Lecture 22: 04/09/2024

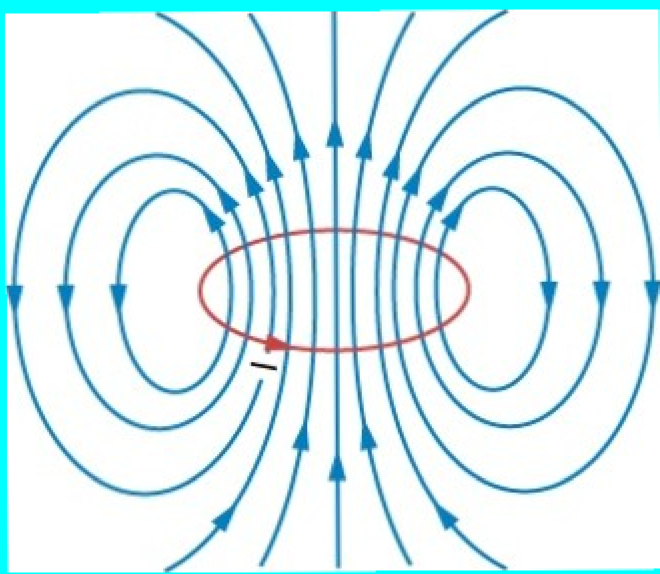
- Announcements
  Grade Change day is April 10 (tomorrow)
  Exams back today
  Updated grades.
  It is unlikely that your final grade will increase by more than a letter.

