

- Announcements

Grade Change day is April 10 (tomorrow)

Exams back today

Updated grades.

It is unlikely that your final grade will increase by more than a letter.

- Last Time

Magnetism

Motors

- Today

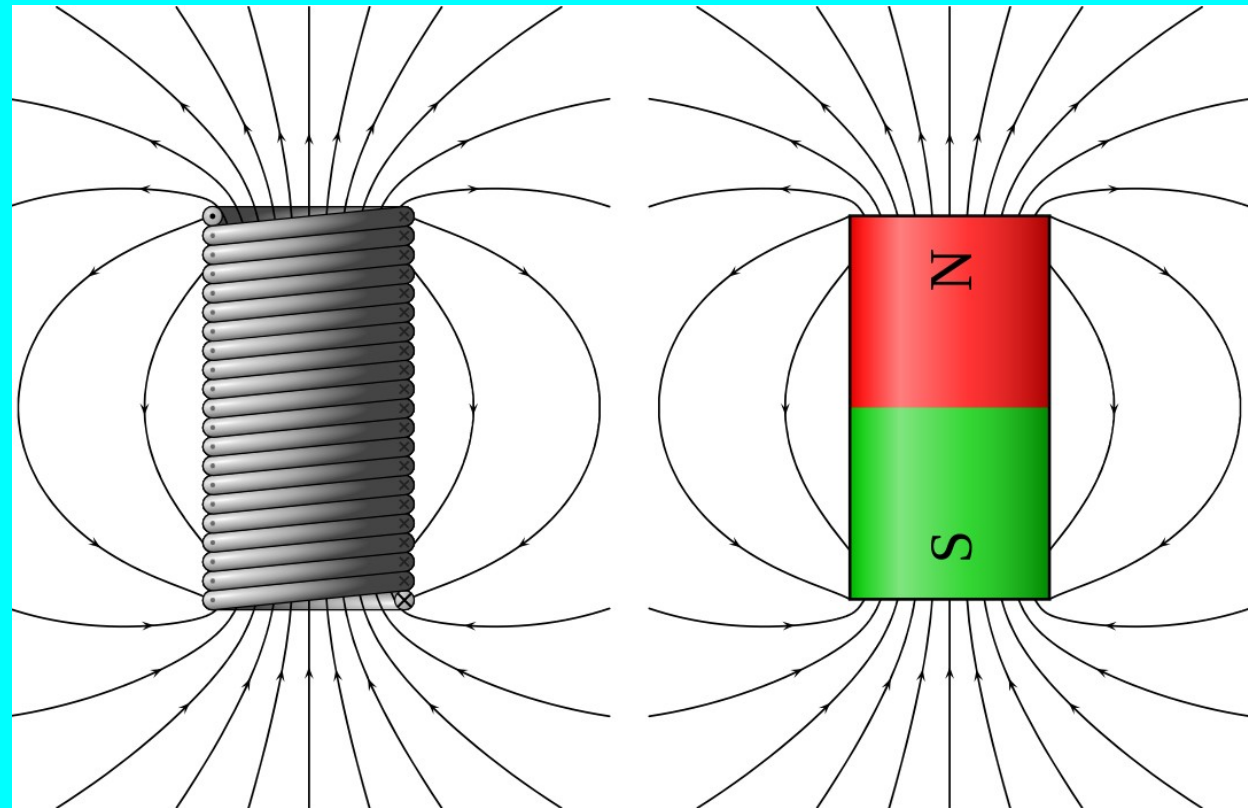
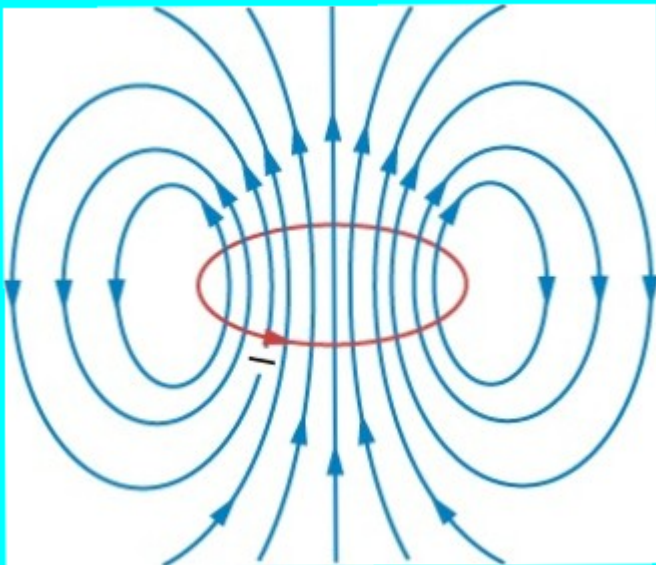
Motors

