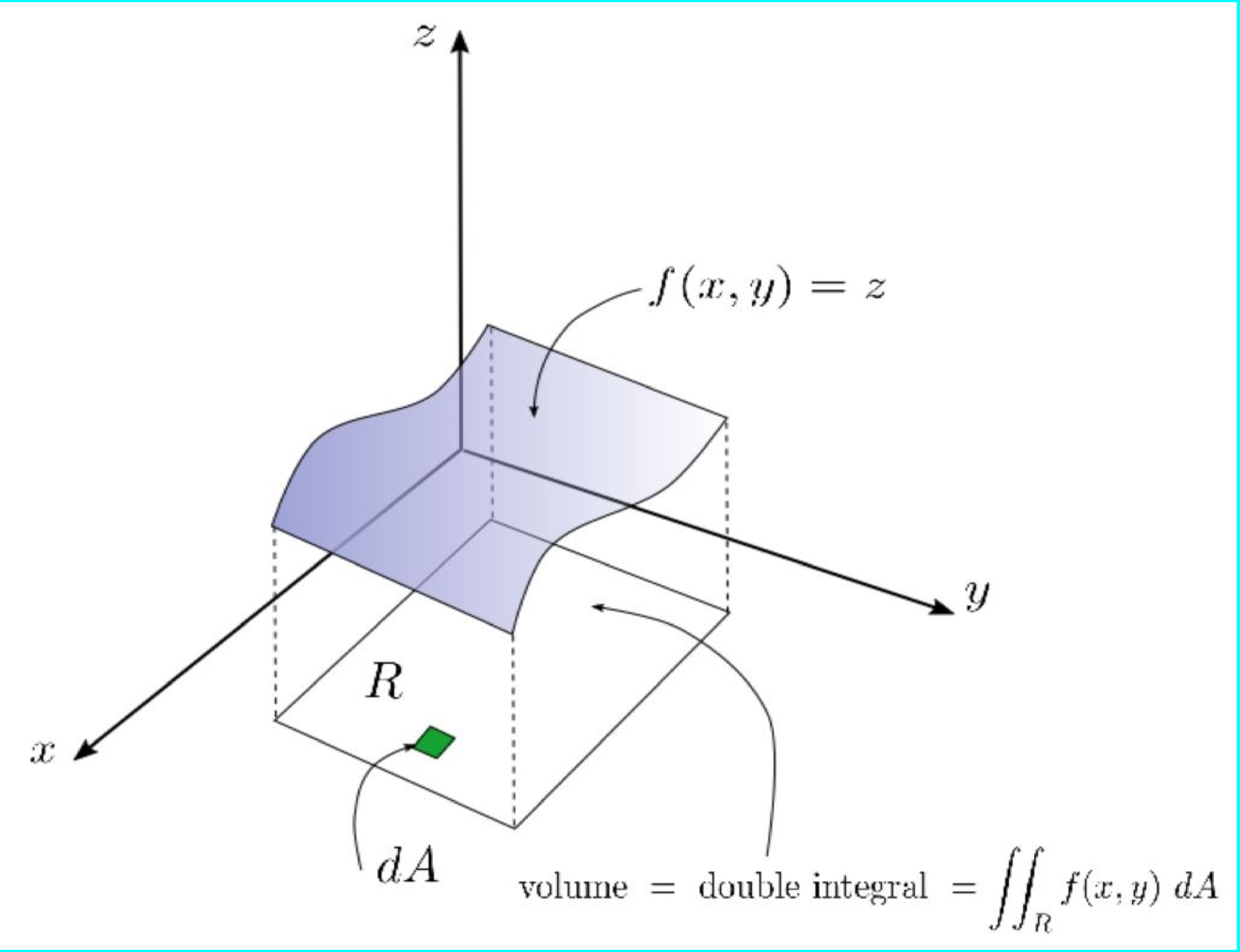
$\oint \vec{F}(u,v)\cdot\hat{n}\,dA$ Vector closed surface integral

