Lecture 23: 04/11/2024

- Announcements
  Exams back today

- Last Time
  Motors/torque, magnetic moment
  Faraday’s Law

- Today
  The infinite formula sheet
  Circular motion of a charged particle
  Motor review
  Ampere’s Law
  Maxwell equations?

How do I avoid an infinite formula sheet?

The deeper the understanding, the fewer the formulae.

E&M has four equations total (Maxwell's equations)

Maxwell's Equations

$$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$ To calculate E for symettrical charges.

$$\int \vec{B} \cdot d\vec{A} = 0$$ Cannot have North magnet w/o a South pole.

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$ To calculate B for symettrical currents.

$$\int \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$ Magnetic induction! Generators! Light!

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… REALLY??

How do I avoid an infinite formula sheet?

The deeper the understanding, the fewer the formulae.

E&M has four equations total (Maxwell's equations)

… REALLY??

… no not yet

How do I avoid an infinite formula sheet – Physics I

$KE_i + U_i = KE_f + U_f$ Energy conservation.

$F_c = \frac{mv^2}{R}$ Centripetal force in circular motion.

$F_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$ Newton's 2nd

$W = \int \vec{F} \cdot d\vec{l}$ Definition of work.

$U = -W$ Relation between work and Potential Energy

$\vec{\tau} = \vec{r} \times \vec{F} = I\alpha$ Torque definition

How do I avoid an infinite formula sheet – really?

$\Delta V = -\int \vec{E}\cdot d\vec{l}$ Definition of potential.

$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$ Gauss Law (sphere/line/plane).

$Q = CV$ Def. of capacitance

$V = IR$ $R = \rho\frac{L}{A}$ Ohm's Law and origin of resistance

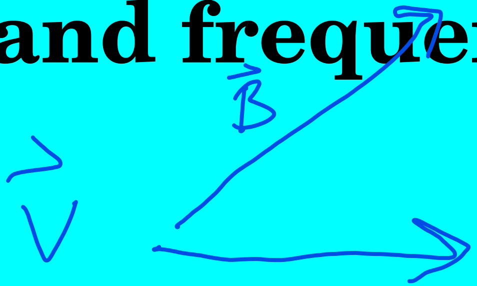
$U = qV$ Relation between potential and potential energy

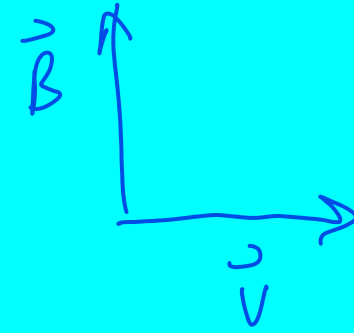
$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$ Lorentz force Law

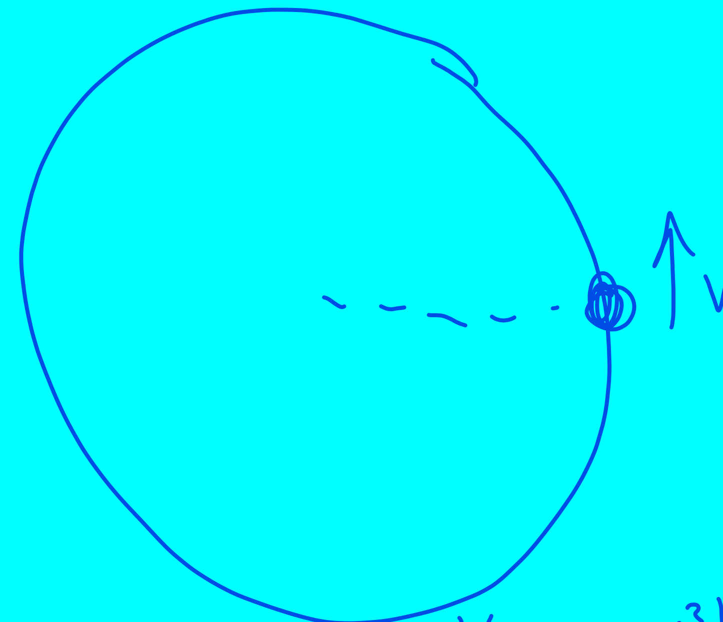
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ Ampere's law (wire/solenoid/current sheet).

How do I avoid an infinite formula sheet – Example … Circular motion in a B-field

Given a proton moving at 1000 km/s in Earth's magnetic field, what is the radius and frequency of the orbit?







$$\vec{F} = q\vec{v} \times \vec{B}$$

$$qvB\sin\theta$$

$$F = qvB$$

$$F_c = \frac{mv^2}{R}$$

$$\frac{mv^2}{R} = qvB$$

$$\frac{mv}{R} = qB$$

$$R = \boxed{208.75}$$

$$R = \frac{mv}{qB} = \frac{(1800)(9.11\times10^{-31})(10^6)}{(1.6\times10^{-19})(50\times10^{-6})}$$

$$= \frac{(1.8\times10^3)(9.11\times10^{-31})(10^6)}{(1.6\times10^{-19})(5\times10^{-5})}$$

$$= (100) \approx 200\ m$$

$$\frac{mv^2}{R} = qvB$$

$$f = ?$$

$$f = \frac{\omega}{2\pi}$$

$$v = \omega R$$

$$\frac{mv}{R} = qB \longrightarrow \frac{m\omega R}{R} = qB$$

$$\omega = \frac{qB}{m}$$

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

$$f = \frac{qB}{2\pi m} = \frac{(1.6\times10^{-19})(5\times10^{-5})}{(6)(2\times10^{-27})}$$