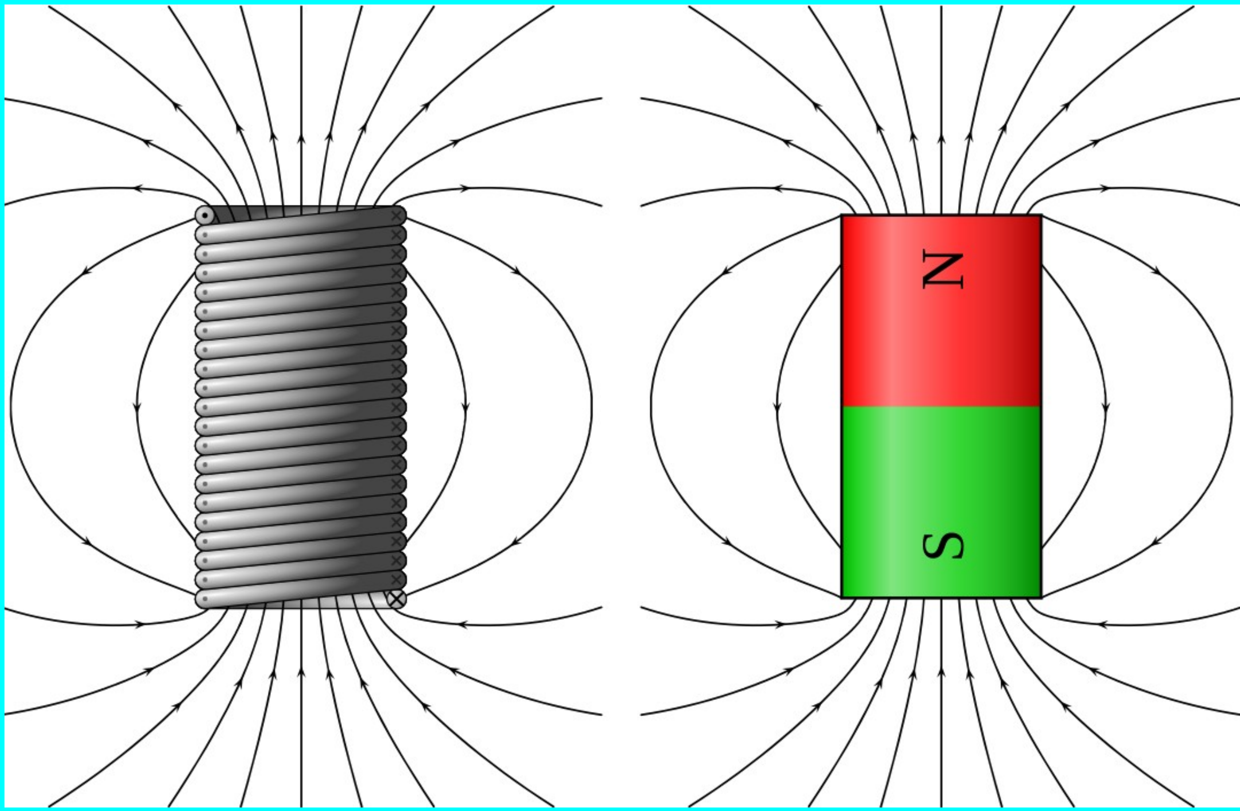
- Last Time
  Magnetism
  Motors

- Today
  Motors
  Ampere's Law
  Faraday's Law

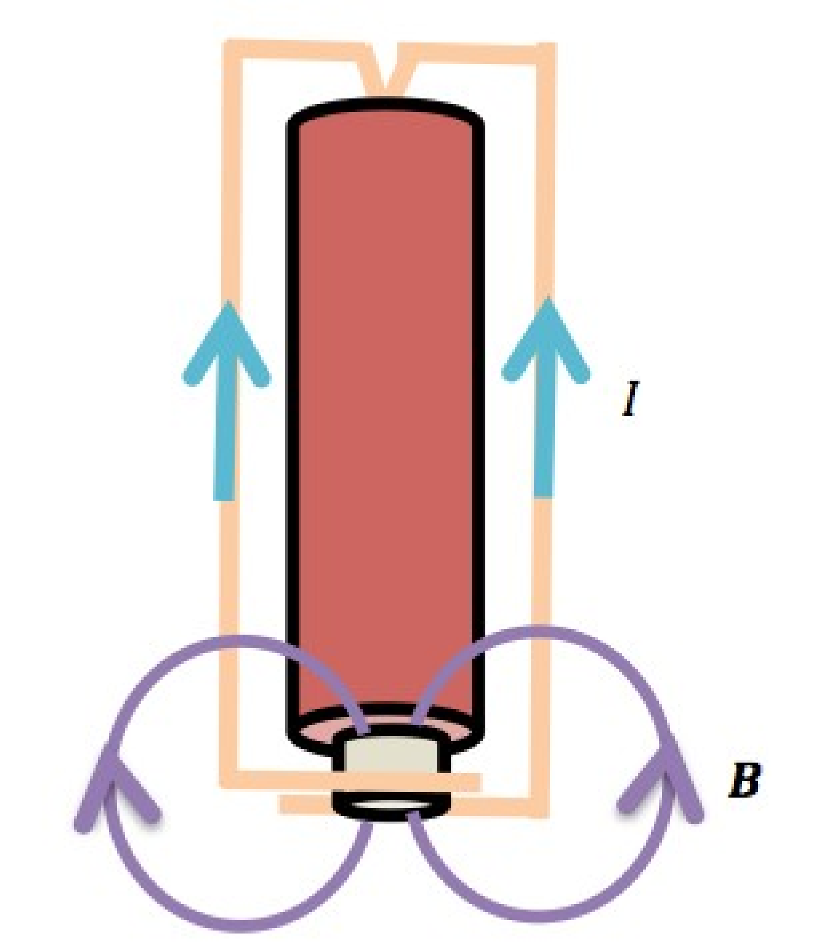
Electromagnets

- A loop of wire is a basic electromagnet
- Many loops of wire (a coil or "solenoid") is a better electromagnet.