$$\oint \vec{E}\cdot d\vec{A}=\frac{Q}{\epsilon_0}$$

$\oiint$ du dv

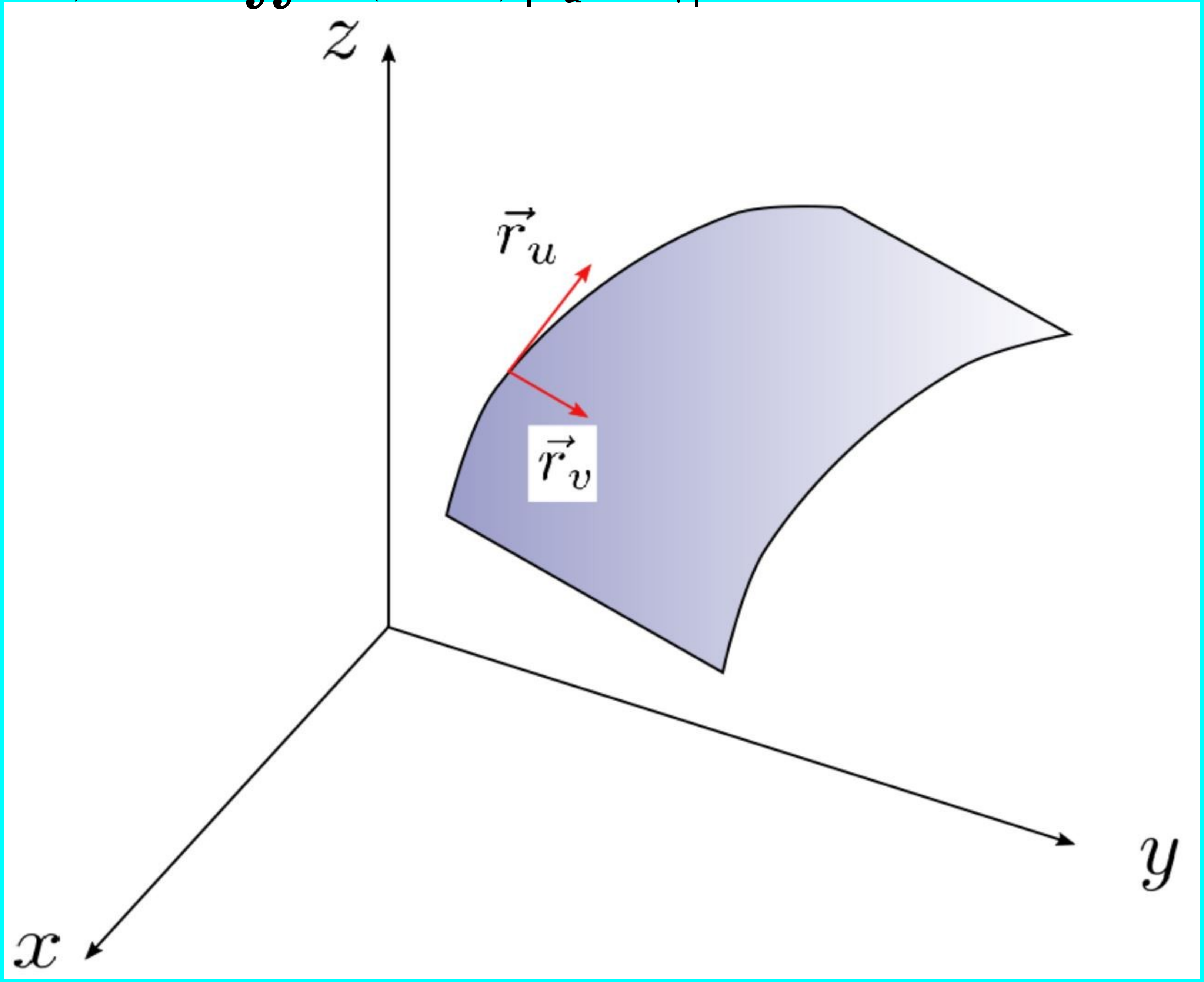


$\int f(x,y)\,dA = \iint f(x,y)\,dx\,dy$ Volume under an area



http://hartleymath.com/calculus3/double-integrals-rectangular-regions

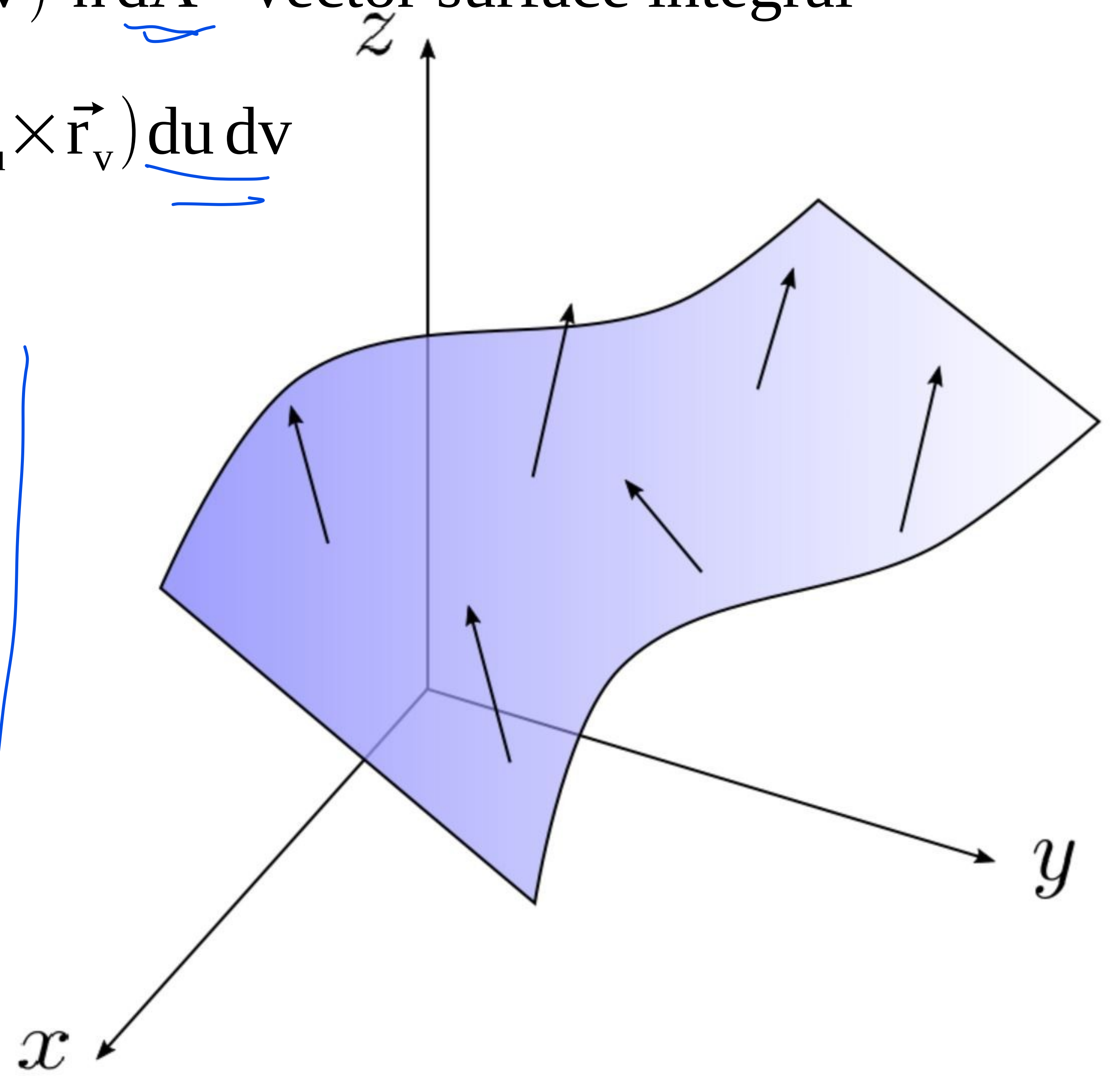
$$\int f(u,v)\,dA = \iint f(u,v)\left|\vec{r}_u \times \vec{r}_v\right| du\,dv$$ Surface integral