$$= \frac{(1.6)(5)}{6\cdot 2}\left(\frac{10^{-24}}{10^{-27}}\right)$$

$$v = \omega R$$

$$f = \frac{\omega}{2\pi}$$

$$v = 2\pi f R \rightarrow f = \frac{v}{2\pi R}$$

$$\frac{8}{6} = \frac{4}{3}\,10^{3} = 1333\ \text{Hz}$$

$$m_{e^-} = \frac{1}{1800}\,m_p$$

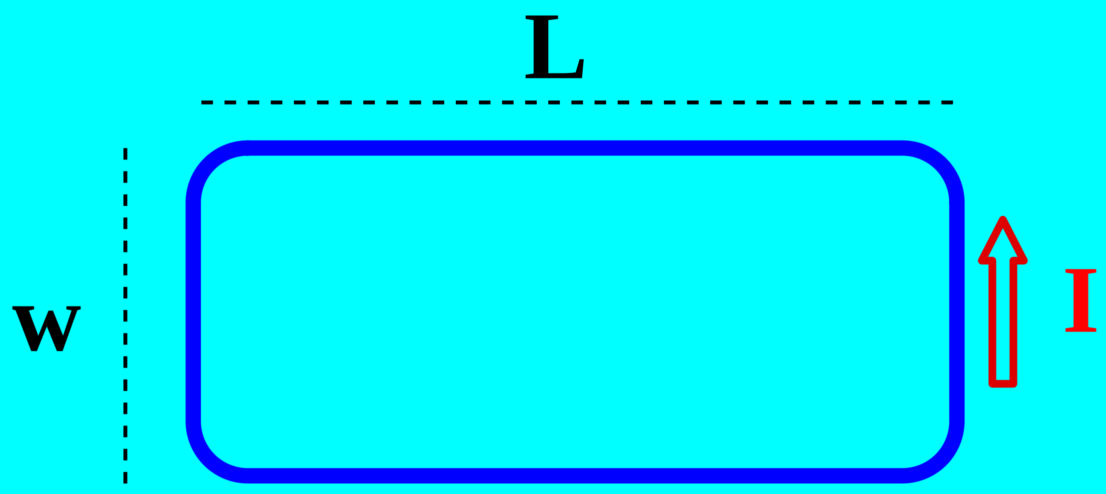
$$\frac{10^{6}}{(6)(208)}$$

2.5 mHz

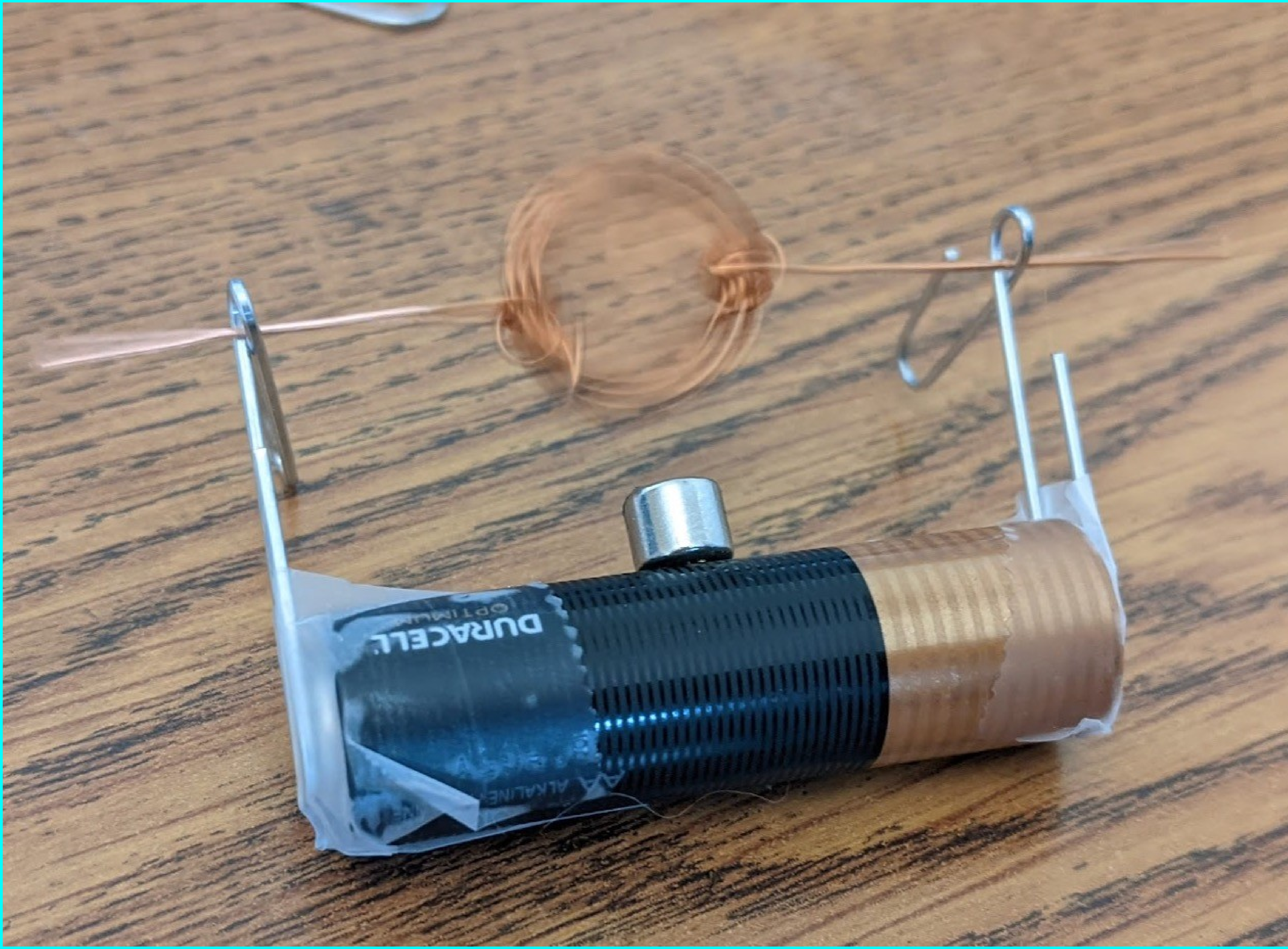
Motor Review

Motors

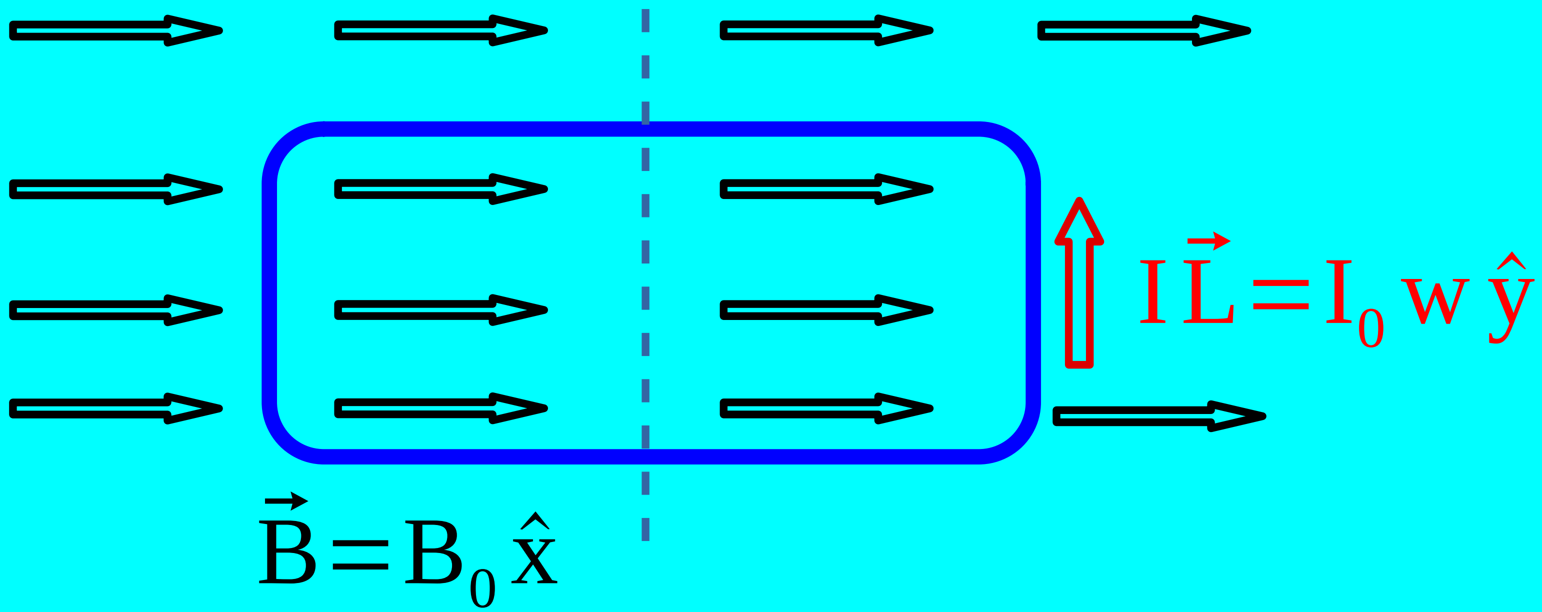
Are just clever loops of wire in magnetic fields



$$\vec{F} = I\,\vec{L} \times \vec{B}$$ Force on current I


DURACELL

Try B-field to the right



$\vec{F} = I\vec{L} \times \vec{B}$ Force on current I

Equations for motors

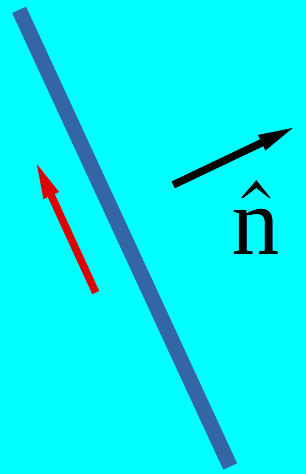
$\vec{F} = I\,\vec{L} \times \vec{B}$ Force on current I

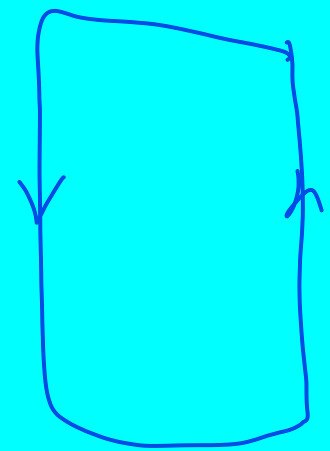
$\vec{\tau} = \vec{r} \times \vec{F}$ Torque definition

$\vec{m} = N\,I\,\vec{A}$ Def. of Magnetic Moment

$\vec{\tau} = \vec{m} \times \vec{B}$ Torque on a Magnetic Moment

This is a side view of a wire loop. Which direction should B point for maximum torque?





$\vec{m} = I\vec{A}$

$A\hat{n} = \vec{A}$

$\vec{\tau} = \vec{m} \times \vec{B}$

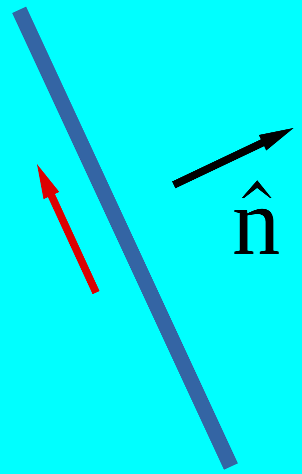
(A) To the right

(B) Up

(C) Along n̂

(D) Perpendicular to n̂

This is a side view of a wire loop. Which direction should B point for zero torque?



$\vec{\tau} = \vec{m} \times \vec{B}$

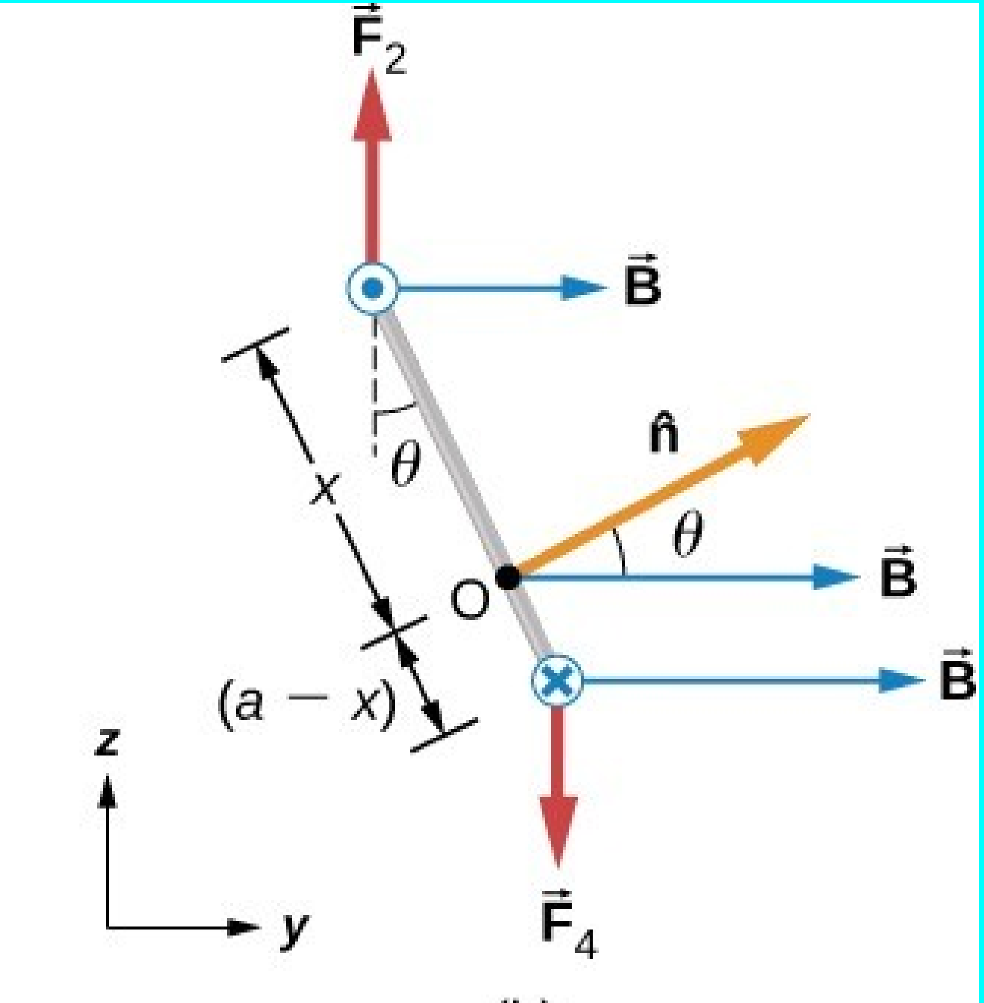
(A) To the right

(B) Up

(C) Along n̂

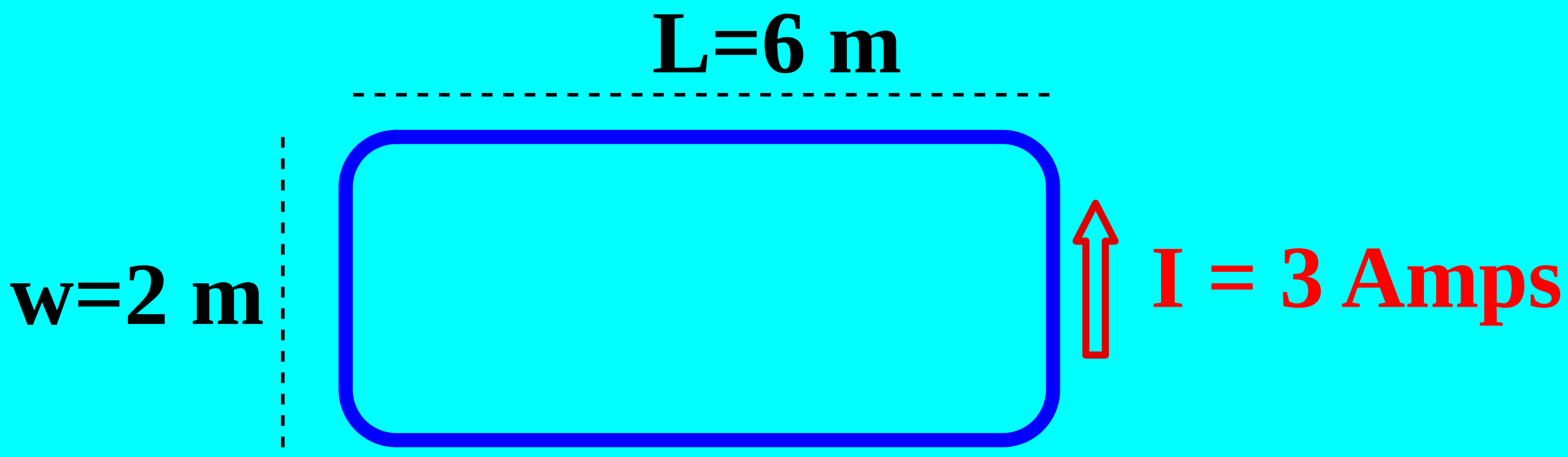
(D) Perpendicular to n̂

$$\hat{n} = \frac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|}$$



(b)

In a 2 T B-field, what is the maximum torque on this loop?