Ampere's Law

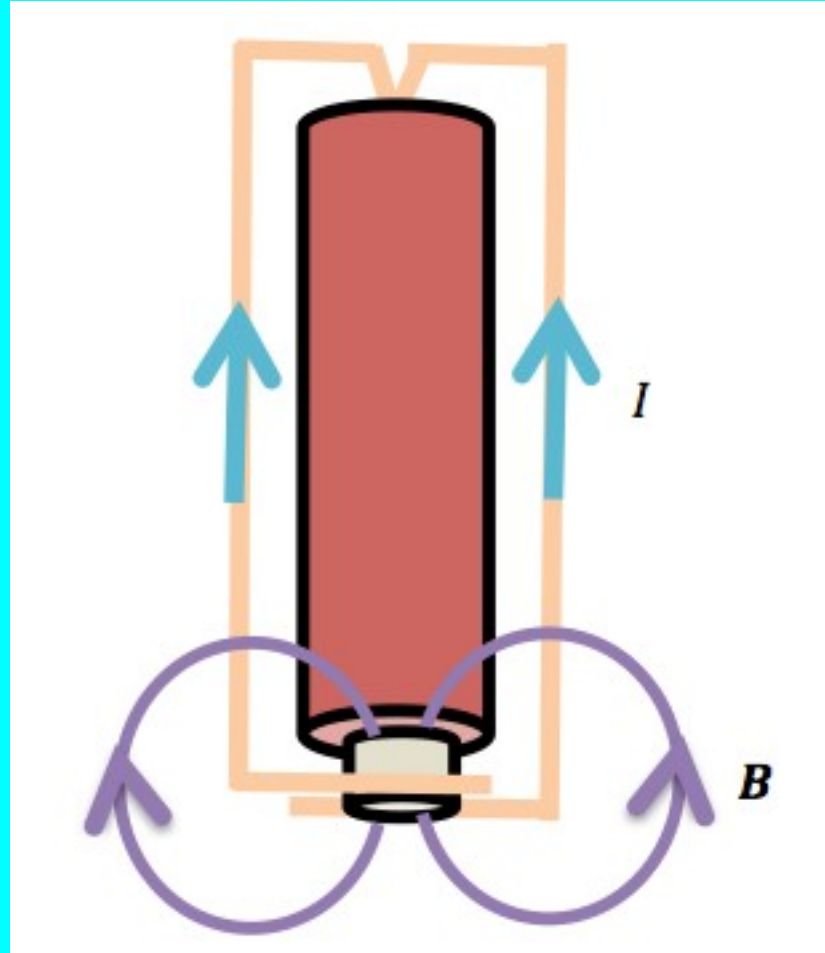
Faraday's Law

Electromagnets

- A loop of wire is a basic electromagnet
- Many loops of wire (a coil or “solenoid”) is a better electromagnet.



“Homopolar” Motor



$$\mathcal{E} = - \frac{d\Phi_B}{dt} = V \rightarrow \Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = - \frac{d}{dt} \Phi_B$$