http://hartleymath.com/calculus3/surface-integrals

$\int \vec{F}(u,v)\cdot\hat{n}\,dA$ Vector surface integral

$\iint \vec{F}\cdot(\vec{r}_u\times\vec{r}_v)\,du\,dv$



$\oint \vec{F}(u,v)\cdot\hat{n}\,dA$ Vector closed surface integral

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

Q

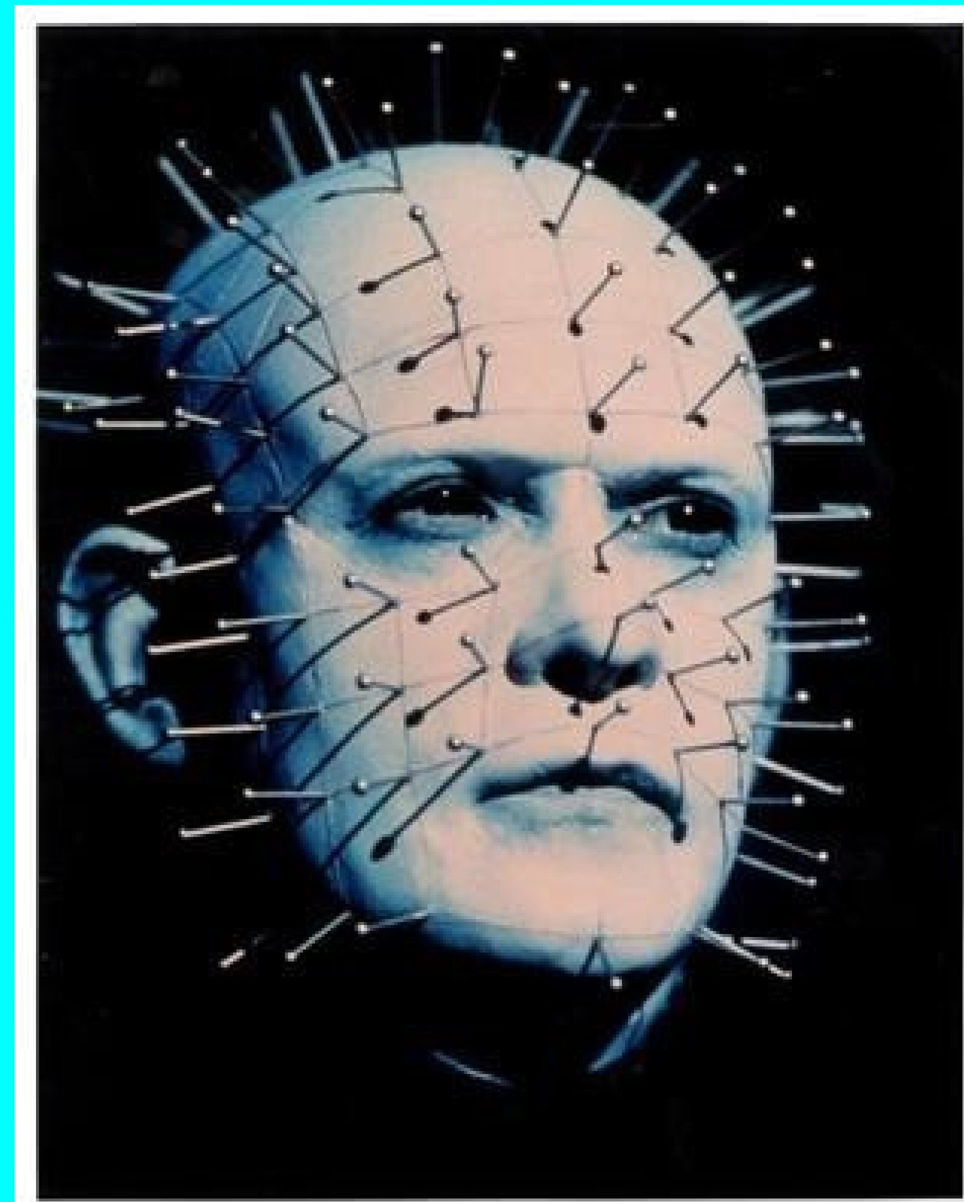
L

r

l

$$2\pi r l E = \frac{\lambda l}{\epsilon_0}$$

$$\lambda = \frac{Q}{L} \qquad E = \frac{\lambda}{2\pi r \epsilon_0}$$

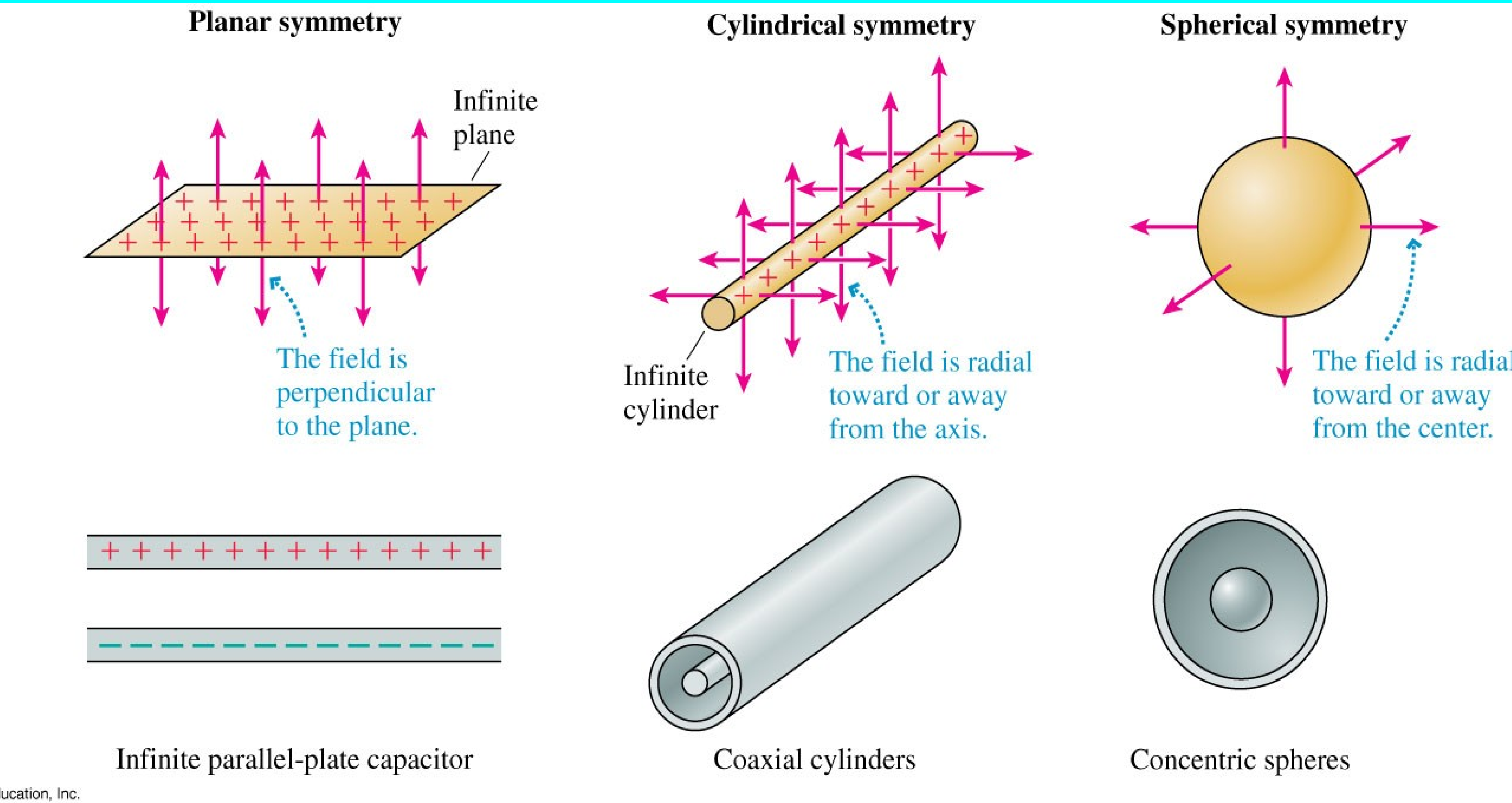


https://math.etsu.edu/multicalc/prealpha/Chap5/Chap5-5/

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

Gauss to calculate E for symettrical charges.

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k} \qquad \vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r} \qquad \vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}$$



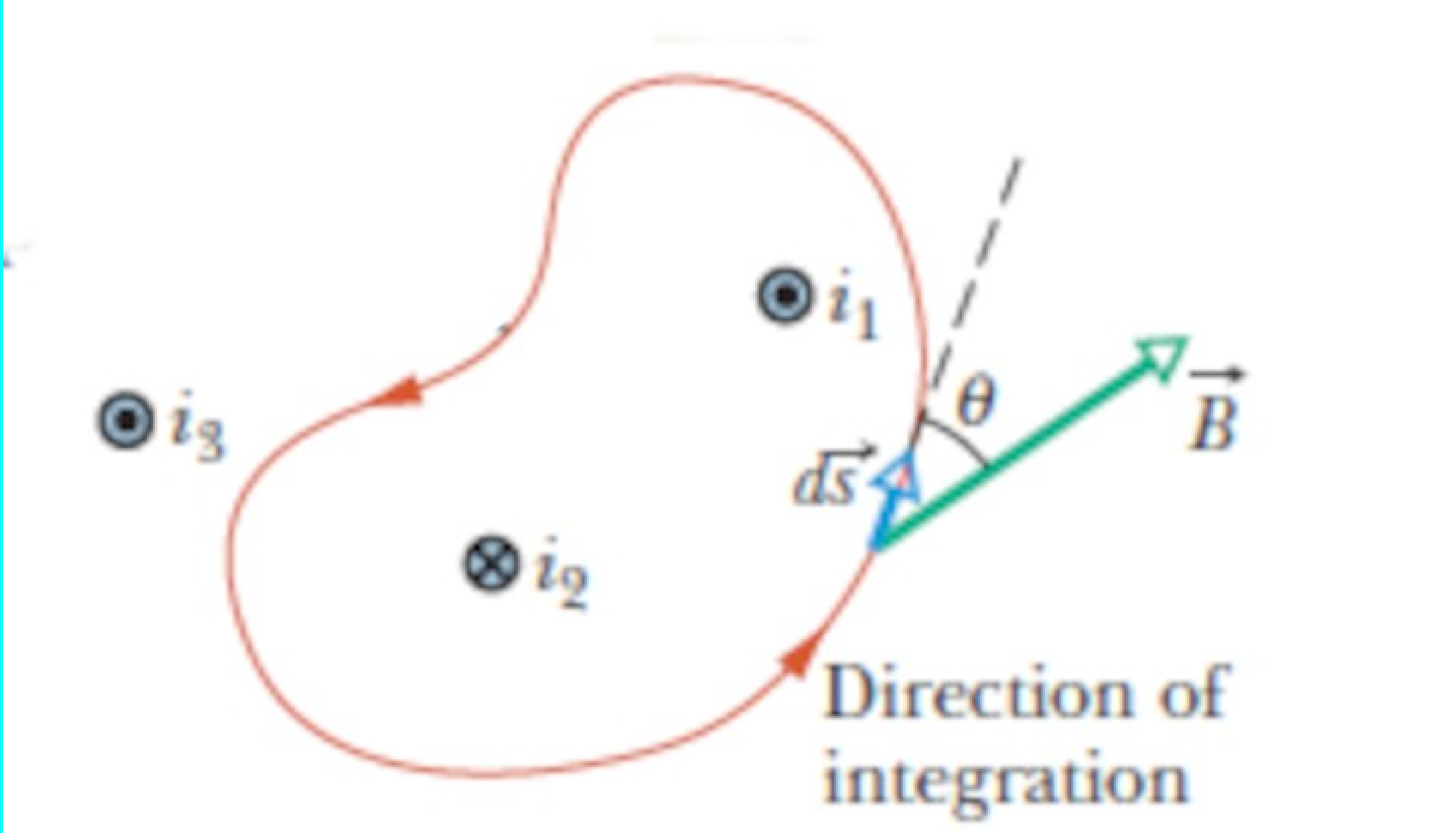
Equations from Ampere's Law

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$ Field of Infinite wire

$$\vec{B} = \frac{\mu_0 I r}{2R^2}\hat{\phi}$$ Field inside a wire of radius R

$$\vec{B} = \mu_0 n I \hat{z}$$ Field of an infinite coil (solenoid)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$ Ampere's Law.



Given $i_1 = 10$ Amps, $i_2 = -5$ A, $i_3 = 5$ A

What is $\int \vec{B} \cdot d\vec{l}$?