(A) $\tau = 6\,\mathrm{N \cdot meter}$

(B) $\tau = 3\,\mathrm{N \cdot meter}$

(C) $\tau = 36\,\mathrm{N \cdot m}$

(D) $\tau = 72\,\mathrm{N \cdot m}$

$\vec{\tau} = \vec{m} \times \vec{B}$

$\vec{m} = I\vec{A}$

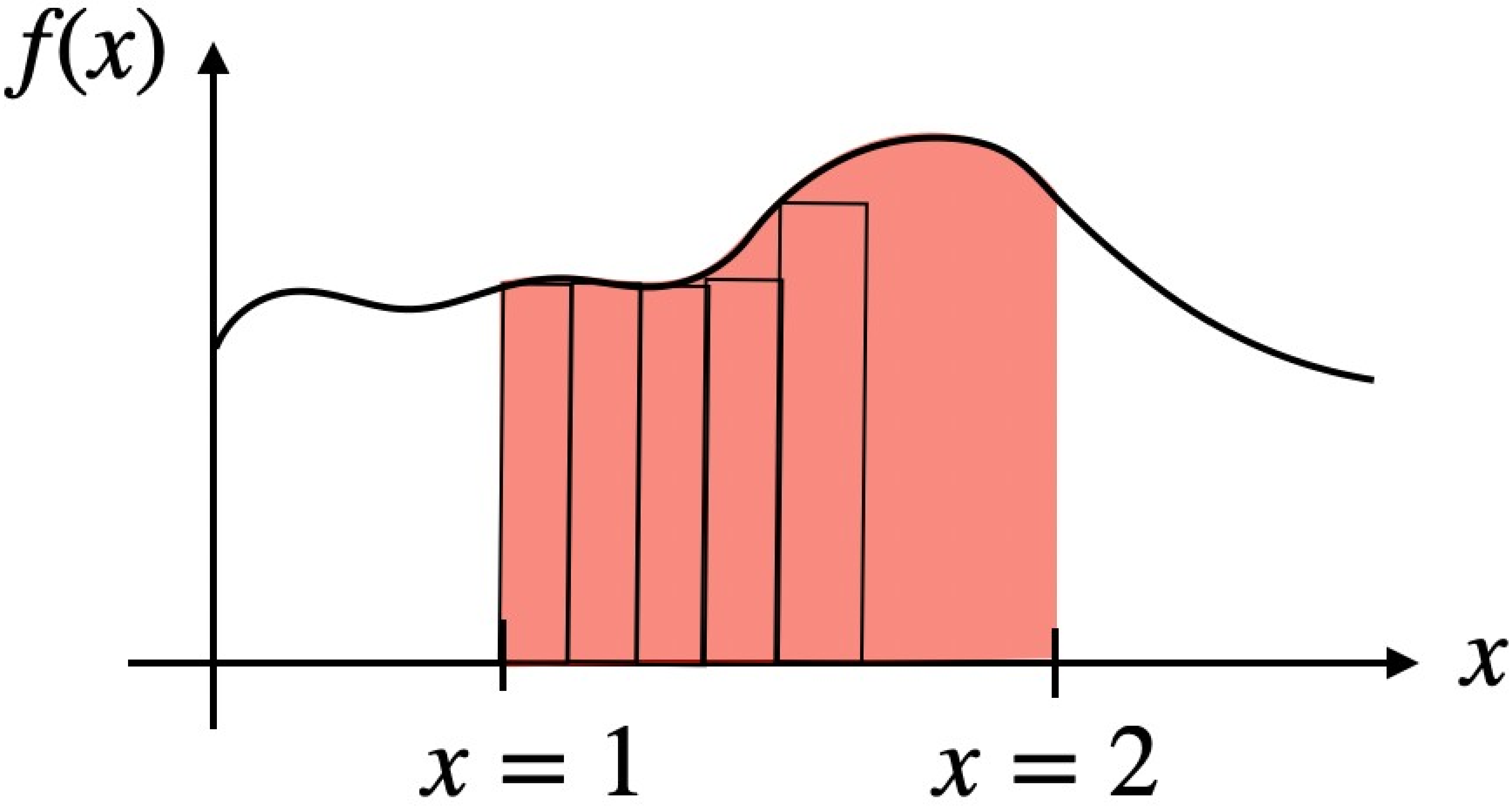
$\vec{m} = IA\hat{n}$

Ampere's Law

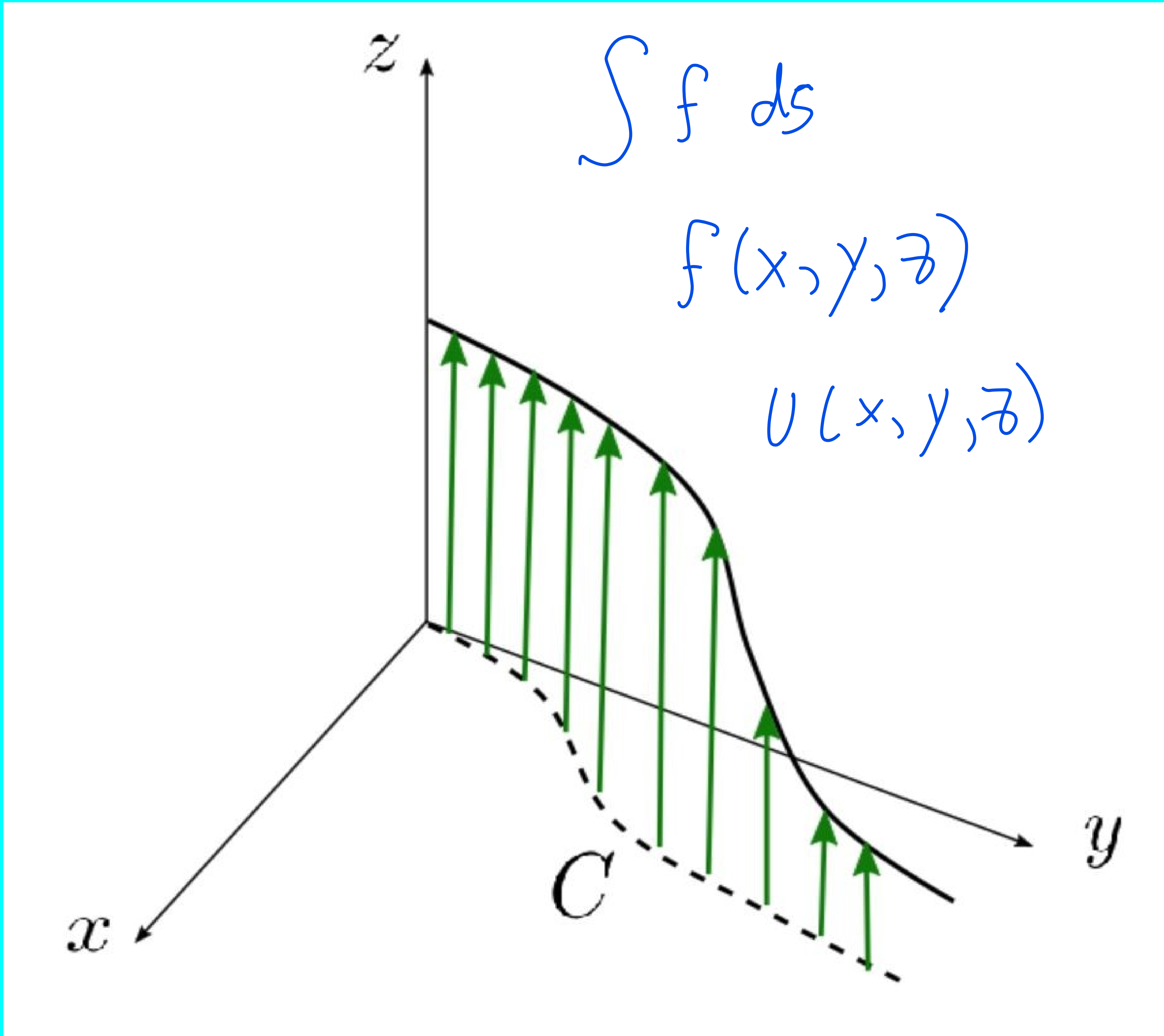
Is always true and is one of the Maxwell Equations

Allows you to calculate the magnetic field in highly symmetric cases of currents

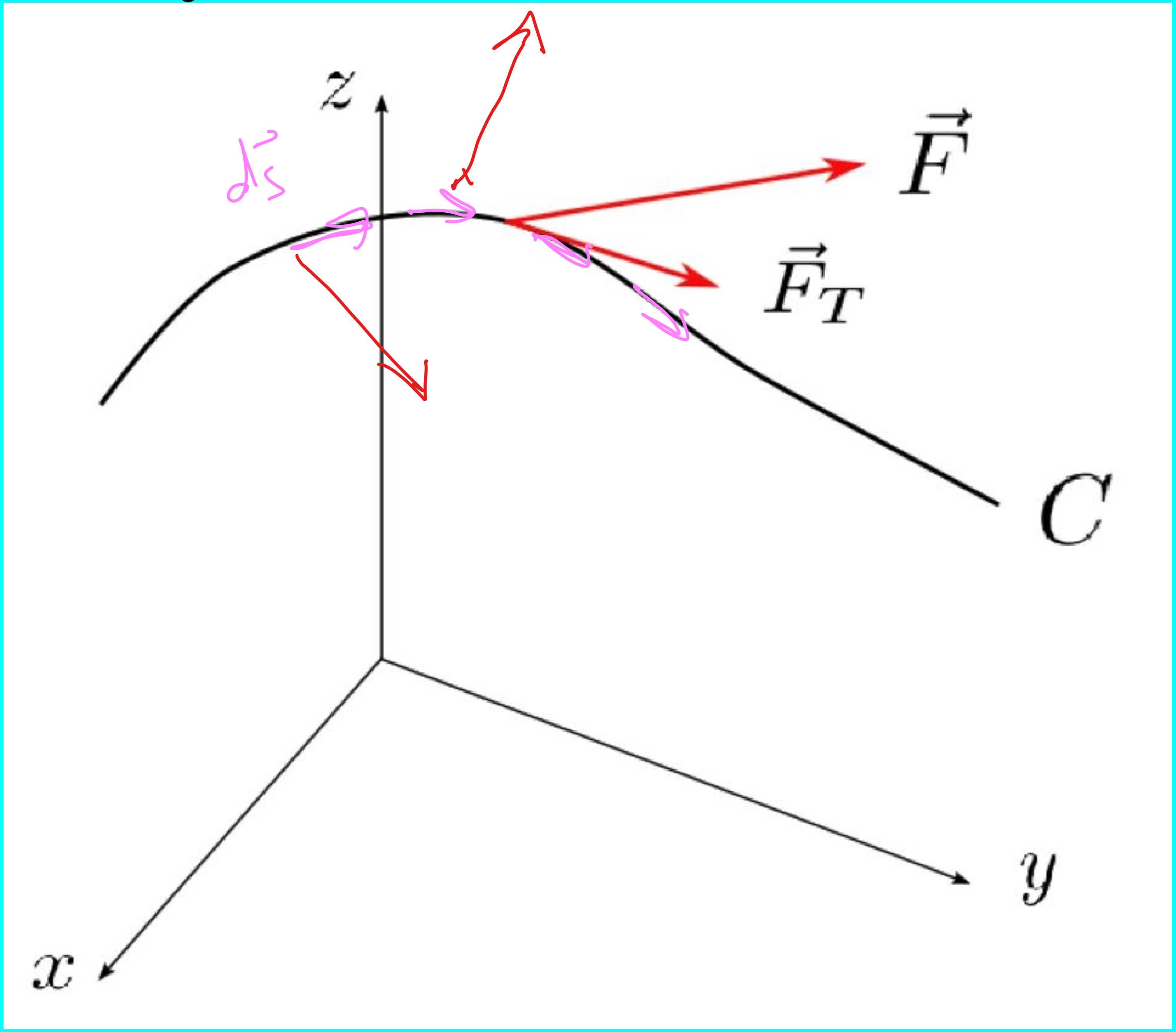
$\int f(x)\,dx$ Area under y-axis



$\int f(x)dx$ Area under arbitrary line

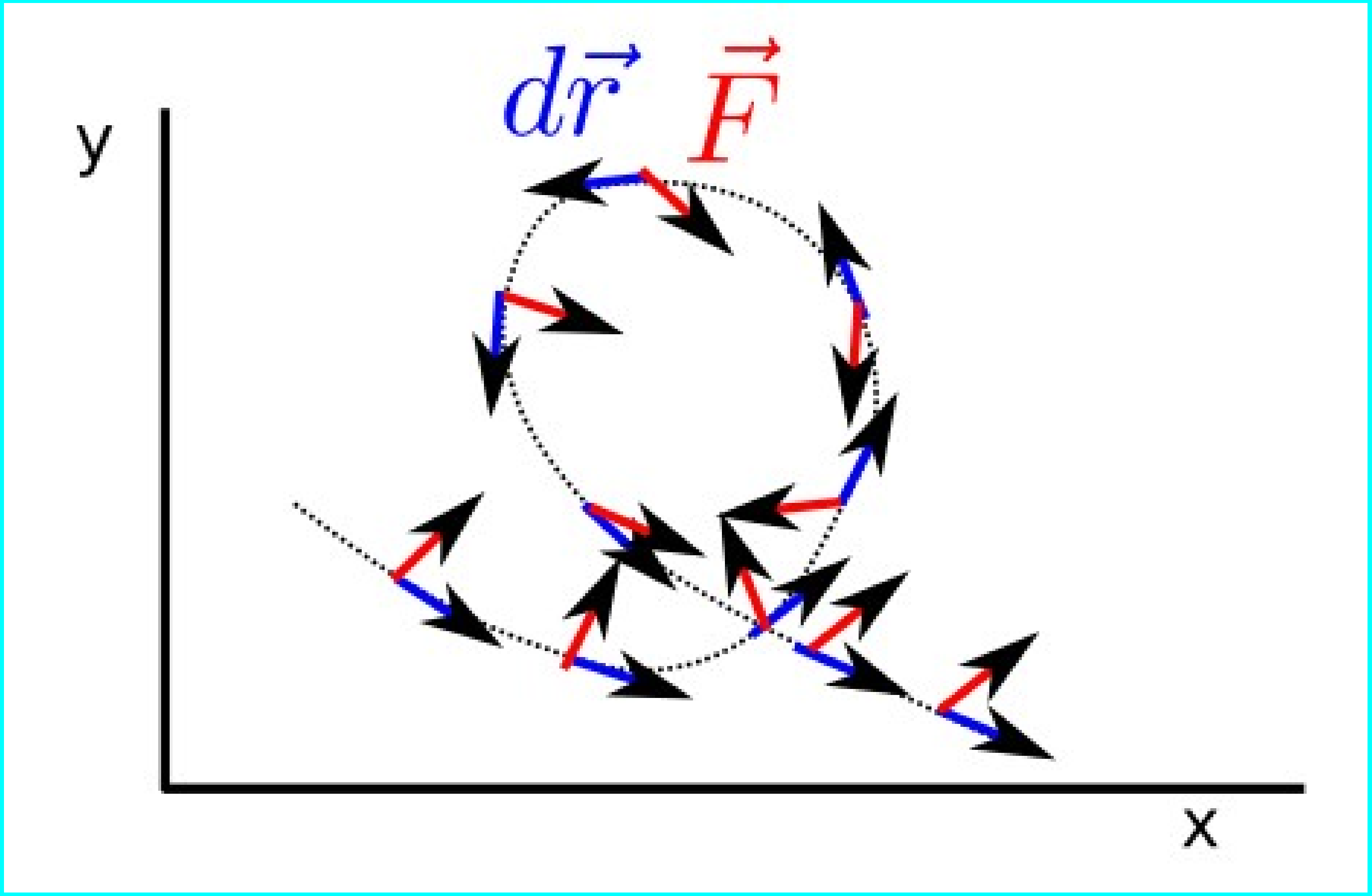


$\int \vec{F} \cdot d\vec{l}$ Vector line integral



http://hartleymath.com/calculus3/line-integrals

$\int \vec{F} \cdot d\vec{r}$ Vector line integral



https://personales.unican.es/garciapa/fisica_general/1_vectores/integral_linea_vector.html

Clicker

- Where have you seen a vector line integral before?