$$\Phi_B = \vec{B} \cdot \vec{A} \leftarrow \text{Need}$$

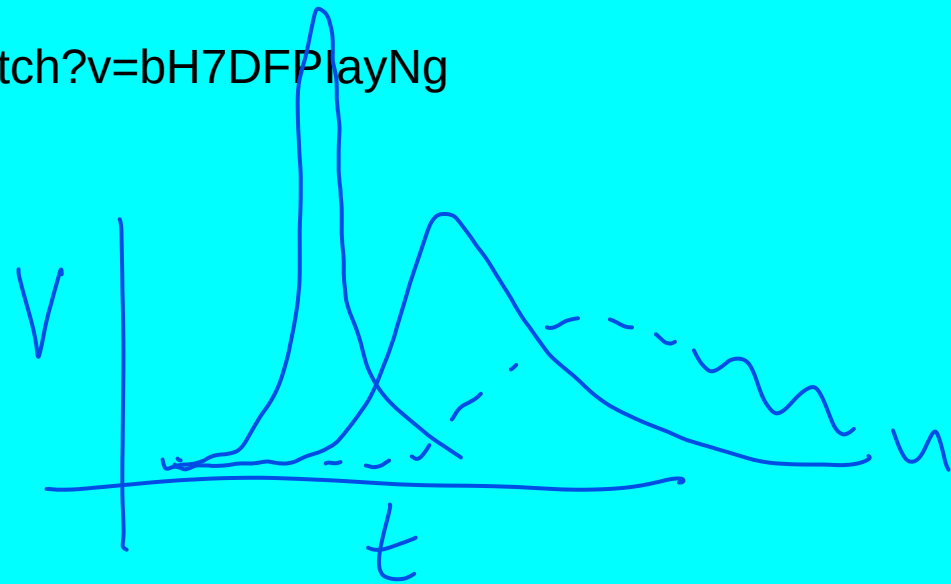
$$\Phi_E = \vec{E} \cdot \vec{A} \text{ reminder}$$

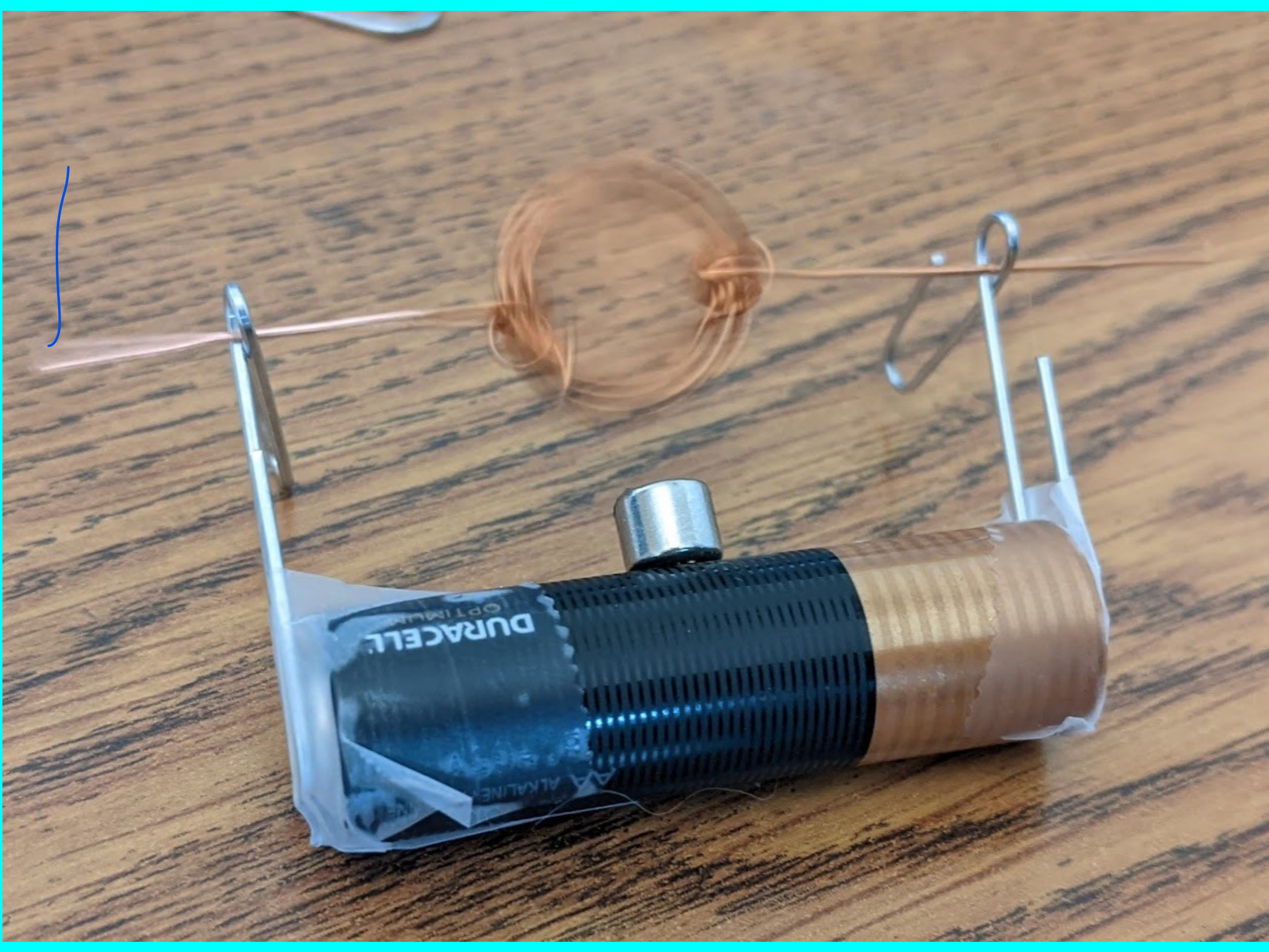
<https://www.youtube.com/watch?v=bH7DFPlayNg>