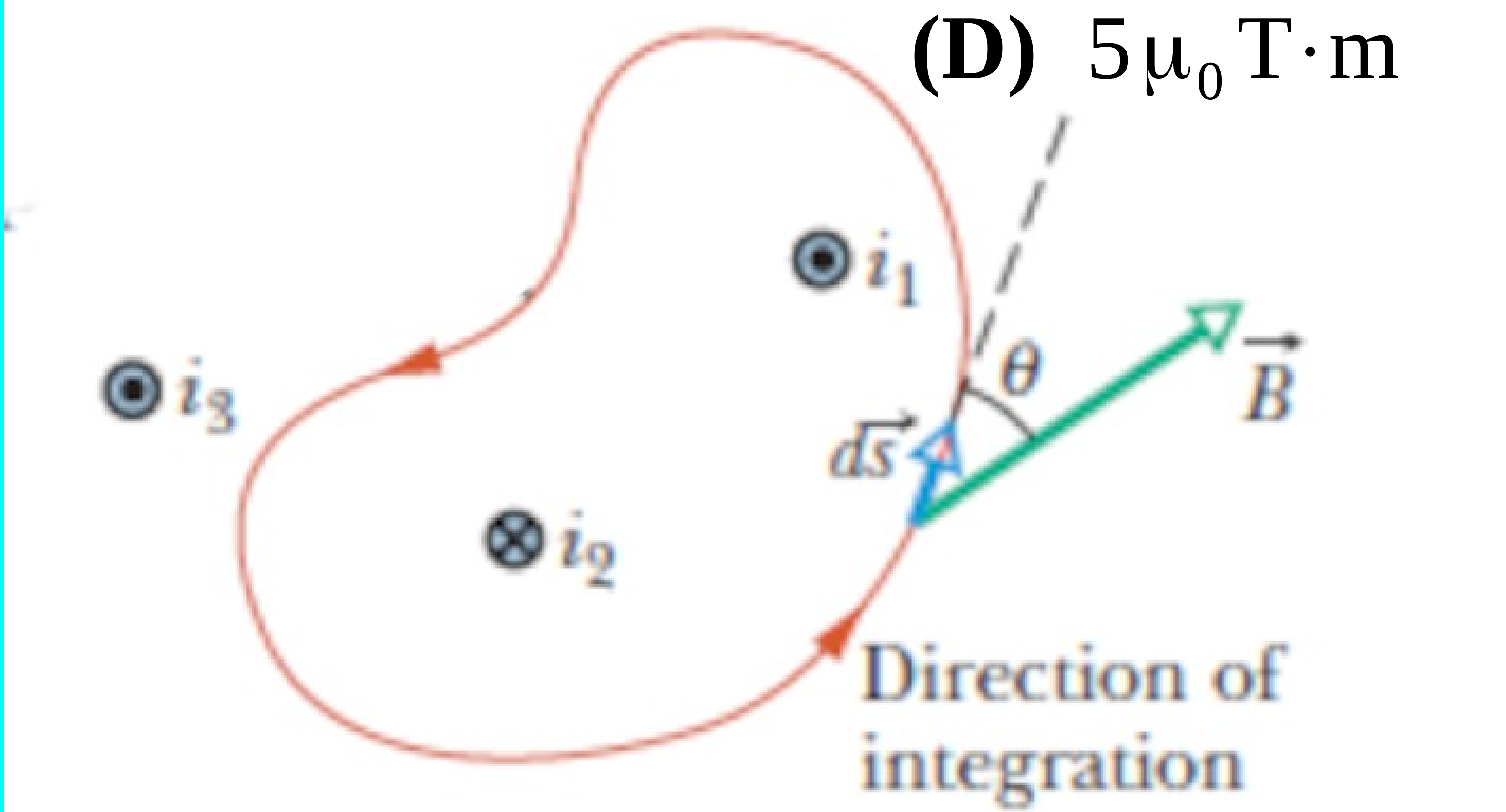
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$