(A) Plan 9 from outer space

(B) Friday the 13 th part 4

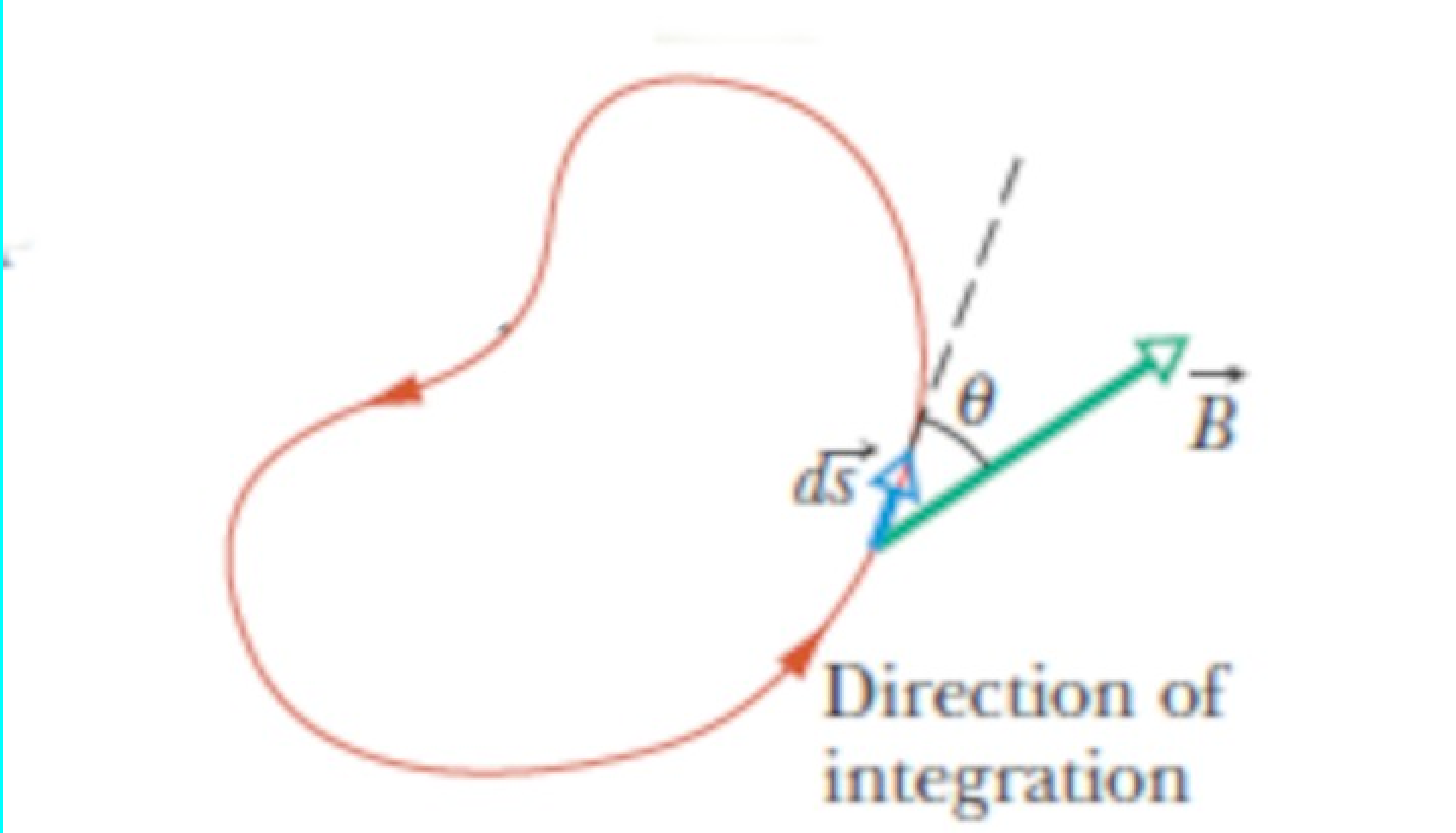
(C) The definition of work

(D) The definition of electric potential

(E) Two of the above

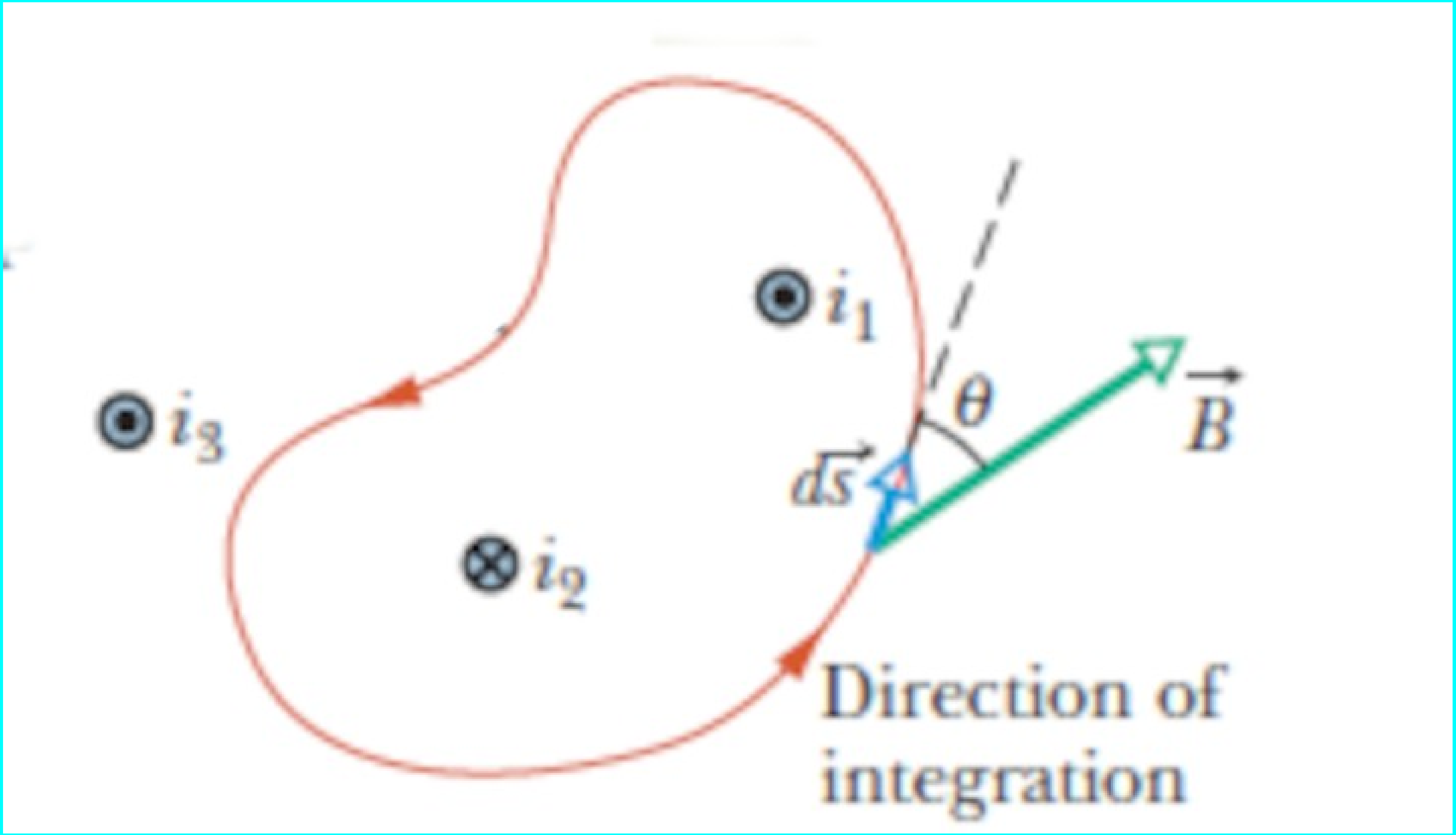
$\int \vec{F} \cdot d\vec{\ell}$

$\oint \vec{F} \cdot d\vec{s}$ Vector closed-loop line integral

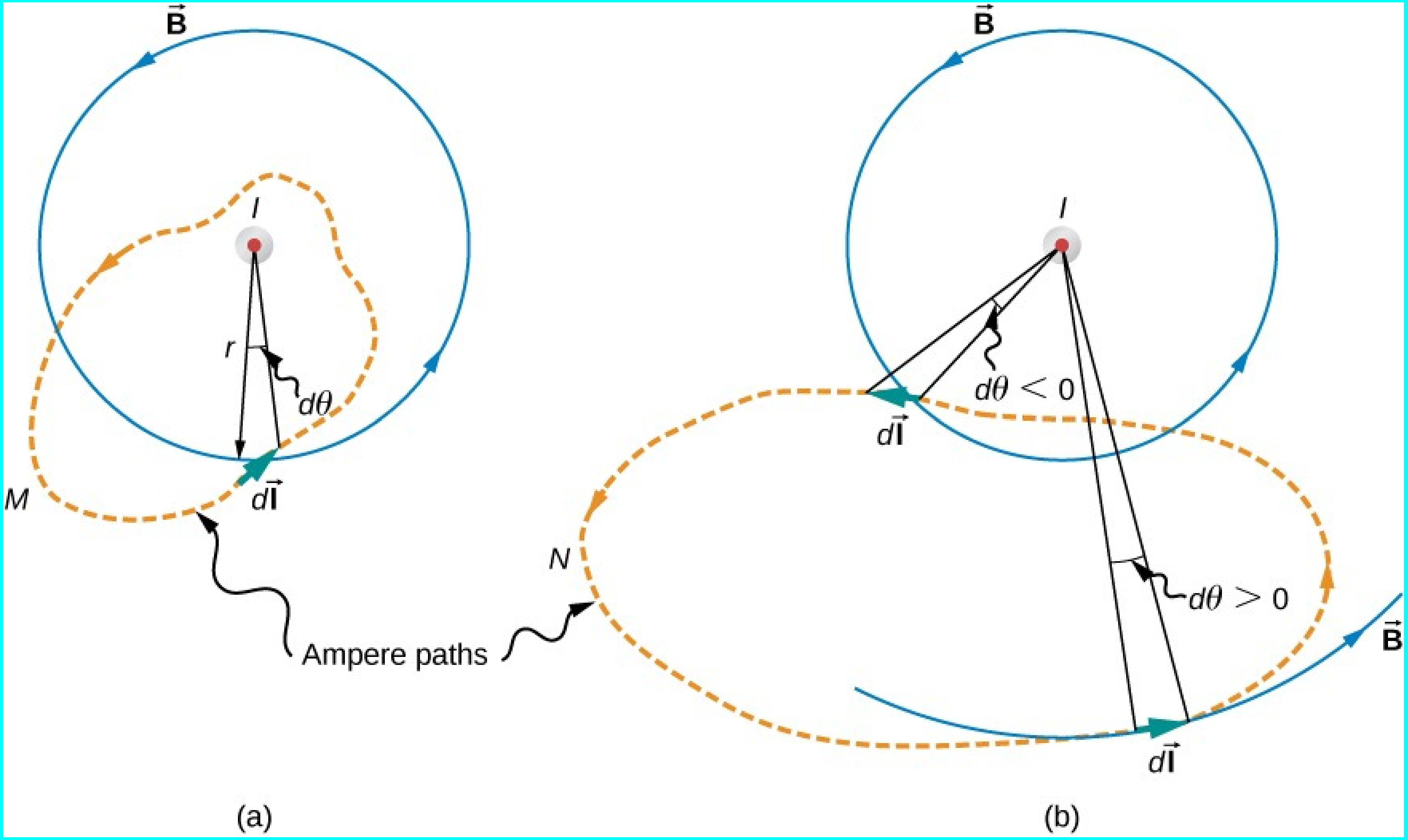


$\oint \vec{F} \cdot d\vec{r}$ Vector closed-loop line integral

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$ Ampere's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad \text{Ampere's Law}$$