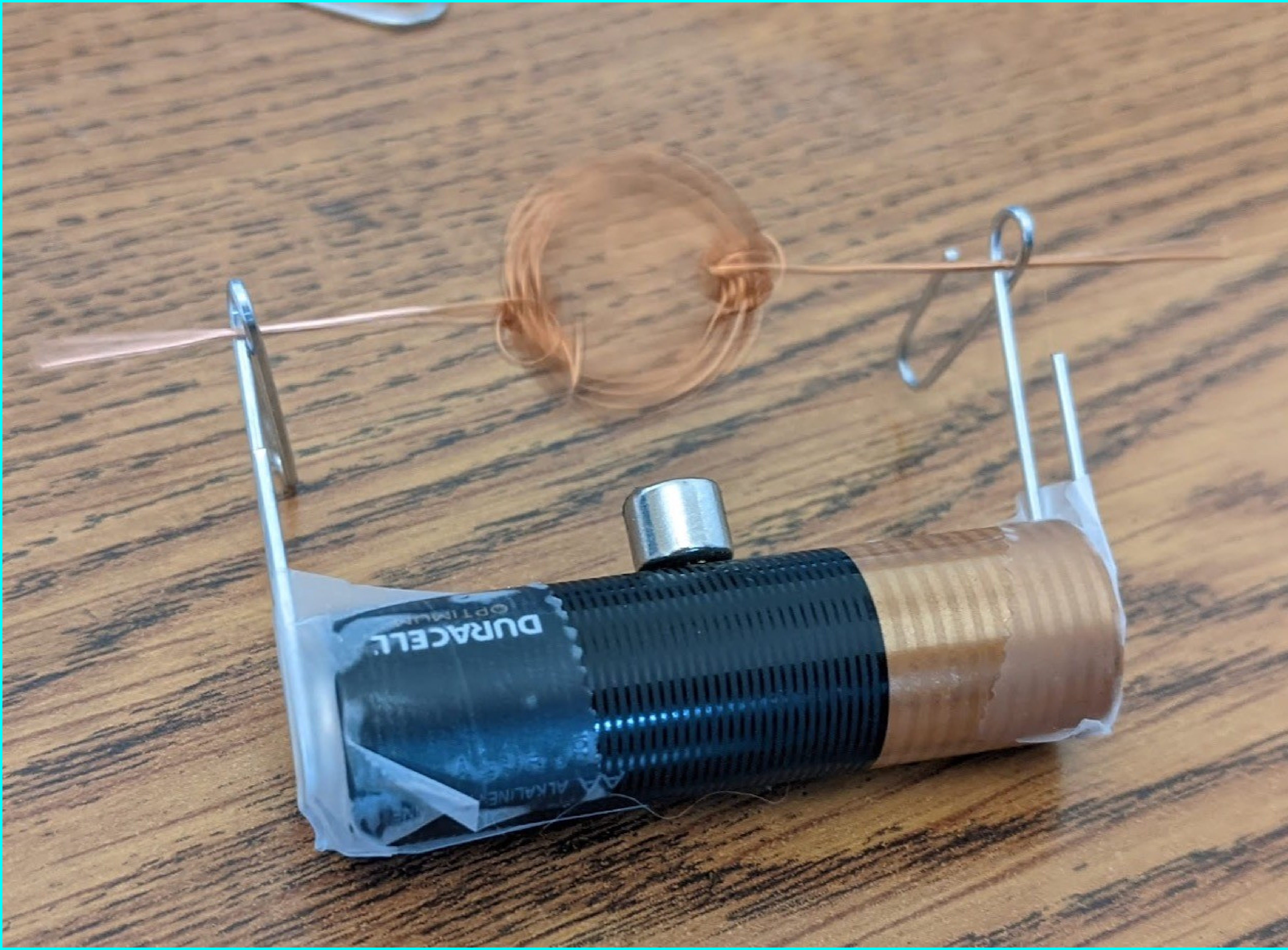




“Homopolar” Motor

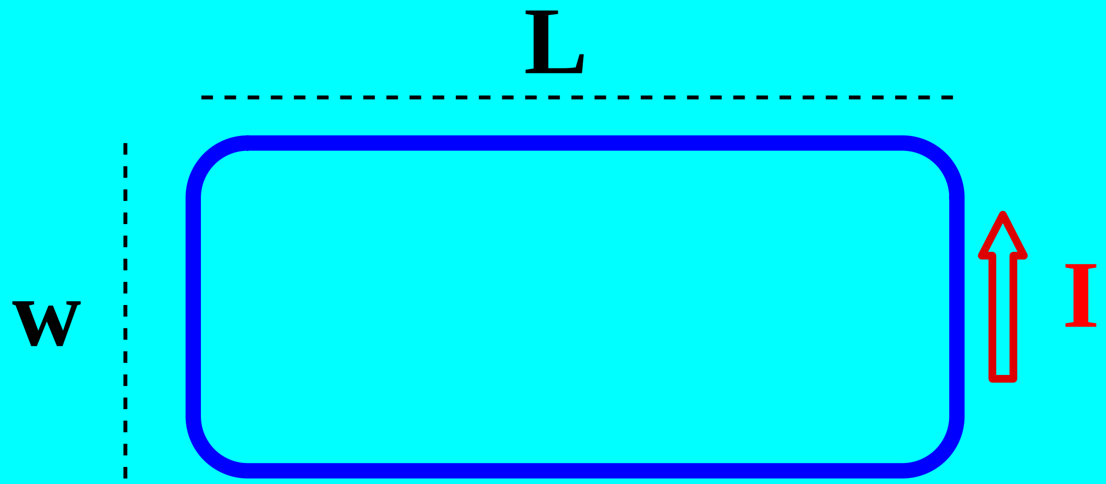


https://www.youtube.com/watch?v=bH7DFPIayNg


DURACELL

Motors

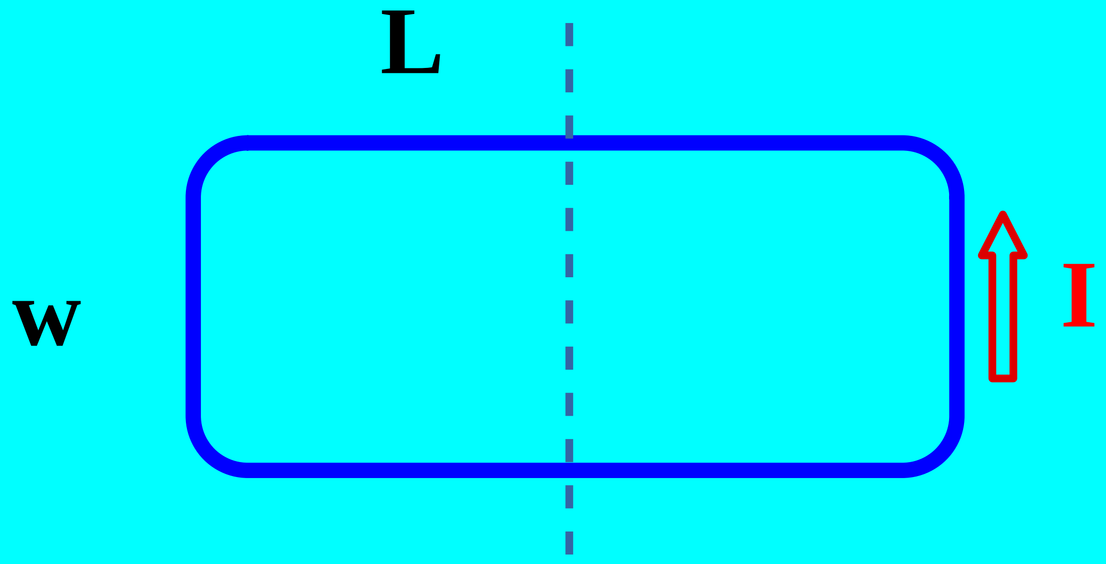