(A) Plan 9 from outer space

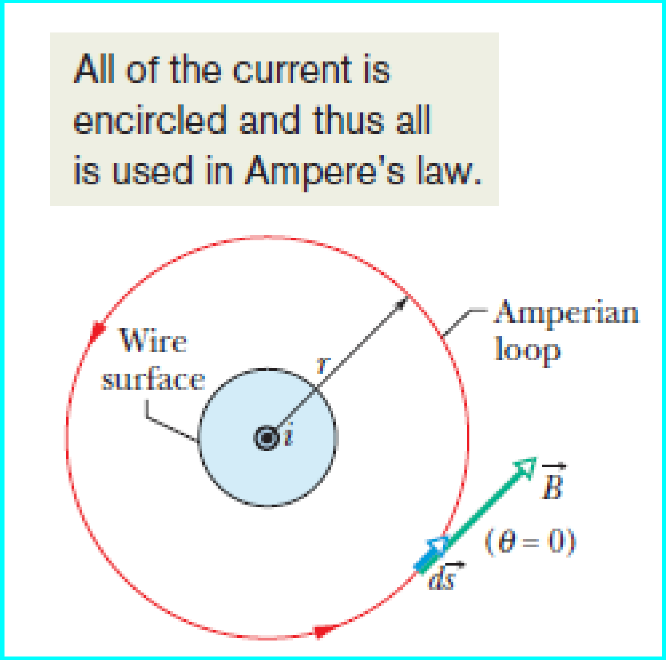
(B) $10\mu_0\, T \cdot m$

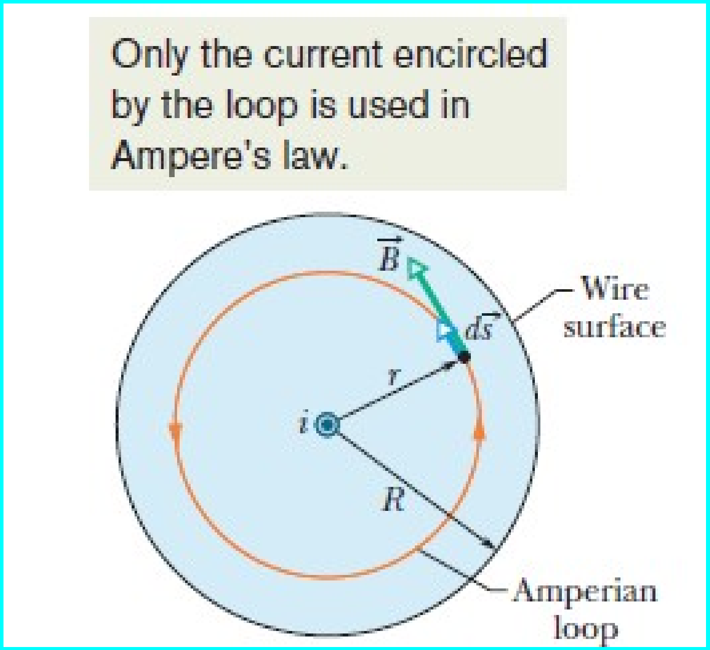
(C) $15\mu_0\, T \cdot m$

(D) $5\mu_0\, T \cdot m$

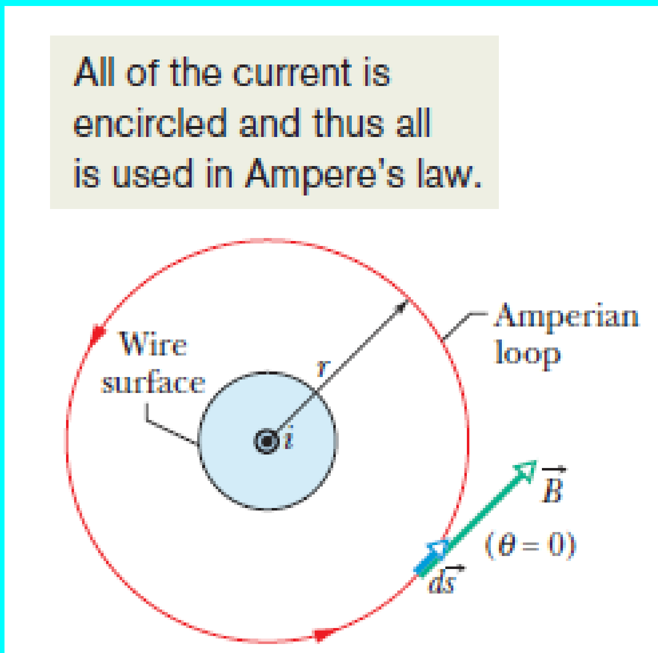


Ampere applied inside and outside of a wire





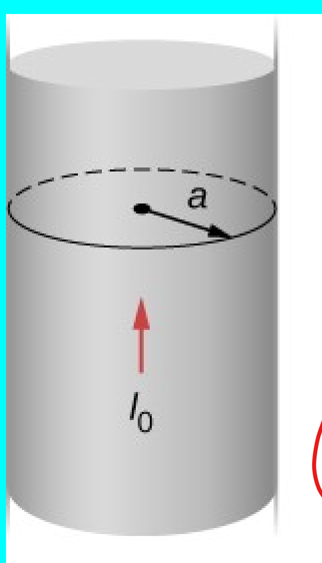
Ampere applied outside of a wire



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

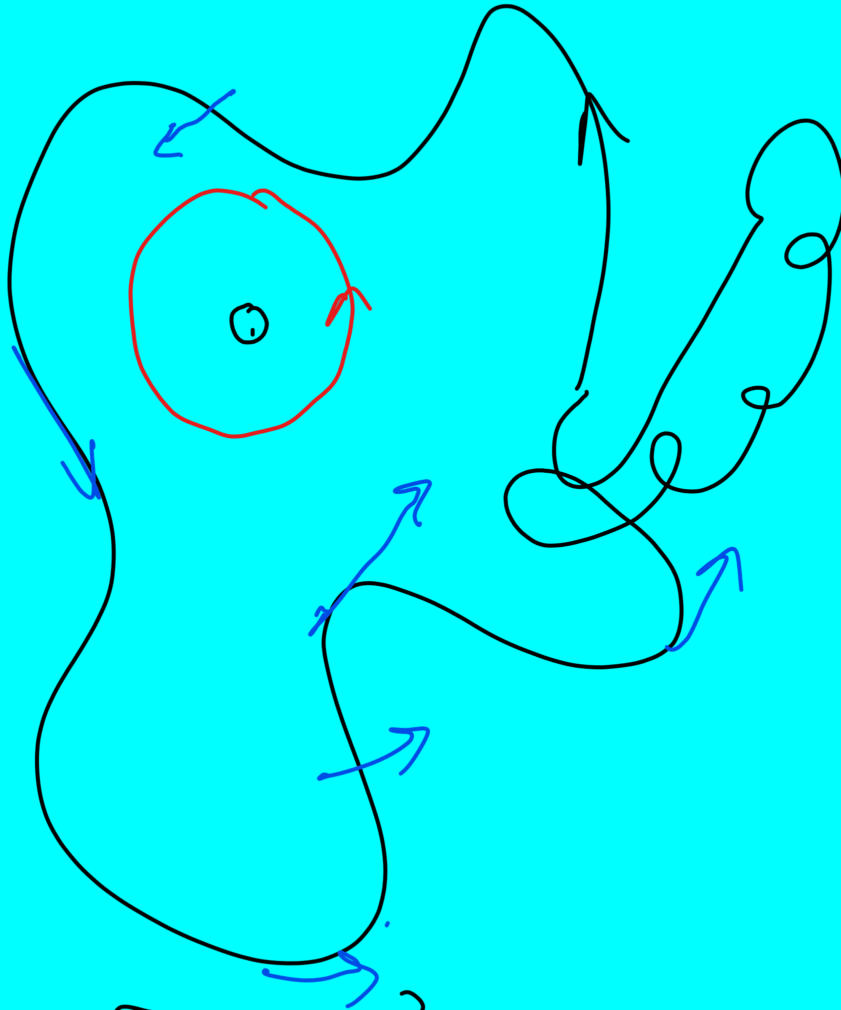
$$B\,2\pi r = \mu_0 I$$

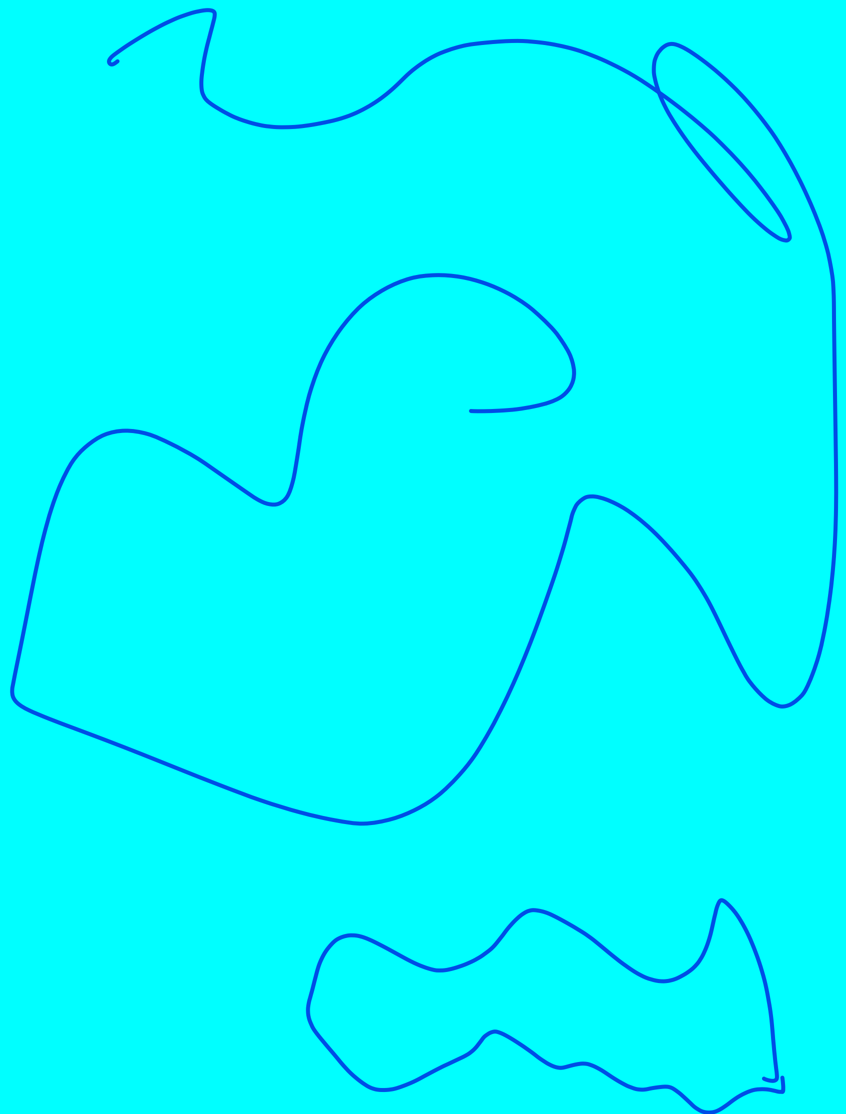
$$B = \frac{\mu_0 I}{2\pi r}$$



(a)

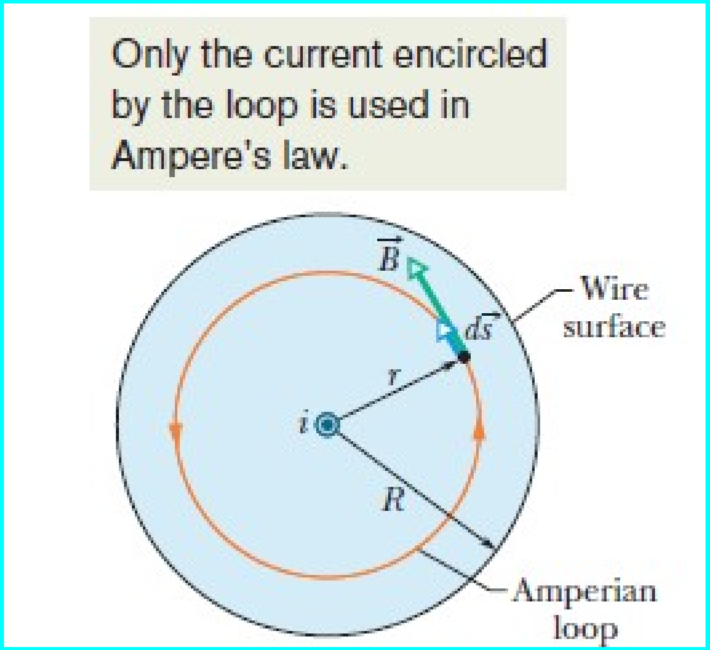
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$ Field outside long wire

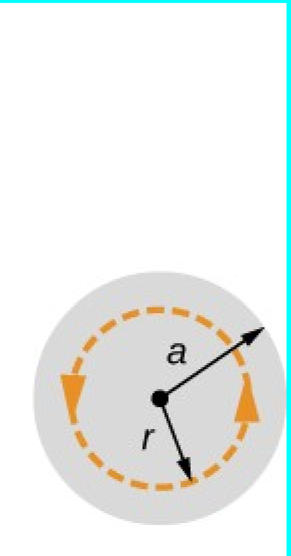




$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Ampere applied inside of a wire