$$\int \mathcal{E}(t) dt = \int - \frac{d}{dt} \Phi_B dt$$

$$= \Phi_B$$

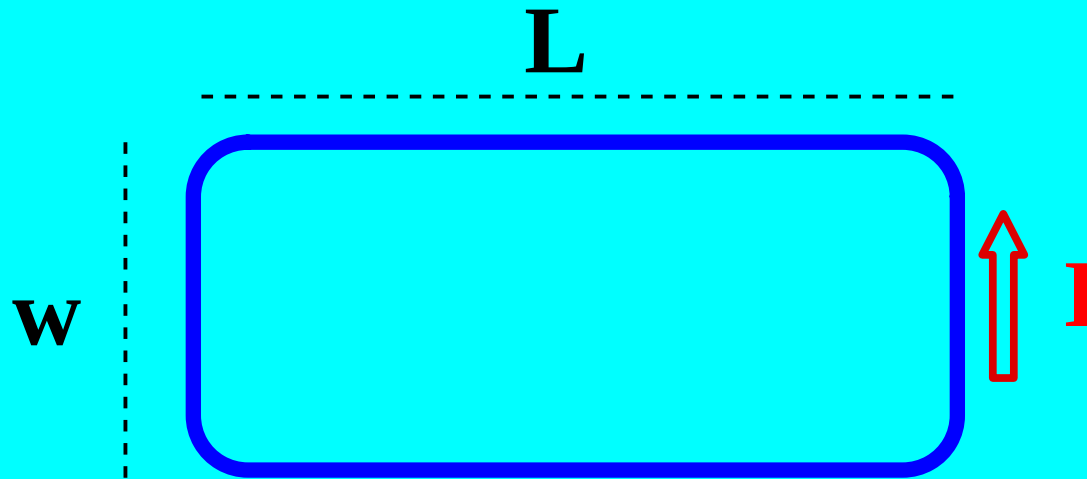
$\vec{B} \cdot \vec{A}$
 $\uparrow \quad \uparrow$





Motors

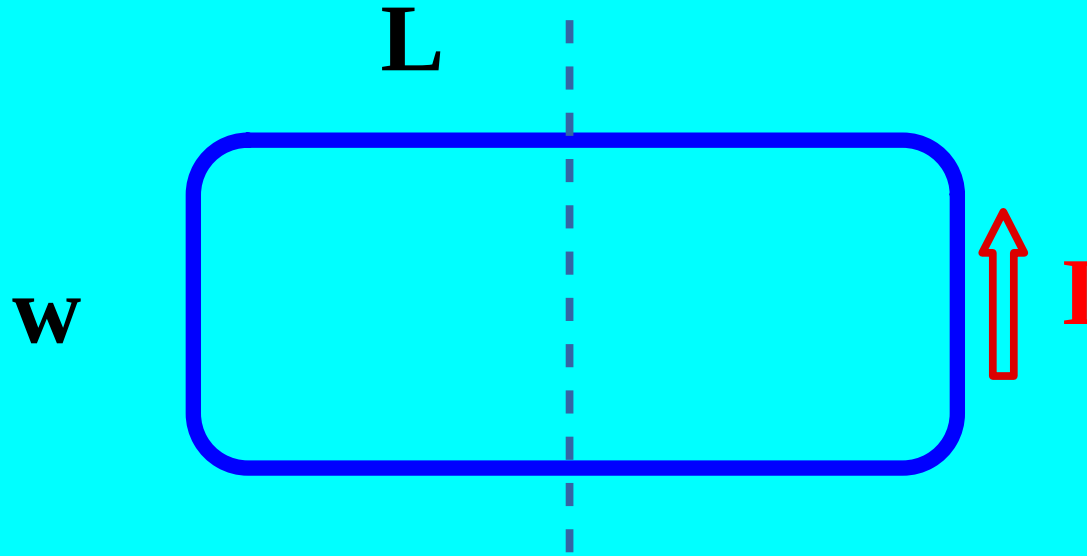
Are just clever loops of wire in magnetic fields