Are just clever loops of wire in magnetic fields



$\vec{F} = I\,\vec{L} \times \vec{B}$ Force on current I

Wire loop

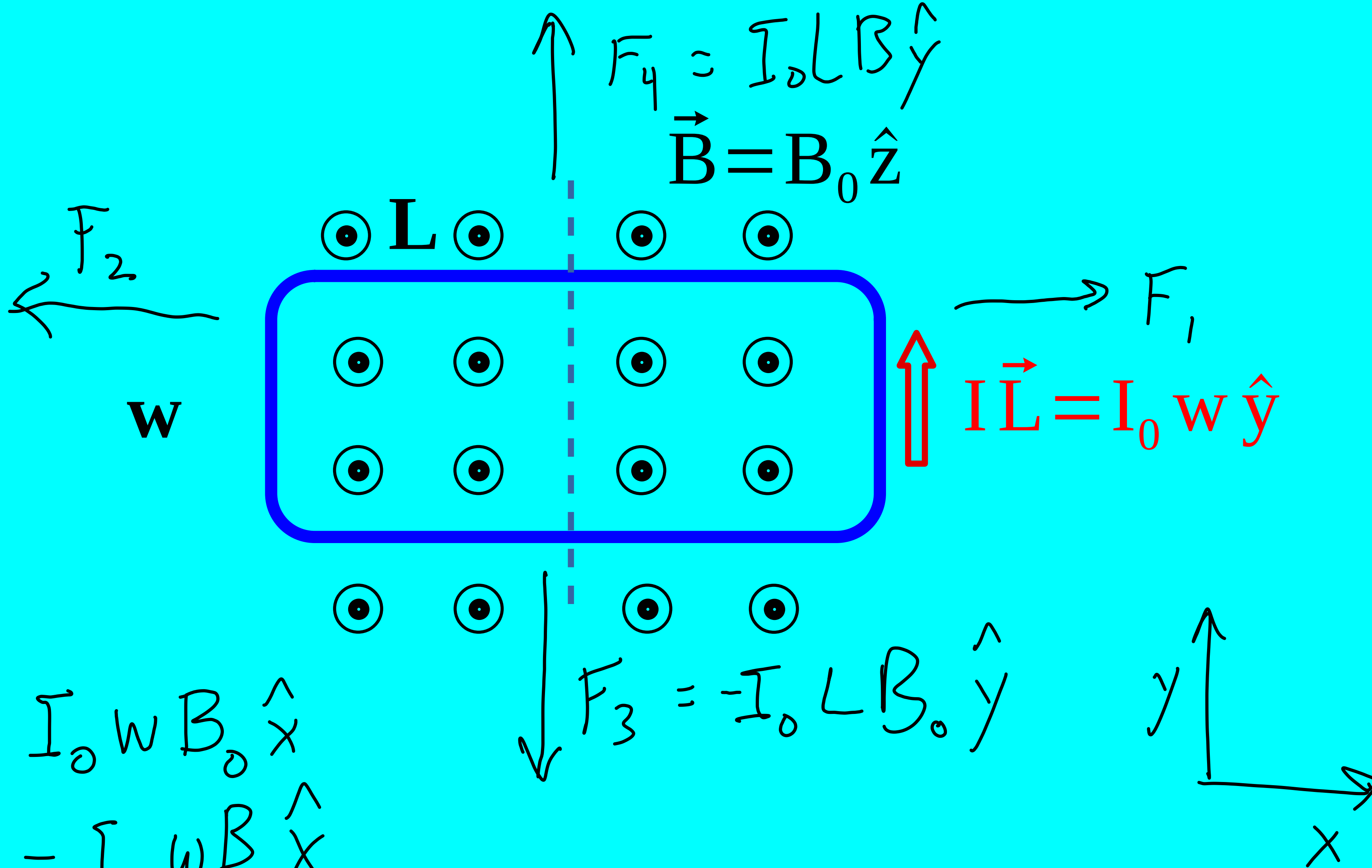
Is free to rotate about vertical axis (dotted line)



Can we calculate net force and torque?