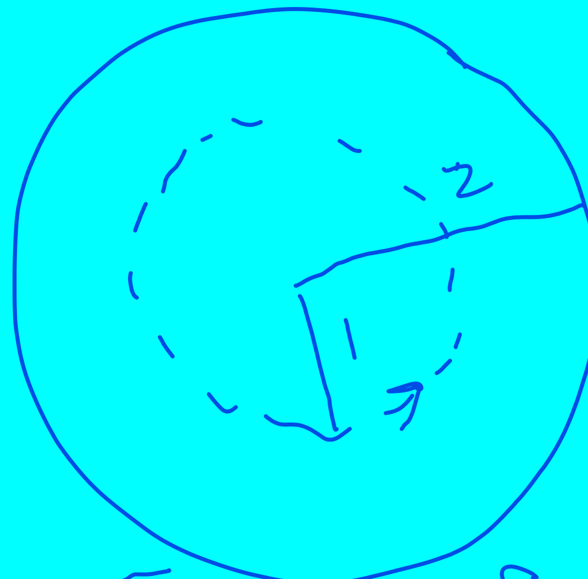




(b)

$$\vec{B} = \frac{\mu_0 I r}{2 R^2} \hat{\phi}$$ Field inside wire of radius R

A wire with radius 2 cm carries 8 Amperes How many Amps are contained within 1 cm radius?



$$J = \frac{I}{\pi R^2} = \frac{8}{\pi (2\,cm)^2}$$

$$I_{loop} = \pi r^2 J = \pi r^2 \frac{8}{\pi (2)^2}$$

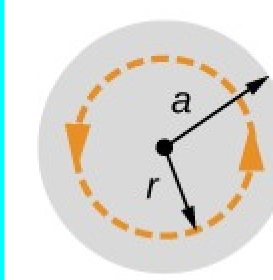
$$I = \pi (1^2) \frac{8}{\pi (2)^2}$$

(A) 16

(B) 8

(C) 4

(D) 2



(b)

A wire with radius “r” carries “I” Amperes How many Amps are contained within radius “a”?

$$J = \frac{I}{\pi a^2}$$

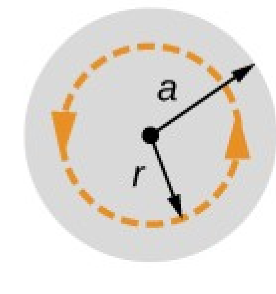
$$I_{enclosed} = \pi r^2 \frac{I}{\pi a^2}$$

(A) I

(B) $\frac{I}{4}$

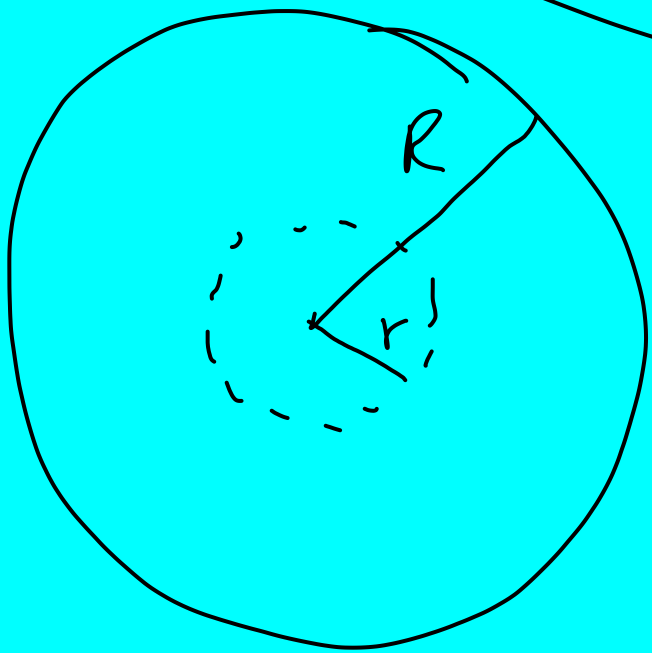
(C) $I\frac{a}{r}$ $\frac{r}{a}$

(D) $I\frac{a^2}{r^2}$ $\frac{r^2}{a^2}$ ✓



(b)

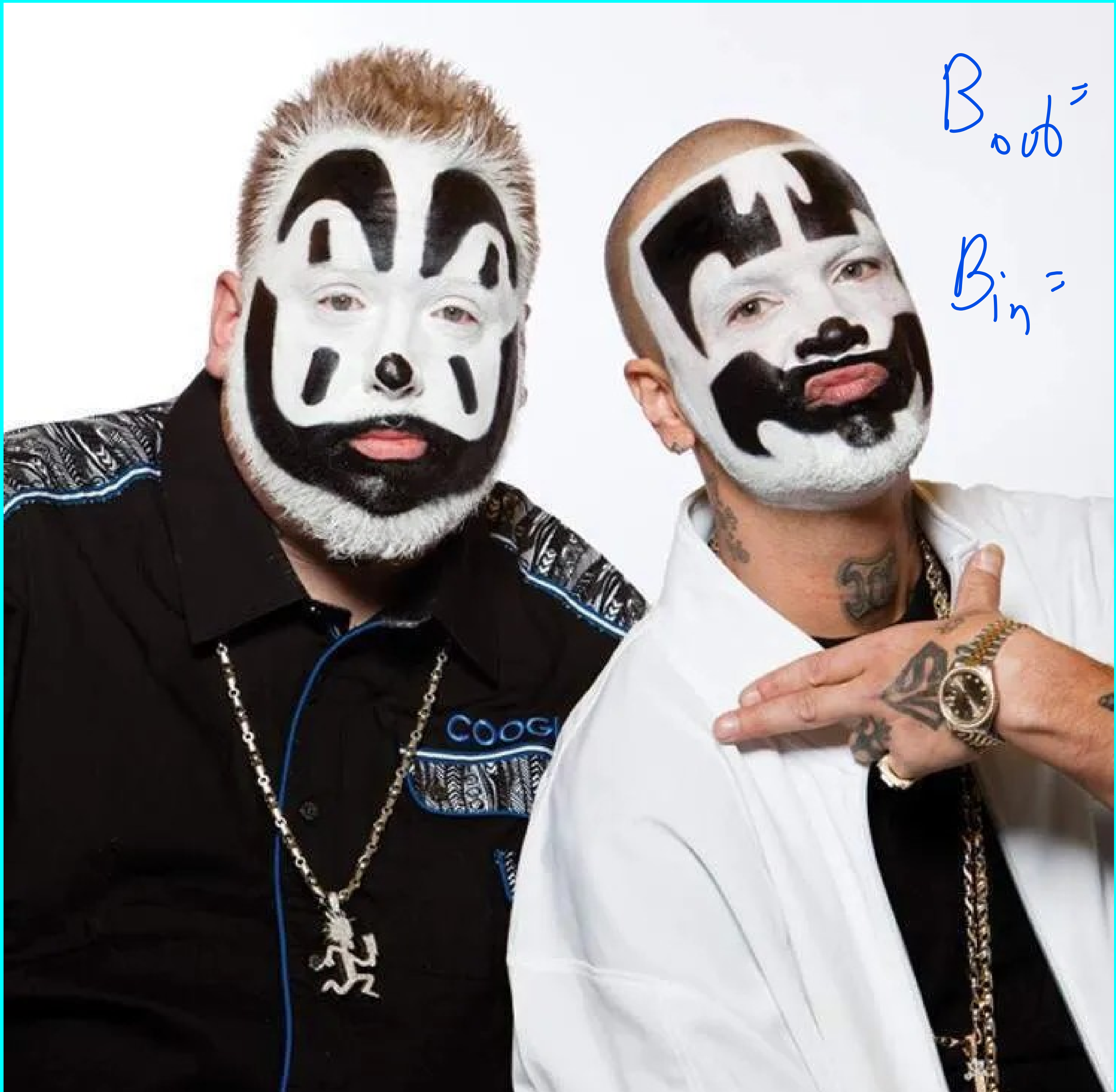
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$



$$2\pi r B = \mu_0 I \frac{\pi r^2}{\pi R^2}$$

$$\vec{B} = \frac{\mu_0 I r}{2\pi R^2}\,\hat{\phi}$$

Sanity Check: What happens at r = R?