Given $i_1 = 10$ Amps, $i_2 = -5$ A, $i_3 = 5$ A

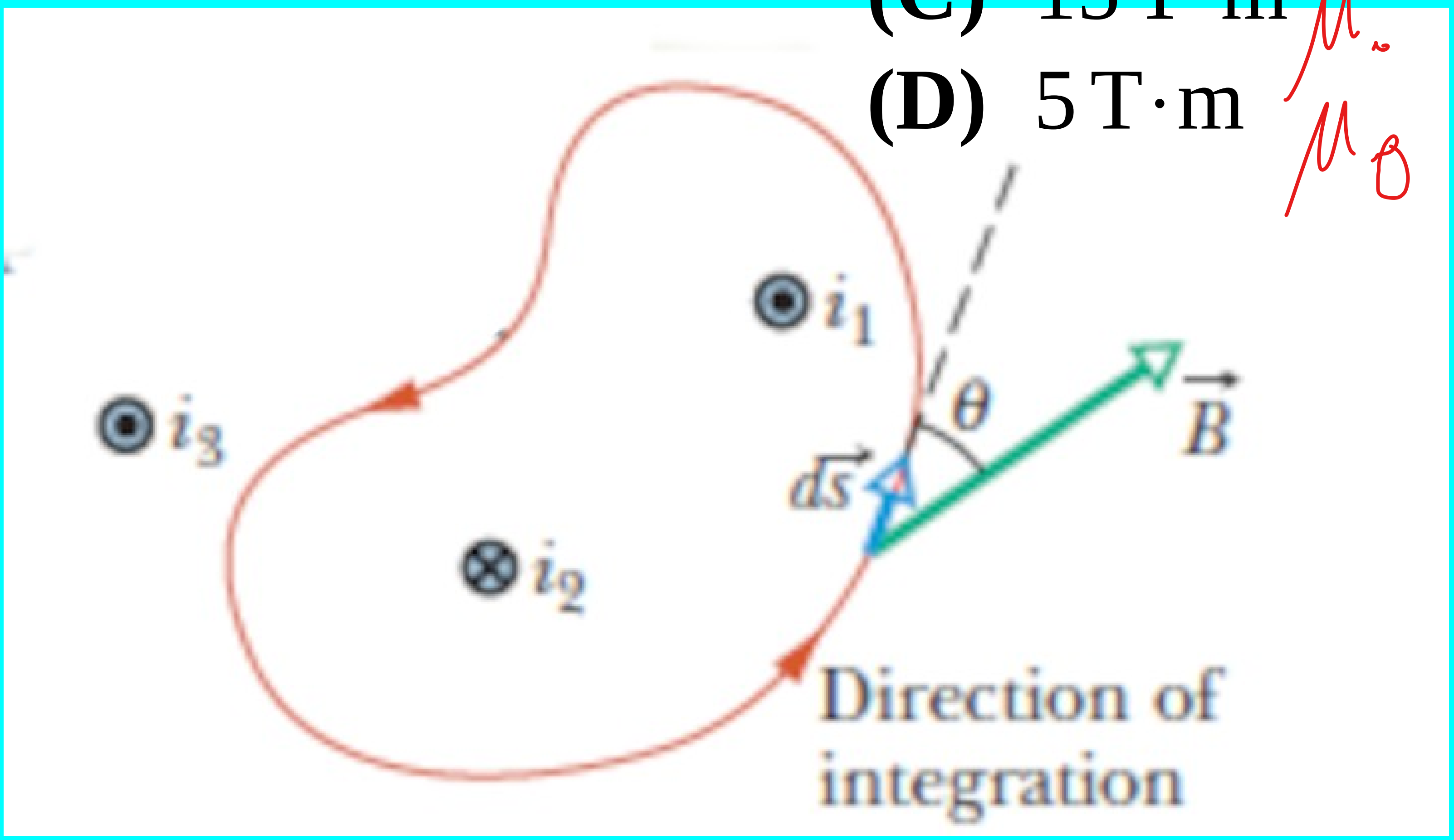
What is $\oint \vec{B} \cdot d\vec{l}$?

(A) Plan 9 from outer space

(B) $10\ \text{T}\cdot\text{m}$

(C) $15\ \text{T}\cdot\text{m}$

(D) $5\ \text{T}\cdot\text{m}$



Equations of Magnetism

$$\vec{F} = Q\,\vec{v} \times \vec{B}$$ Force on charge Q

$$\vec{F} = I\,\vec{L} \times \vec{B}$$ Force on current I

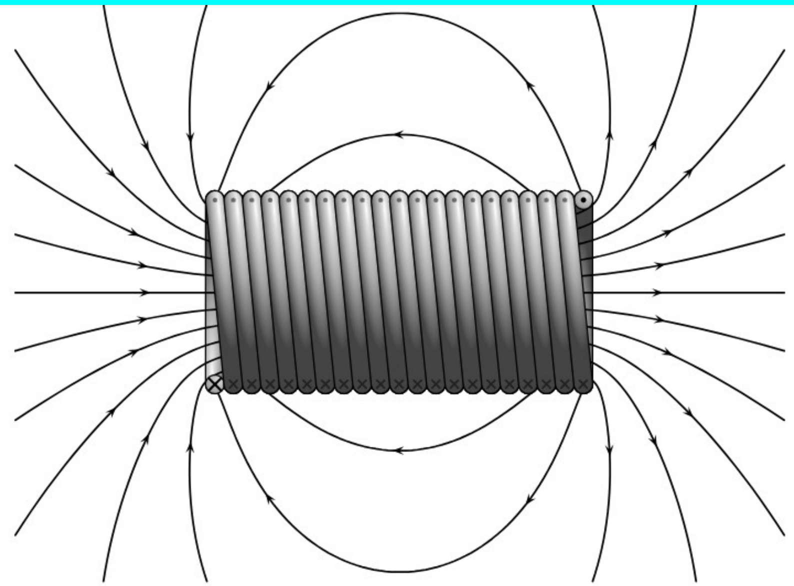
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$$ Field of Infinite wire

$$\vec{B} = \frac{\mu_0 I}{2a}\,\hat{z}$$ Field in center of wire loop

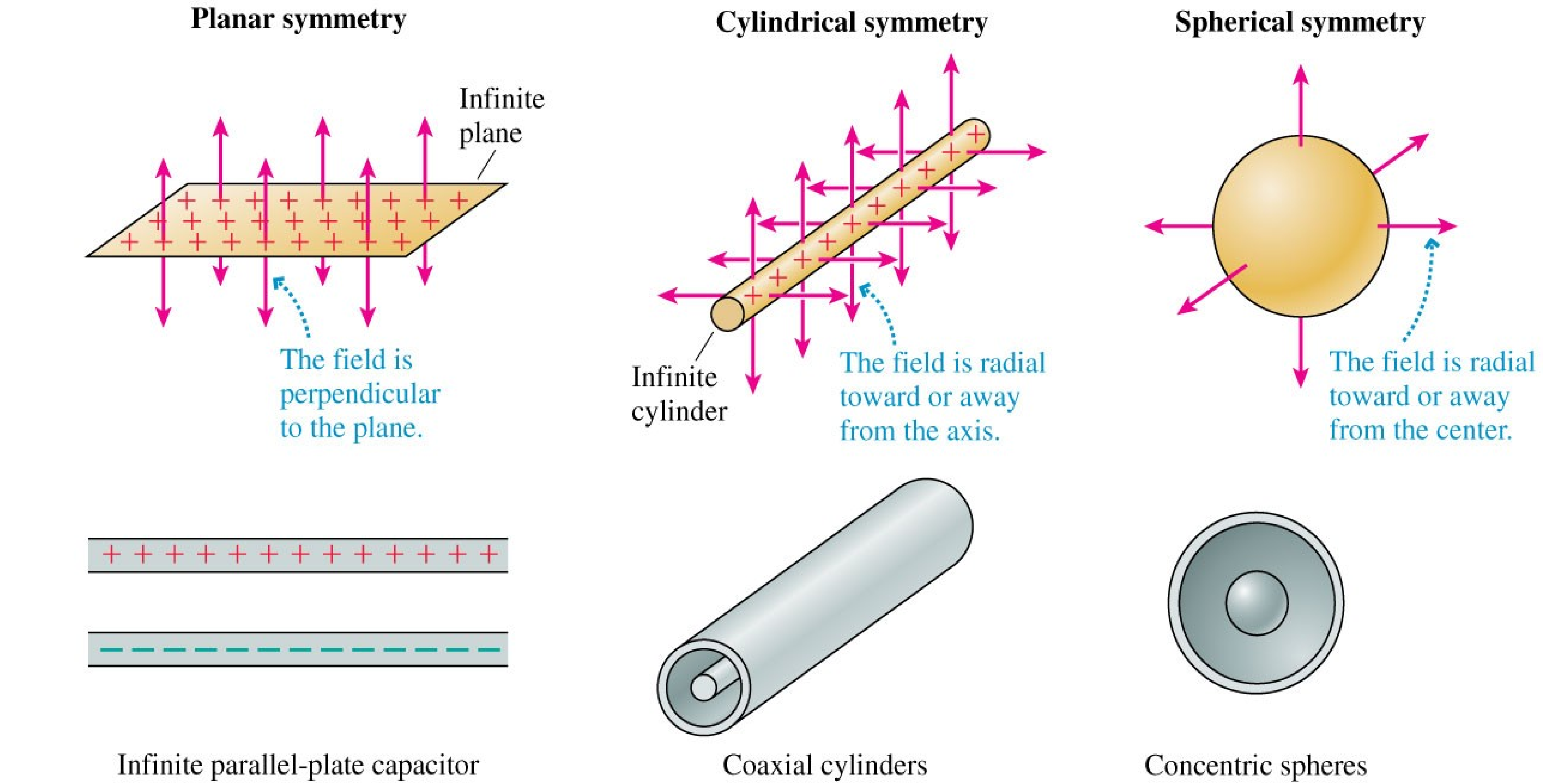
$$\vec{B} = \mu_0 n I \hat{z}$$ Field of an infinite coil (solenoid)

Derive field around a wire

Derive field of a solenoid



$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k} \qquad \vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r} \qquad \vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}$$



Maxwell's Equations

$$\int \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$ To calculate E for symettrical charges.

$$\int \vec{B}\cdot d\vec{A} = 0$$ Cannot have North magnet w/o a South pole.

$$\int \vec{B}\cdot d\vec{l} = \mu_0 I$$ To calculate B for symettrical currents.

$$\int \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$ Magnetic induction! Generators! Light!

Faraday’s Law

A changing magnetic flux makes a voltage.
Moving a magnet in a loop of wire makes a voltage.
Spinning a wire in a magnetic field makes a Voltage
Turning on an electromagnet near a coil of wire makes a voltage.

Faraday's Law Lab

You will yank a loop of wire out of a magnetic field. This will allow you to measure the magnetic field.

You will run an AC current through a coil. This will cause a voltage to appear in a different coil.

Electric fields are calculated with Coulomb's law or Gauss's Law

Magnetic Fields are calculated with the Biot-Savart Law or Ampere's Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2} \quad \text{Biot Savart}$$

Maxwell's Equations

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$ To calculate E for symettrical charges.

$$\oint \vec{B}\cdot d\vec{A} = 0$$ Cannot have North magnet w/o a South pole.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$ To calculate B for symettrical currents.