$\vec{F} = I\,\vec{L} \times \vec{B}$ Force on current I

Try B-field out of page

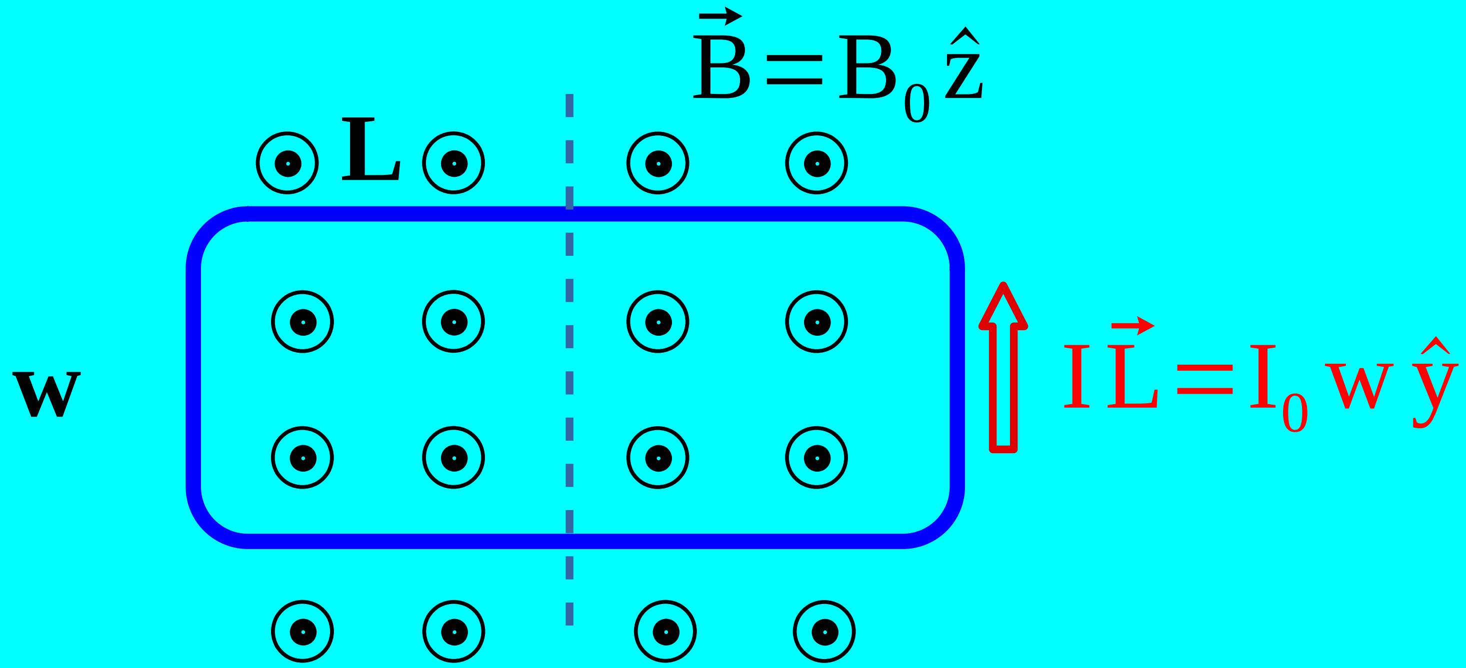


$F_1 = I_0 w B_0 \hat{x}$

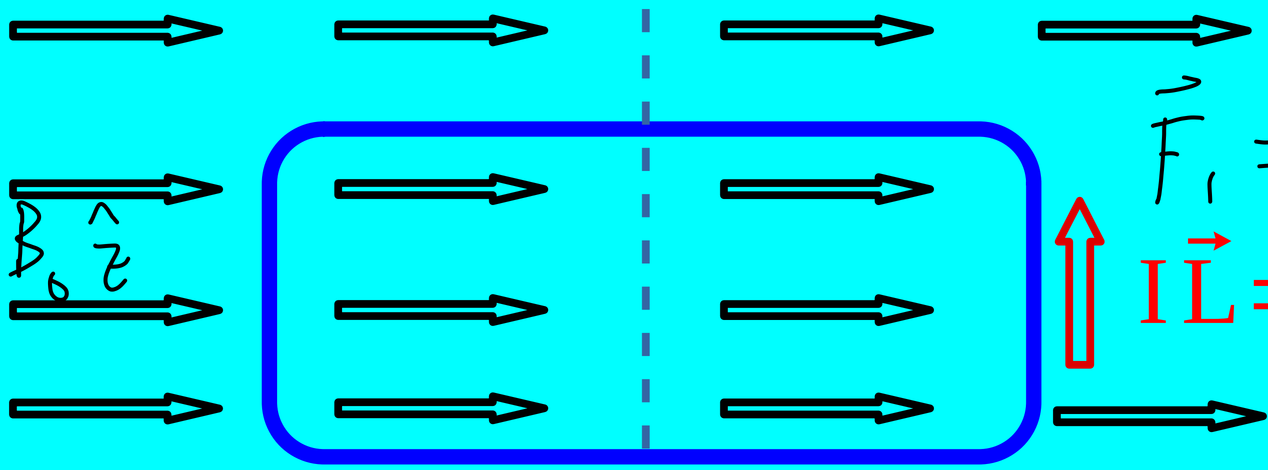
$F_2 = -I_0 w B_0 \hat{x}$

$\vec{F} = I\vec{L} \times \vec{B}$ Force on current I

Try B-field out of page



Try B-field to the right



$\vec{F}_1 = I_0 w B_0 (-\hat{z})$

$\vec{F}_2 = I_0 w B_0 \hat{z}$

$I\vec{L} = I_0 w \hat{y}$

$\vec{B} = B_0 \hat{x}$

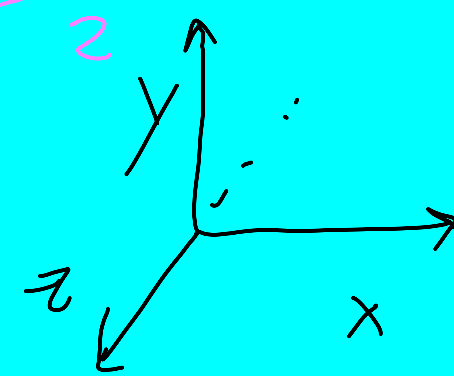
$\vec{r}_1 = r_1 \hat{x} = \frac{L}{2} \hat{x}$

$\vec{r}_2 = -r_2 \hat{x}$

$\vec{r}_2 = -\frac{L}{2} \hat{x}$

$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 \quad \vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2$

$\vec{\tau} = \vec{r} \times \vec{F}$



$\vec{F} = I\vec{L} \times \vec{B}$ Force on current I

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \frac{L}{2}\hat{x} \times I_0 w B_0 \hat{z}$$

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = -\frac{L}{2}\hat{x} \times I_0 w B_0 \hat{z}$$

$$-L w I_0 B_0 \hat{x} \times \hat{z}$$

$$\vec{\tau} = L w I_0 \hat{y} B_0$$

$$\tau = \underline{L w I_0} B_0$$

$$\vec{m} = I_0 \vec{A}$$

$$\vec{m} = I_0 A = I w L$$

$$B = B_0$$

$$\tau = I w L B_0$$

$$\tau = \vec{m} \times \vec{B}$$

$$\vec{m} = N I A$$

$\uparrow$

Equations for motors

$\vec{F} = I\,\vec{L} \times \vec{B}$ Force on current I

$\vec{\tau} = \vec{r} \times \vec{F}$ Torque definition

$\vec{m} = N\,I\,\vec{A}$ Def. of Magnetic Moment

$\vec{\tau} = \vec{m} \times \vec{B}$ Torque on a Magnetic Moment

Torque

- A wrench is 30 cm long and a person exerts 10 N at right angles to it. The torque is?

(A) 30 N·m

(B) 10 N·m

(C) 300 N·m

(D) 3 N·m

(E) 0 N·m

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = rF\sin\theta$

"Give me a moment"

"Moment" is an old physics/engineering word that has NOTHING TO DO with momentum. It is a general way of referring to quantities multiplied by lengths or area.

Thus "Moment of Inertia"
Or "Moment arm"
Or "Magnetic Moment"

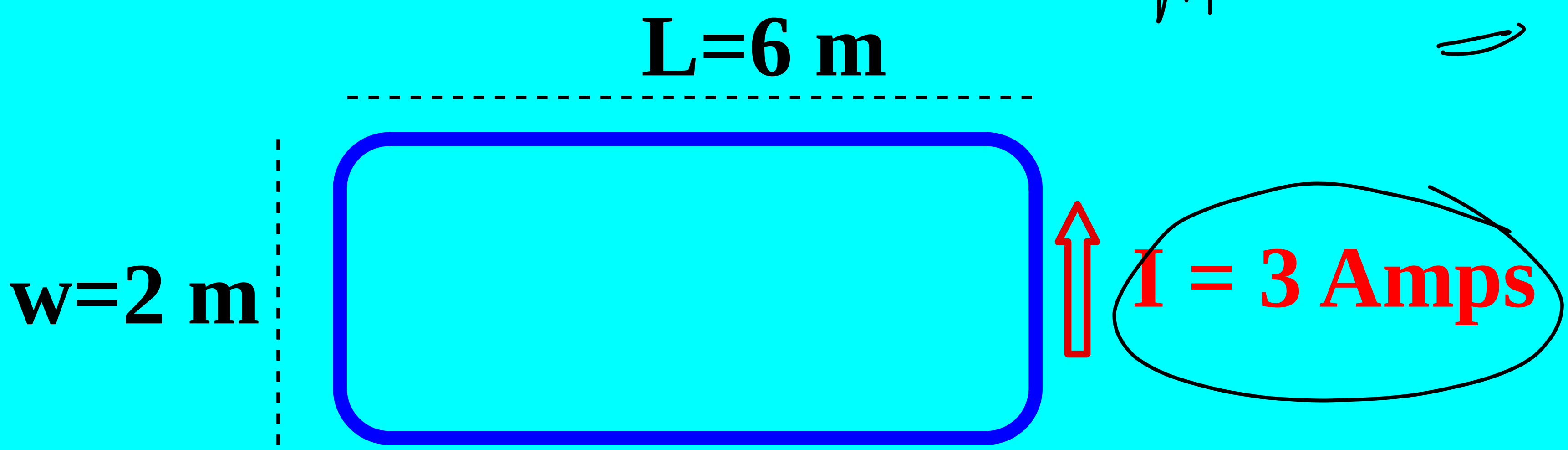
~~$I = MR^2$~~

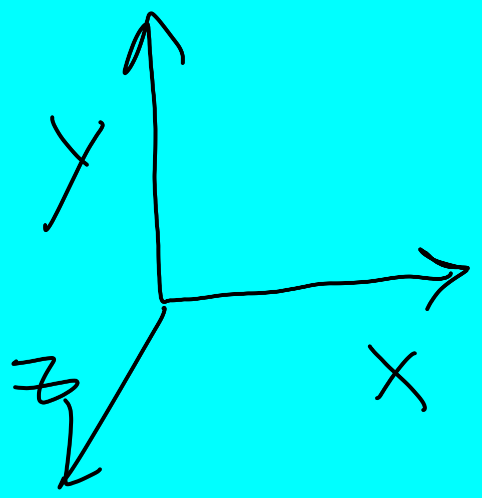
$$\vec{m} = N I \vec{A}$$

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Using magnetic moment makes it much easier to calculate motor torques.