$$B_{out} = \frac{\mu_0 I}{2\pi r}$$

$$B_{in} = \frac{\mu_0 I r}{2\pi R^2}$$

$$r = R$$

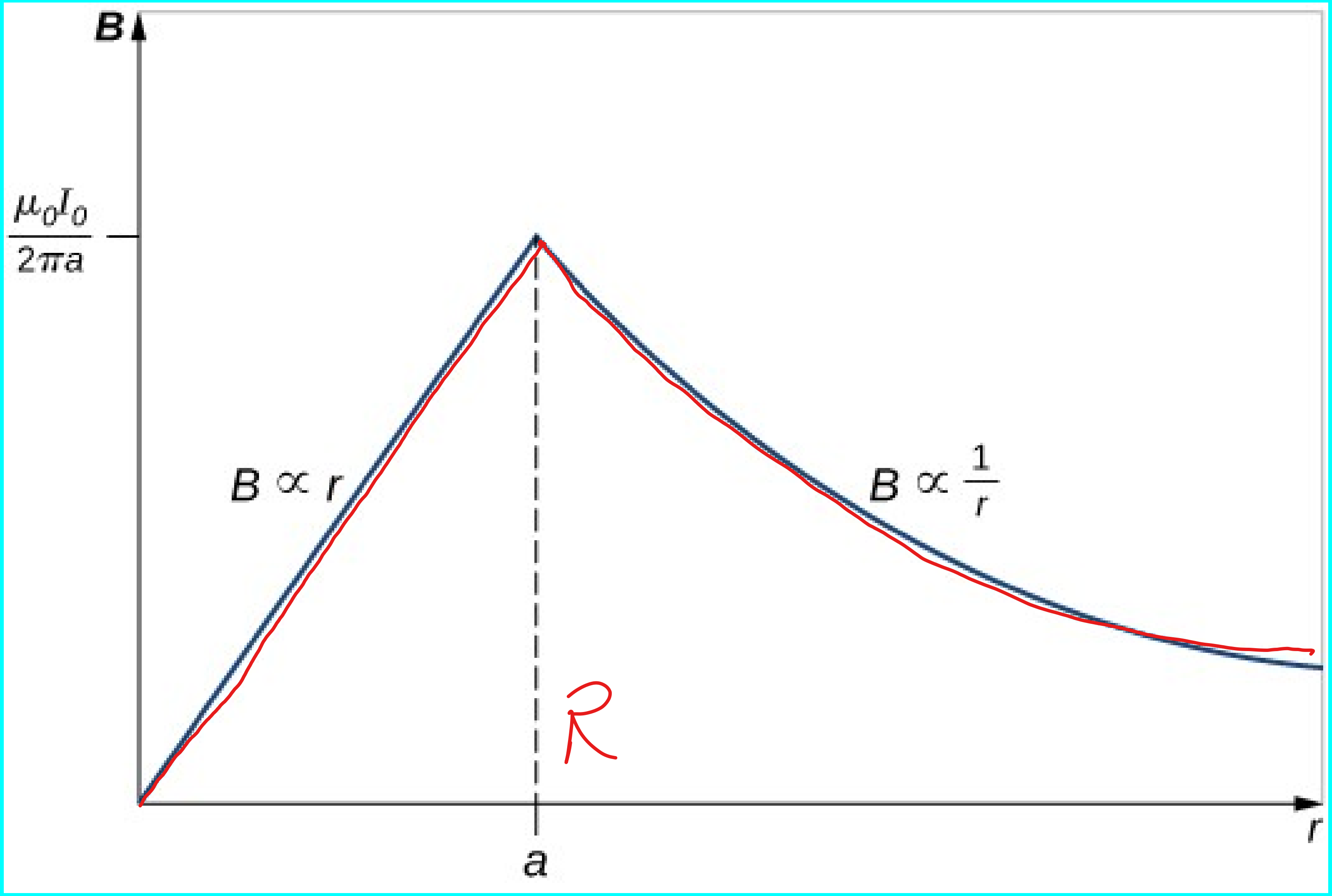
$$\frac{\mu_0 I r}{2\pi r^2}$$

What happens at r = R?

$$\vec{B} = \frac{\mu_0 I r}{\pi 2 R^2}\hat{\phi}$$ Field inside wire of radius R

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$ Field outside long wire

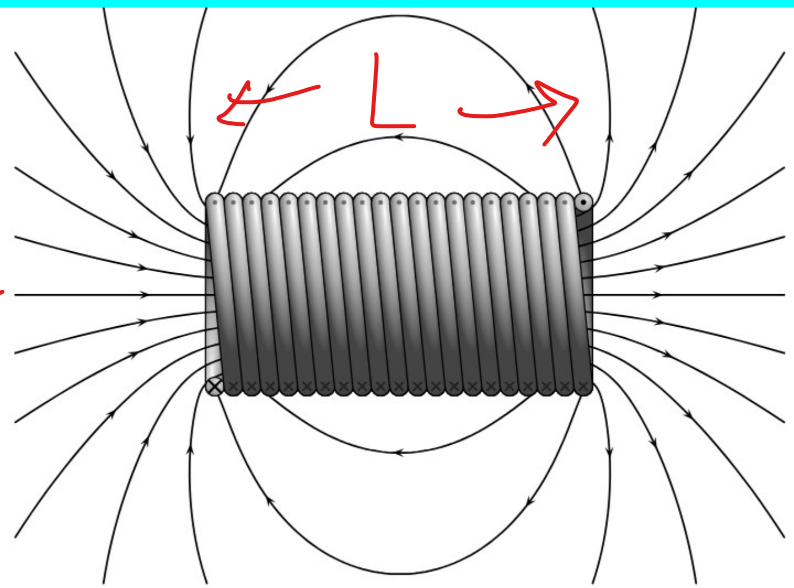
Ampere result inside and outside of a wire



Derive field of a solenoid

$n = \frac{N}{L} = \frac{20}{3} = 6.6\overline{6}$

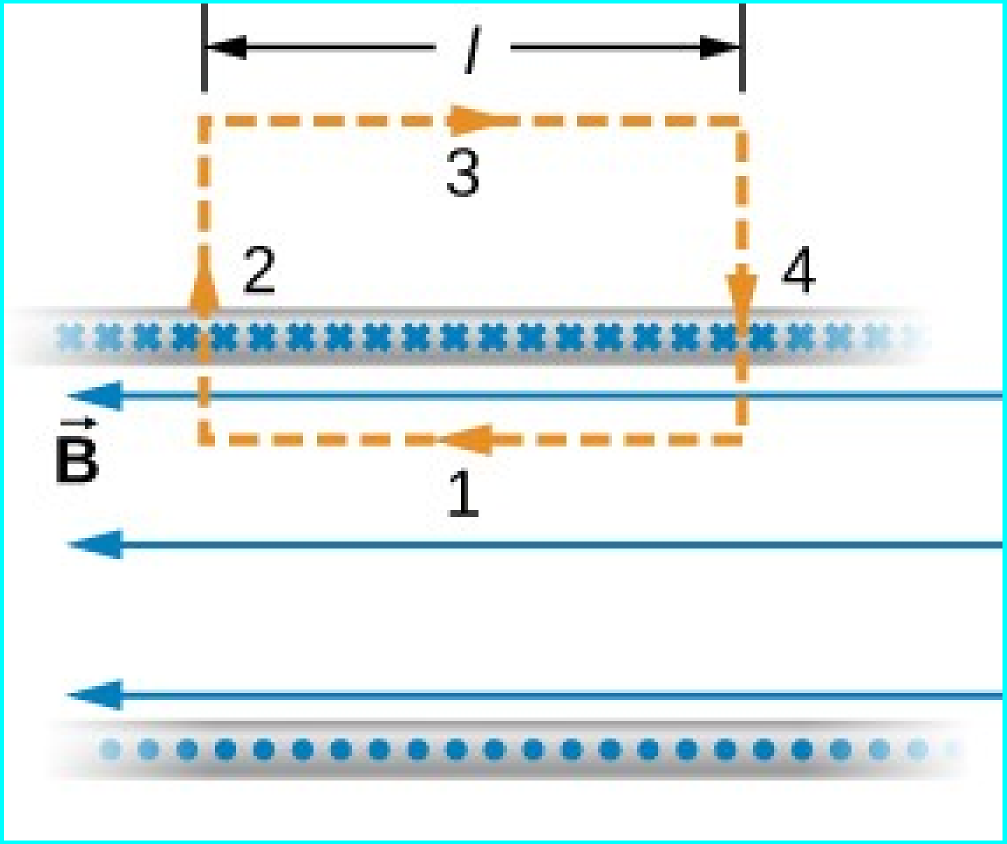
$n = .06$[illegible] $\frac{1}{m}$



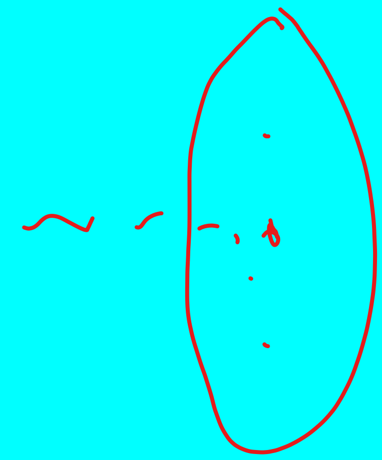
$$\vec{B} = \mu_0 n I \hat{z}$$

Field of an infinite coil (solenoid)