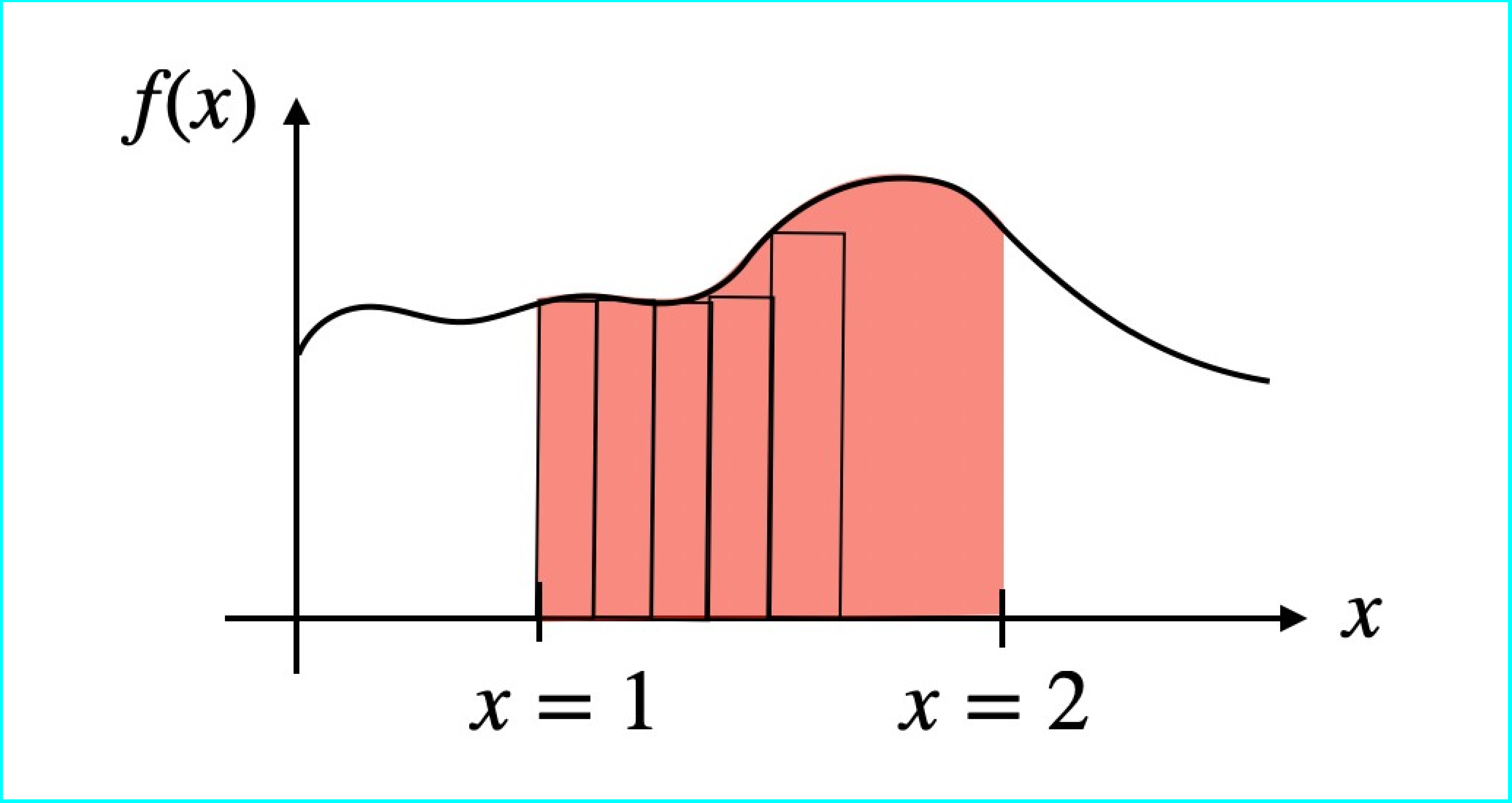
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ Magnetic induction! Generators! Light!

$$\Phi_B = \int B\cdot d\vec{A}$$

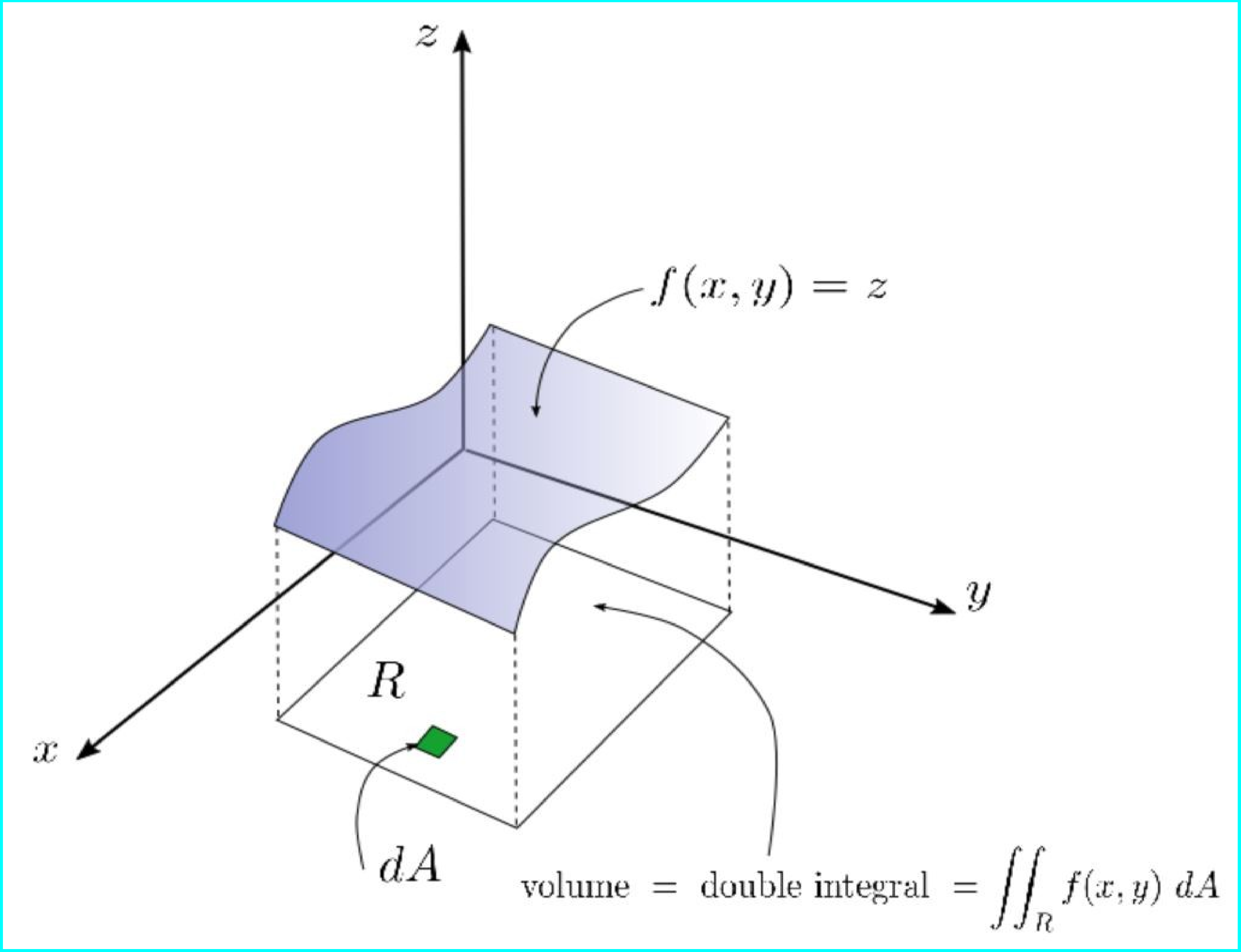
Recap Lecture 23

- Circular motion of charges
- Torque in a motor
- Amperes law
- Field of a wire
- Field of a solenoid
-

$\int f(x)dx$ Area under curve

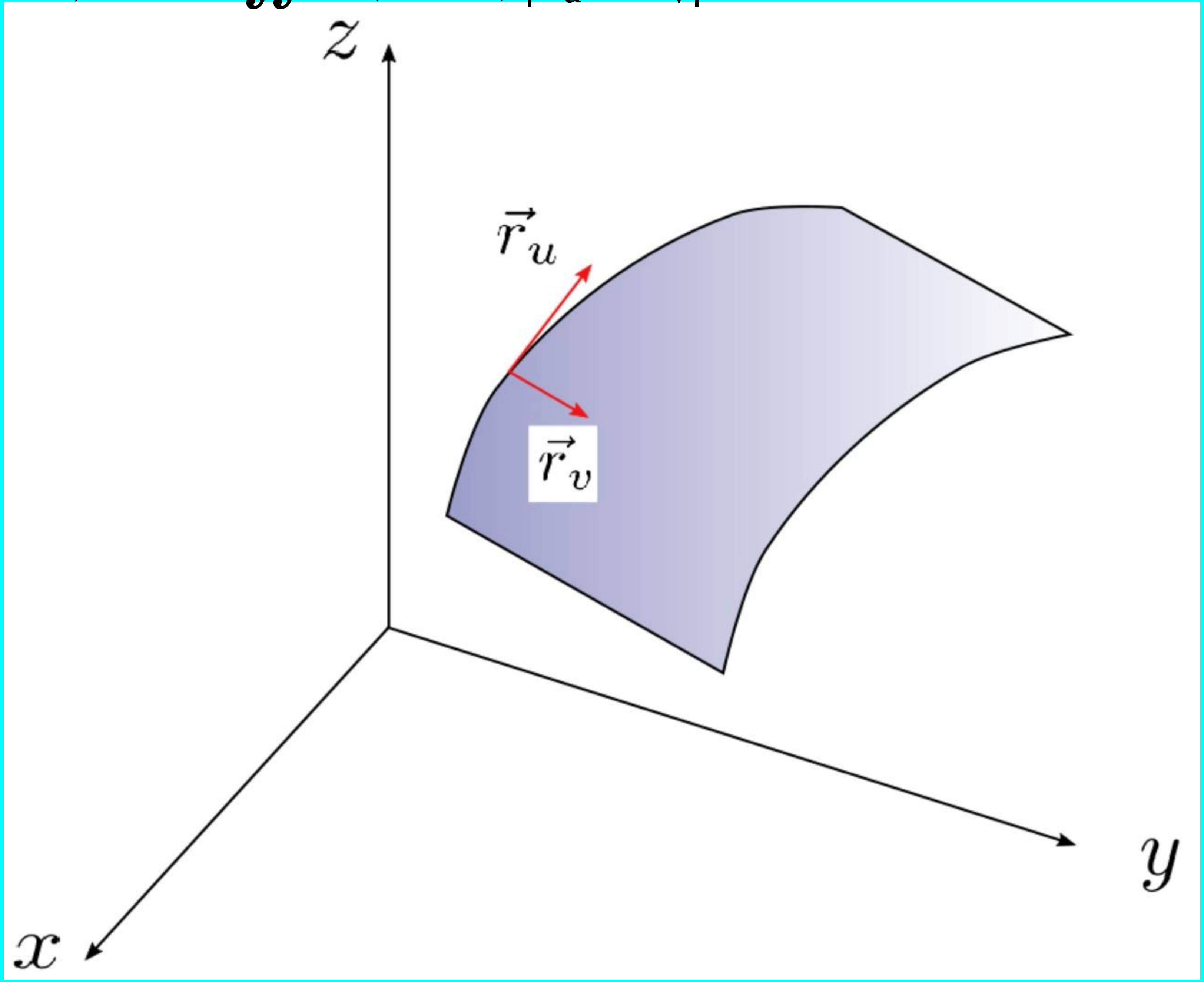


$\int f(x,y)\,dA = \iint f(x,y)\,dx\,dy$ Volume under an area



http://hartleymath.com/calculus3/double-integrals-rectangular-regions

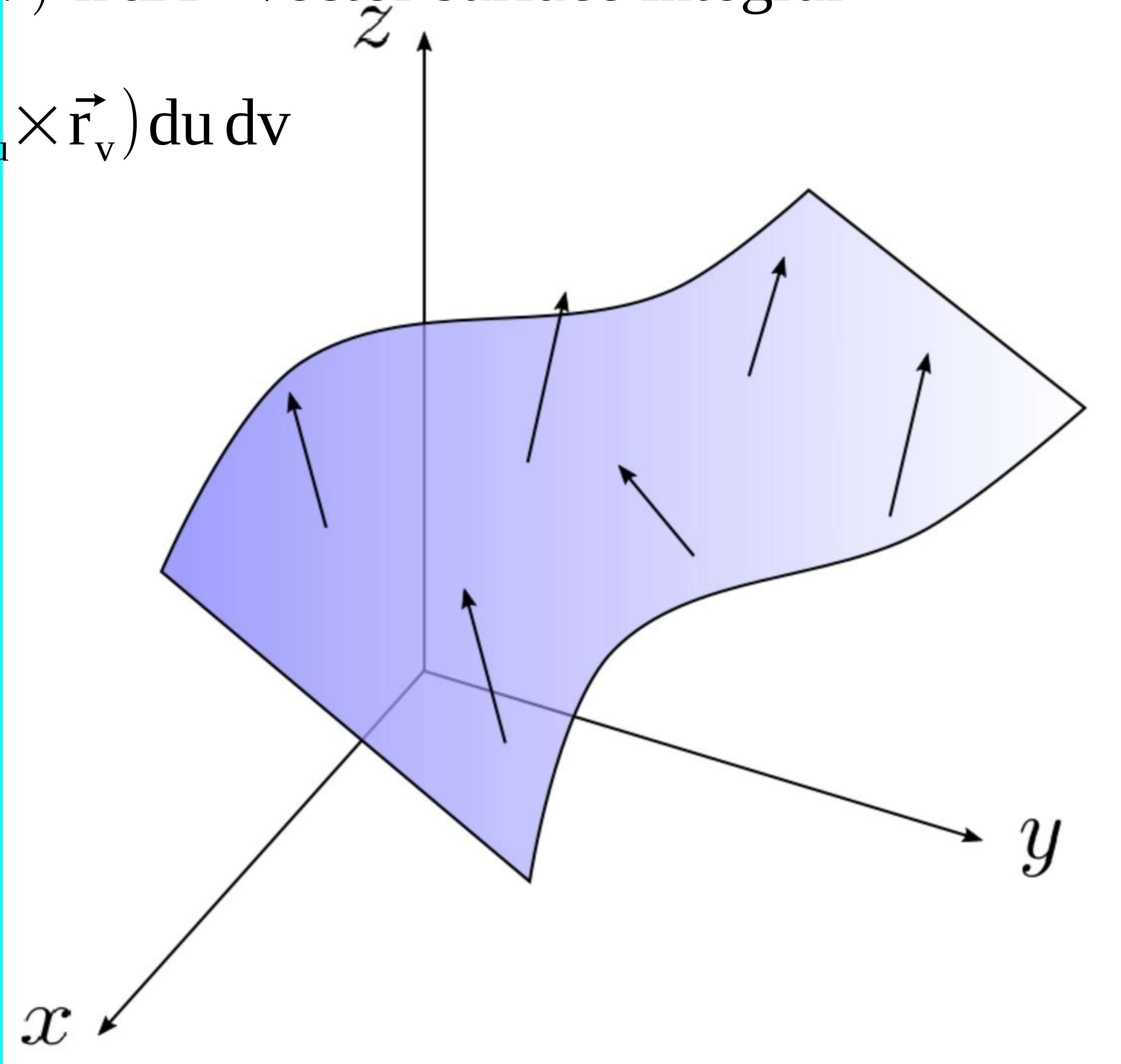
$$\int f(u,v)\,dA = \iint f(u,v)\left|\vec{r}_u \times \vec{r}_v\right| du\,dv \quad \text{Surface integral}$$