What is the magnetic moment of this loop?



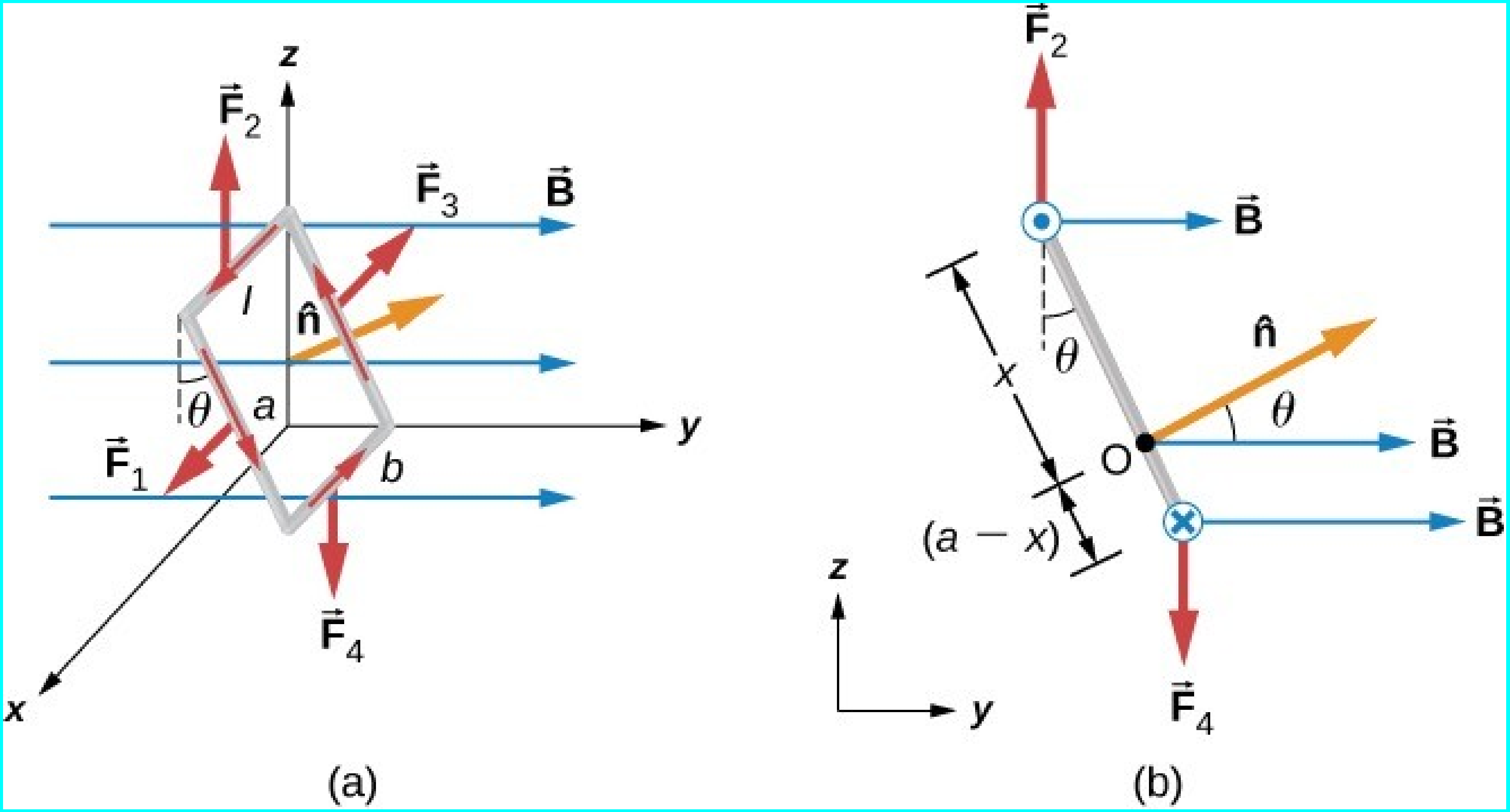


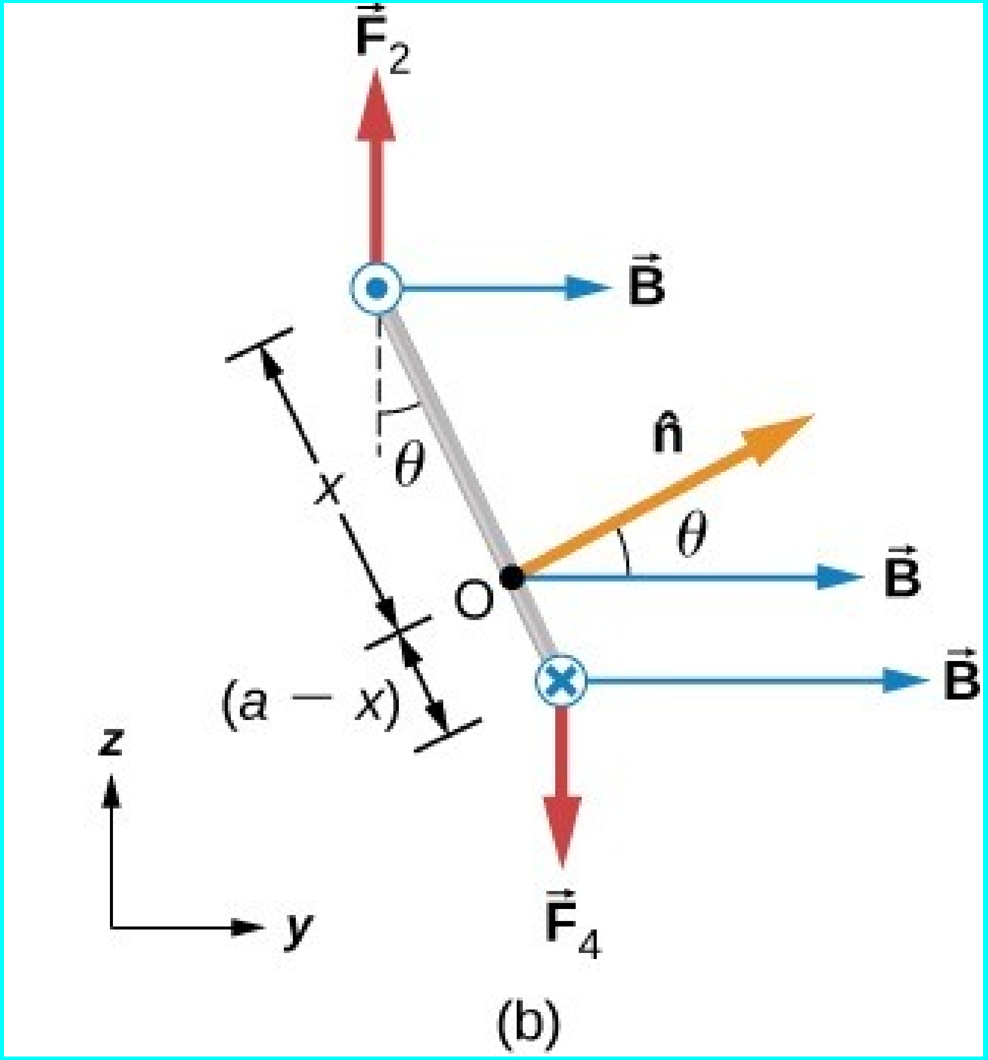
(A) $\vec{m} = 3\,\hat{x}$ Ampere·meter^2