$B = \frac{\mu_0 I N}{2a}$



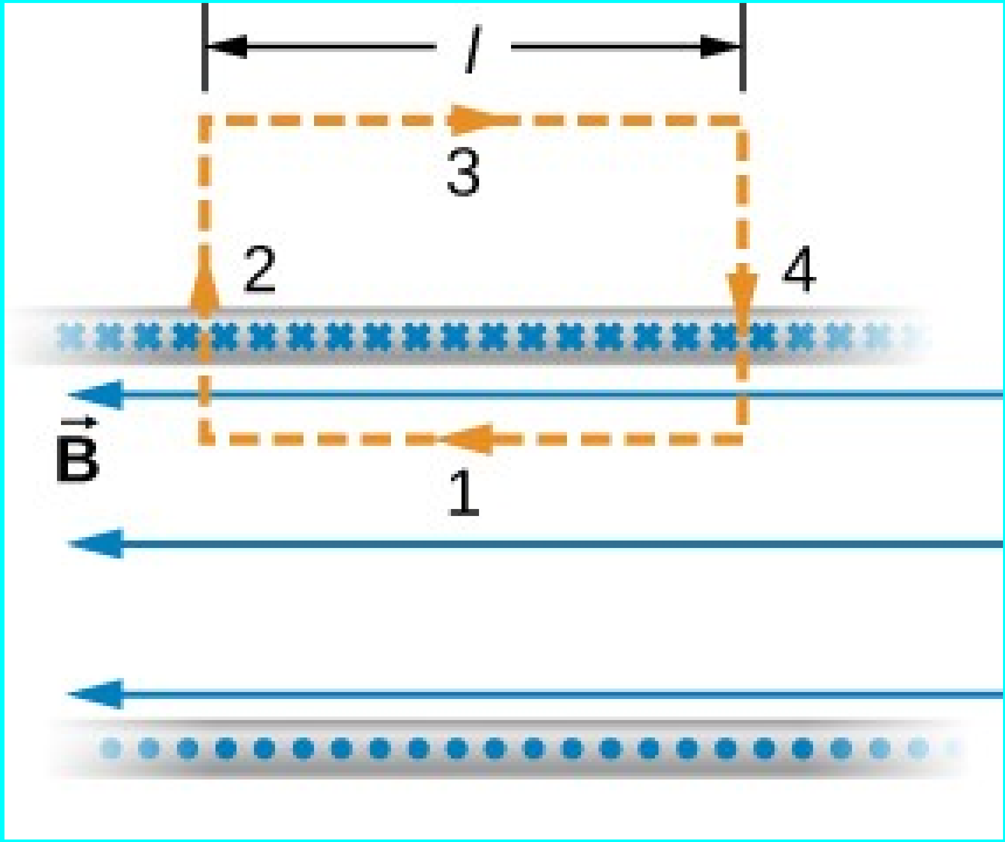
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$



$\vec{B} = \mu_0 n I \hat{z}$ Field of an infinite coil (solenoid)

If the solenoid carries current "I" and there are "N" windings inside the amperian loop what is $\oint \vec{B}\cdot d\vec{l}$?

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

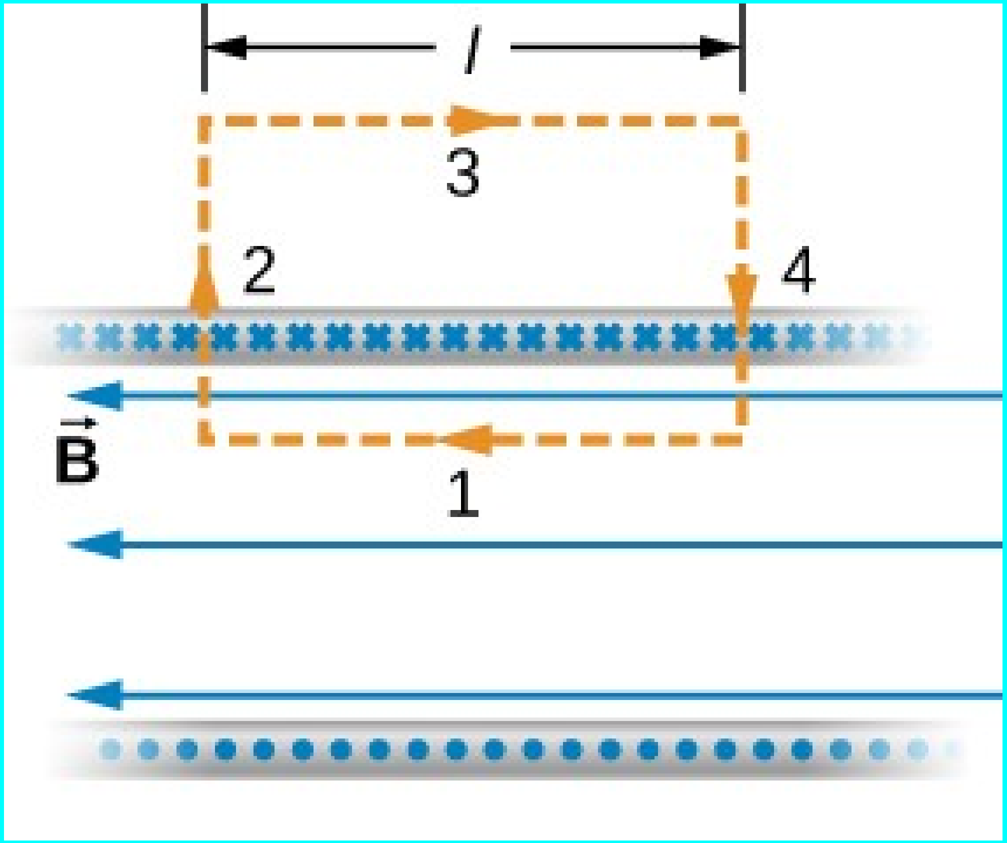


(A) I

(B) $\mu_0 N I$

(C) $\mu_0 I/N$

(D) $-\mu_0 I$



$\vec{B} = \mu_0 n I \hat{z}$ Field of an infinite coil (solenoid)

$\vec{B} = \mu_0 n I \hat{z}$ Field of an infinite coil (solenoid)

If the solenoid carries current "I" and there are "N" windings inside the amperian loop what is $\oint \vec{B} \cdot d\vec{l}$?

(A) I

(B) $\mu_0 N I$

(C) $\mu_0 I/N$

(D) $-\mu_0 I$

If the coil shown is 10 cm long and has a total of 4000 windings and carries two Amperes, what is the magnetic field inside?

(A) 0.25 T

(B) 0.025 T

(C) 1 T

(D) 0.1 T

$$\vec{B} = \mu_0 n I \hat{z}$$

Ferromagnets, Permeability and Susceptibility

$\vec{B} = \mu_0 n I \hat{z}$ Field of an infinite coil (solenoid)

$\vec{B} = \mu_0 (1 + \chi) n I \hat{z}$ With susceptibility

$\vec{B} = \mu n I \hat{z}$ With permeability ¿

Recap Lecture 24

- Amperes law
- Field of a wire
- Field of a solenoid
-

Maxwell's Equations

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$ To calculate E for symettrical charges.

$$\oiint \vec{B}\cdot d\vec{A} = 0$$ Cannot have North magnet w/o a South pole.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$ To calculate B for symettrical currents.

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$ Magnetic induction! Generators! Light!

$$\Phi_B = \int B\cdot d\vec{A}$$

How do I avoid an infinite formula sheet – really?

$\Delta V = -\int \vec{E} \cdot d\vec{l}$ Definition of potential.

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ Gauss Law (sphere/line/plane).

$Q = CV$ Def. of capacitance

$V = IR \quad R = \rho \frac{L}{A}$ Ohm's Law and origin of resistance

$U = qV$ Relation between potential and potential energy

$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ Lorentz force Law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ Ampere's law (wire/solenoid/current sheet).