http://hartleymath.com/calculus3/surface-integrals

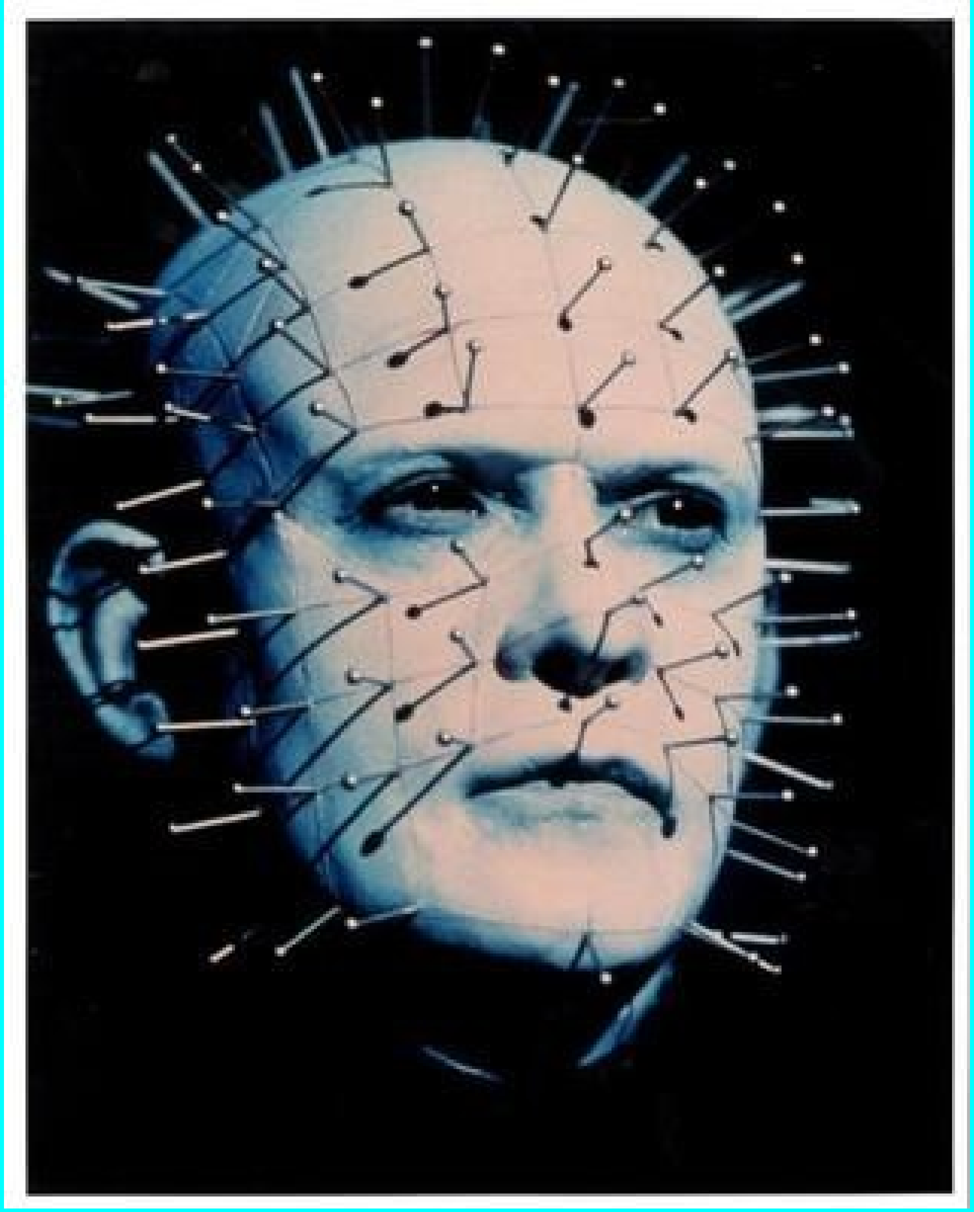
$\int \vec{F}(u,v)\cdot\hat{n}\,dA$ Vector surface integral

$$\iint \vec{F}\cdot(\vec{r}_u \times \vec{r}_v)\,du\,dv$$



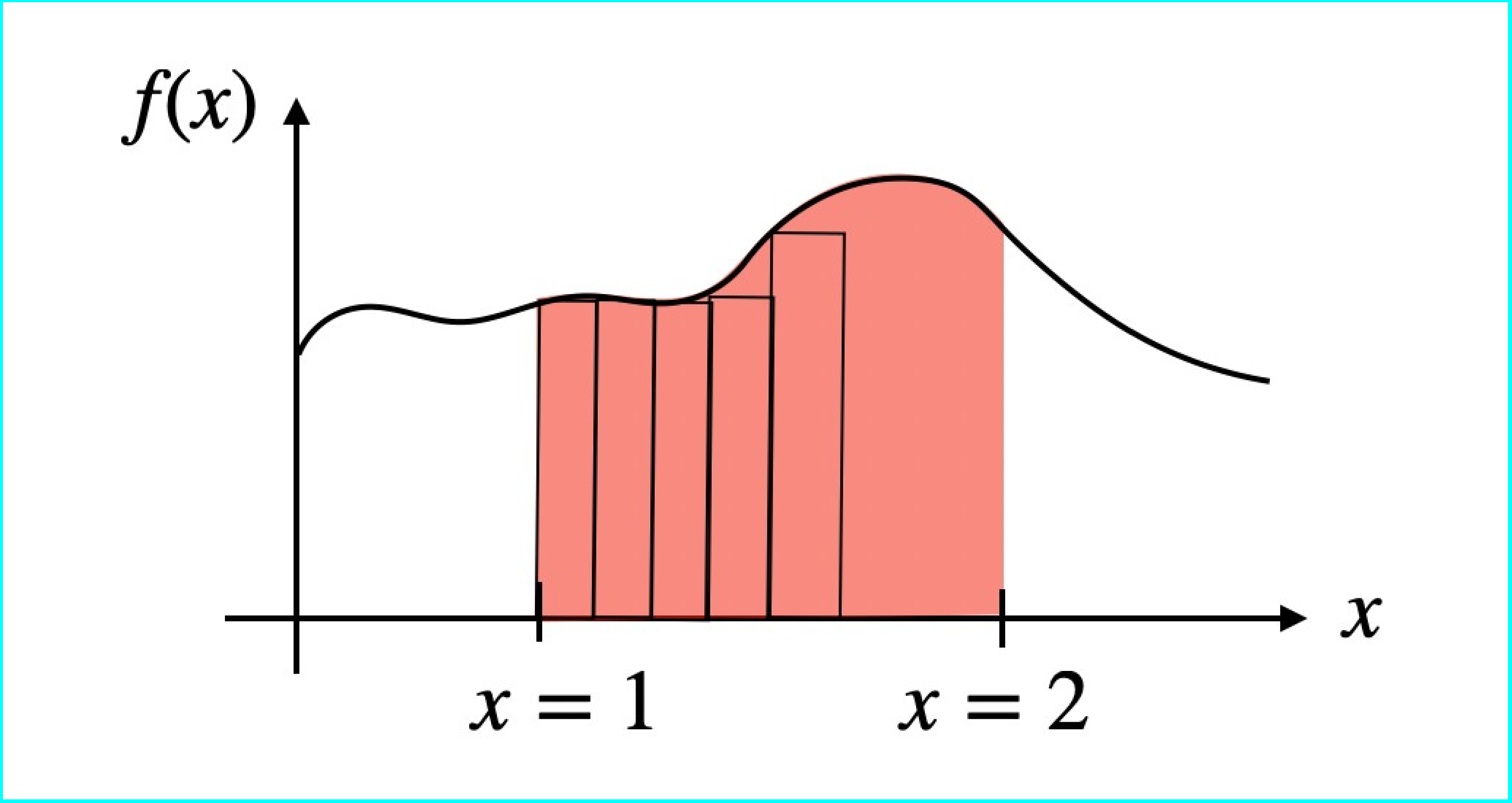
$\oint \vec{F}(u,v)\cdot\hat{n}\,dA$ Vector closed surface integral