(B) $\vec{m} = 3\,\hat{z}$ Ampere·meter^2

(C) $\vec{m} = 36\,\hat{z}$ Ampere·meter^2

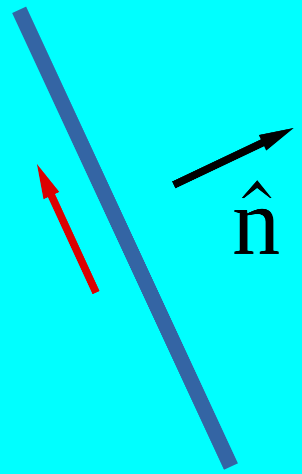
(D) $\vec{m} = 3\,\hat{y}$ Ampere·meter^2


z
F2
F3
B
I
n̂
θ
a
y
F1
b
x
F4
(a)
F2
B
θ
x
n̂
θ
O
B
(a − x)
B
z
y
F4
(b)



$\vec{F}_2$
$\vec{B}$
$\hat{n}$
θ
x
θ
O
$\vec{B}$
$\vec{B}$
$(a - x)$
z
y
$\vec{F}_4$
(b)

This is a side view of a wire loop. Which direction should B point for maximum torque?



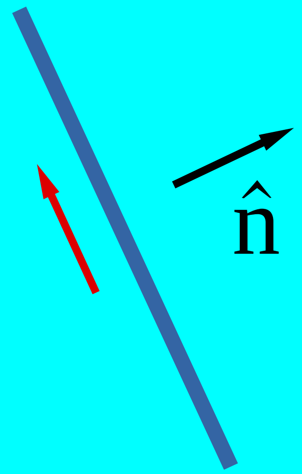
(A) To the right

(B) Up

(C) Along $\hat{n}$

(D) Perpendicular to $\hat{n}$

This is a side view of a wire loop. Which direction should B point for zero torque?

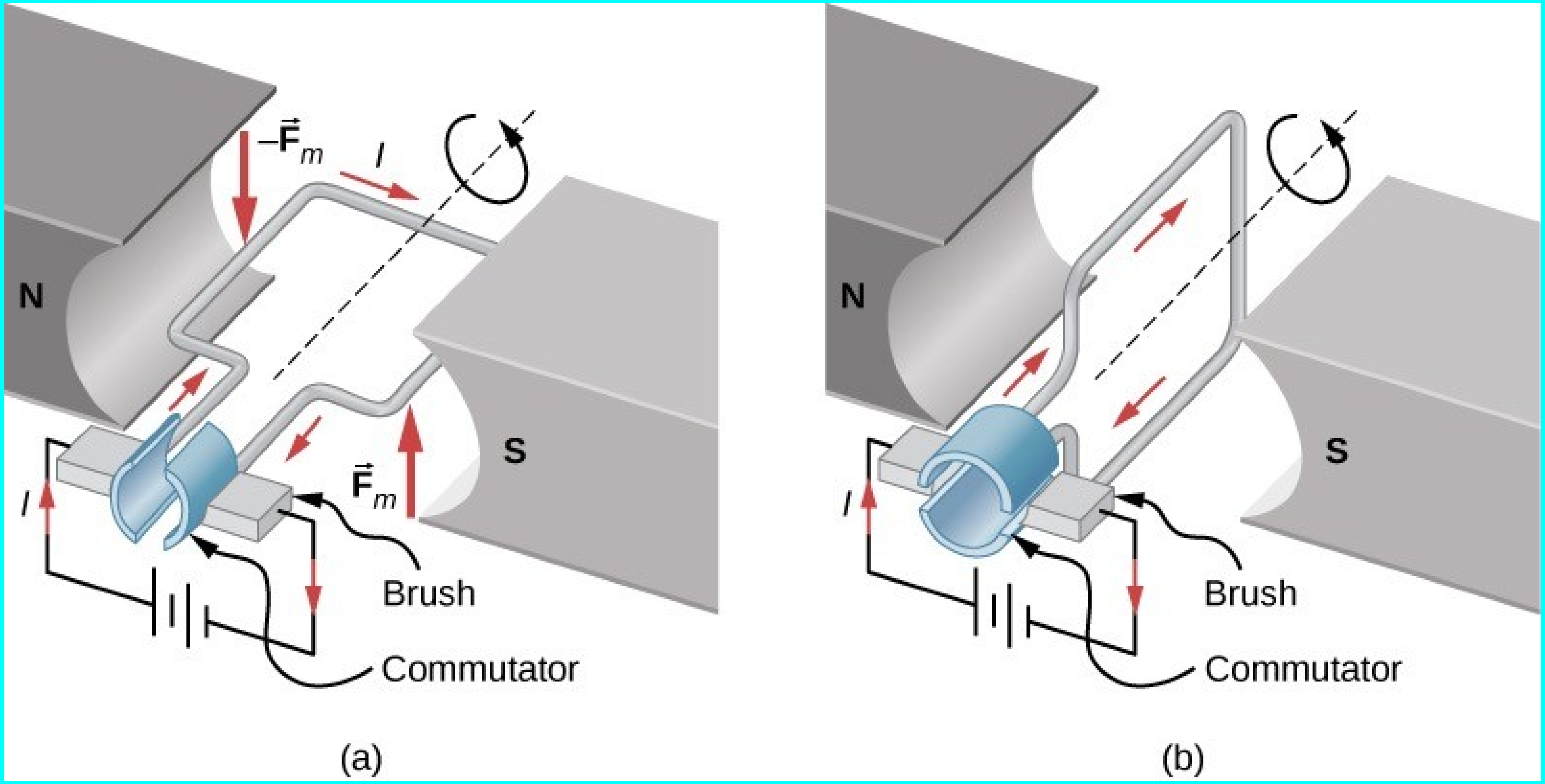


(A) To the right

(B) Up

(C) Along $\hat{n}$

(D) Perpendicular to $\hat{n}$


$-\vec{F}_m$
I
N
S
I
$\vec{F}_m$
Brush
Commutator
(a)
N
S
I
Brush
Commutator
(b)

Equations of Magnetism

$\vec{F} = Q\,\vec{v} \times \vec{B}$ Force on charge Q

$\vec{F} = I\,\vec{L} \times \vec{B}$ Force on current I

$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ Field of Infinite wire

$\vec{B} = \frac{\mu_0 I}{2a}\hat{z}$ Field in center of wire loop

$\vec{B} = \mu_0 n I \hat{z}$ Field of an infinite coil (solenoid)

Maxwell's Equations

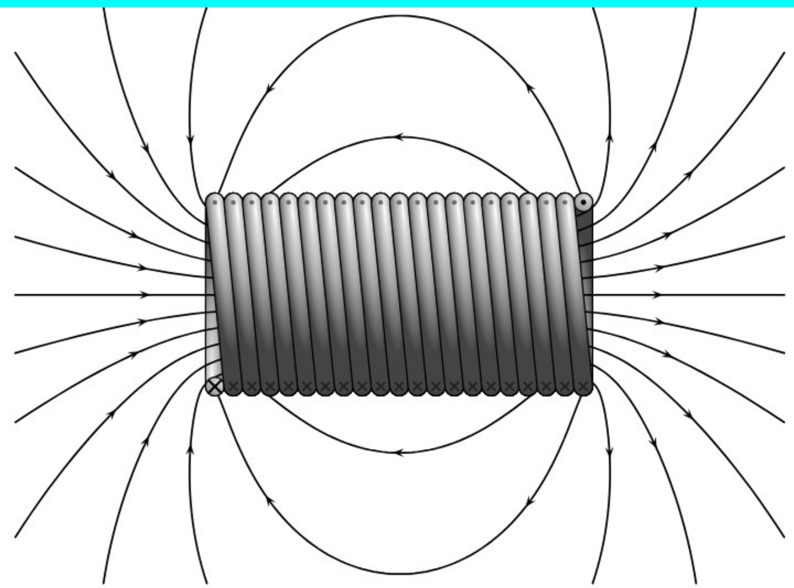
$$\int \vec{E}\cdot d\vec{A}=\frac{Q}{\epsilon_0}$$ To calculate E for symettrical charges.

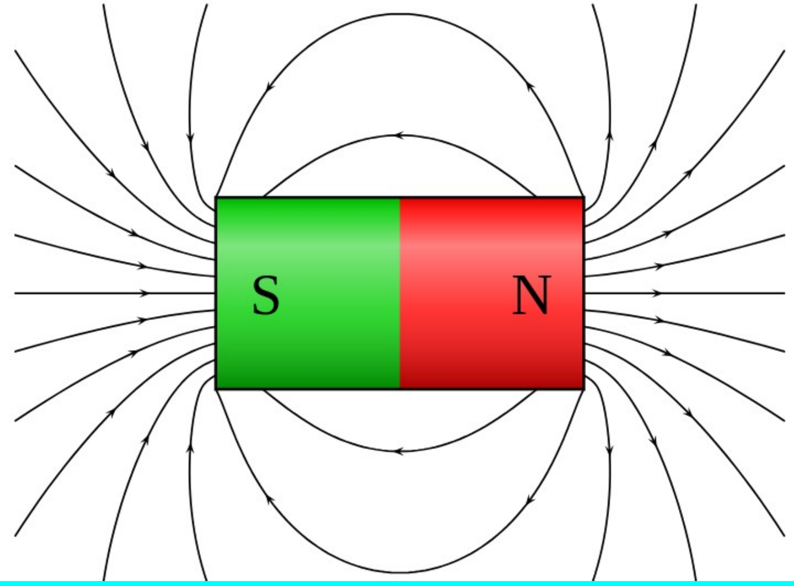
$$\int \vec{B}\cdot d\vec{A}=0$$ Cannot have North magnet w/o a South pole.

$$\int \vec{B}\cdot d\vec{l}=\mu_0 I$$ To calculate B for symettrical currents.

$$\int \vec{E}\cdot d\vec{l}=-\frac{d\Phi}{dt}$$ Magnetic induction! Generators! Light!

A solenoid is like a bar magnet





Faraday's Law

A changing magnetic flux makes a voltage.
Moving a magnet in a loop of wire makes a voltage.
Spinning a wire in a magnetic field makes a Voltage
Turning on an electromagnet near a coil of wire makes a voltage.

Faraday's Law Lab

You will yank a loop of wire out of a magnetic field. This will allow you to measure the magnetic field.

You will run an AC current through a coil. This will cause a voltage to appear in a different coil.

Electric fields are calculated with Coulomb's law or Gauss's Law

Magnetic Fields are calculated with the Biot-Savart Law or Ampere's Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2} \quad \text{Biot Savart}$$

Recap Lecture 22

- Magnetic forces are at right angles to velocity and field
- Superposition works for magnetic fields just like electric fields
- Motors are clever combos of wire and B
- Solenoids are "simple" … they have uniform B-fields inside.
- Charged particles go in circles in magnetic fields
- Torque is cross product of magnetic moment and B
- Ampere's Law and Biot Savart law are used to calculate magnetic fields