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

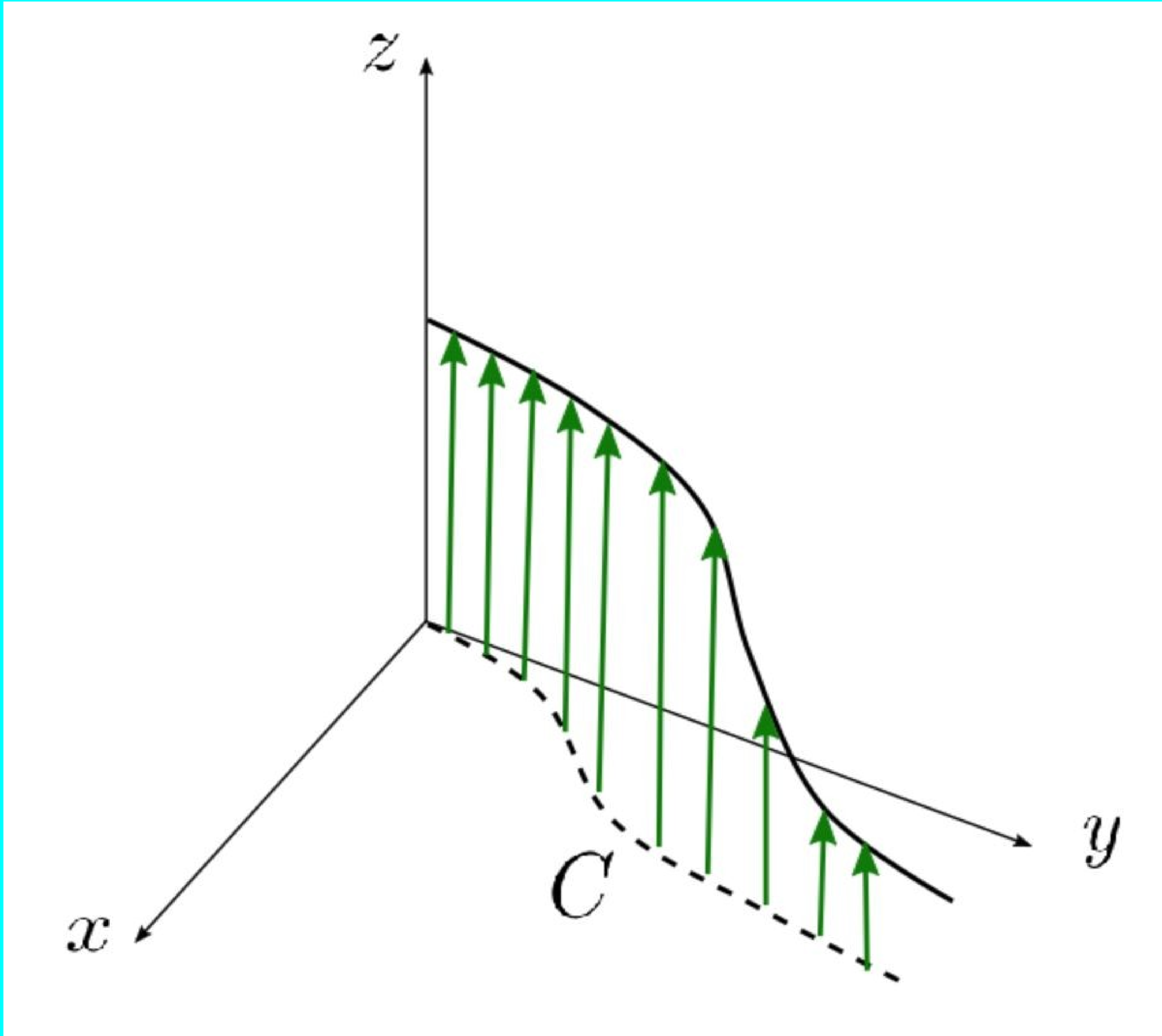


https://math.etsu.edu/multicalc/prealpha/Chap5/Chap5-5/

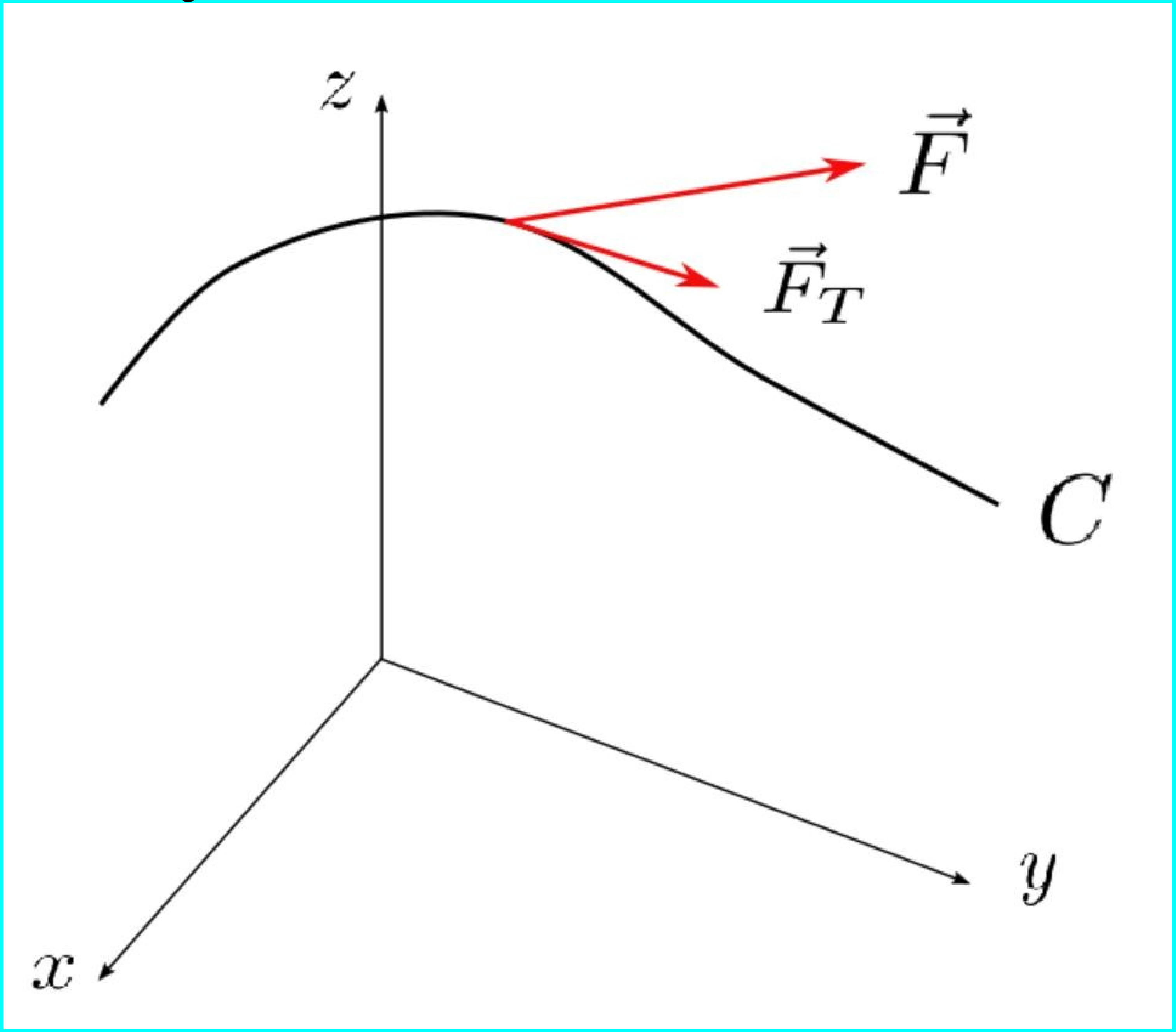
$\int f(x)dx$ Area under y-axis



$\int f(x)\,dx$ Area under arbitrary line



$\int \vec{F} \cdot d\vec{l}$ Vector line integral



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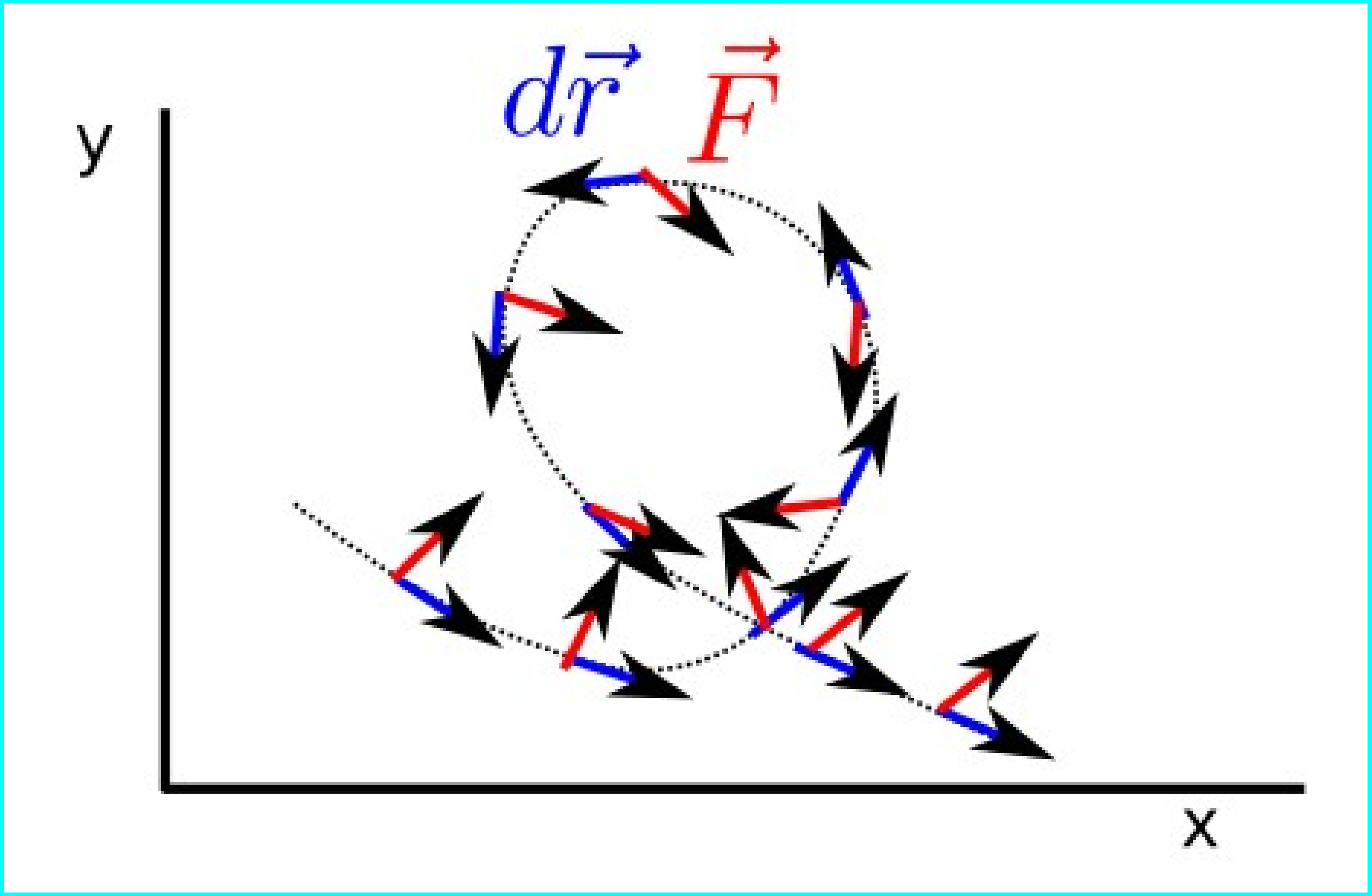
$\int \vec{F} \cdot d\vec{r}$ Vector line integral



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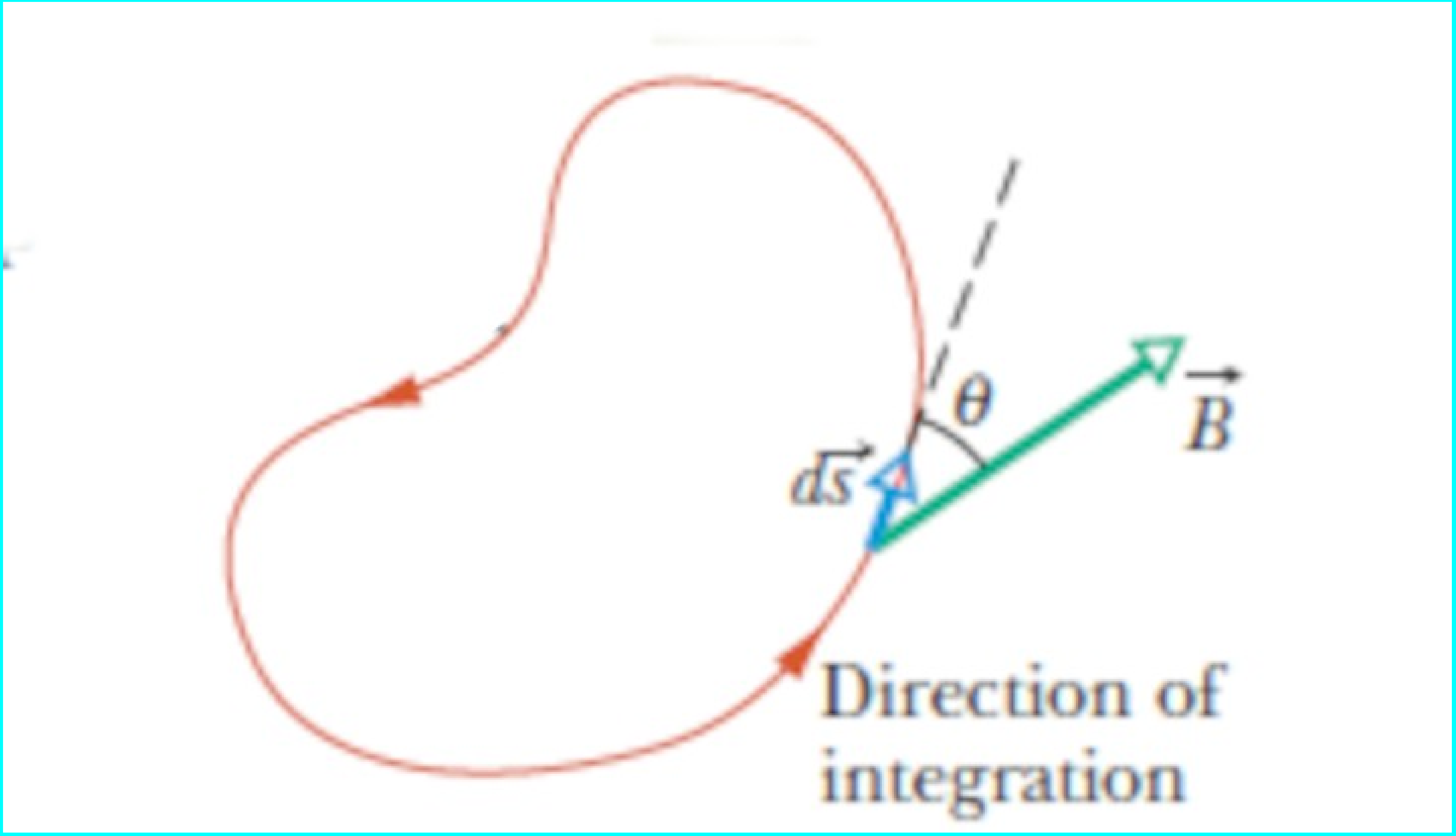
$\oint \vec{B} \cdot d\vec{s}$ Vector closed-loop line integral



Torque

- A wrench is 30 cm long and a person exerts 10 N at right angles to it. The torque is?

(A) $30\,N\cdot m$

(B) $10\,N\cdot m$

(C) $300\,N\cdot m$

(D) $3\,N\cdot m$

(E) $0\,N\cdot m$