$$\vec{F} = I \vec{L} \times \vec{B} \quad \text{Force on current } I$$

Wire loop

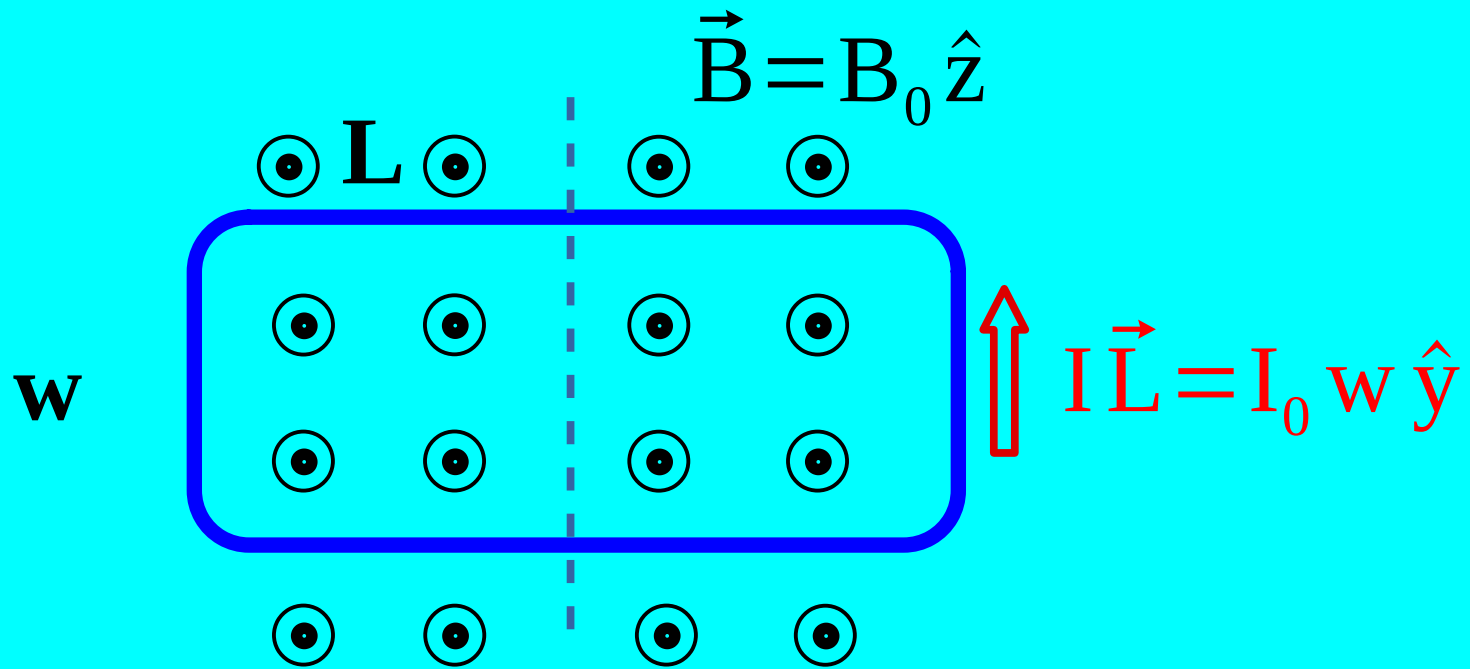
Is free to rotate about vertical axis (dotted line)



Can we calculate net force and torque?

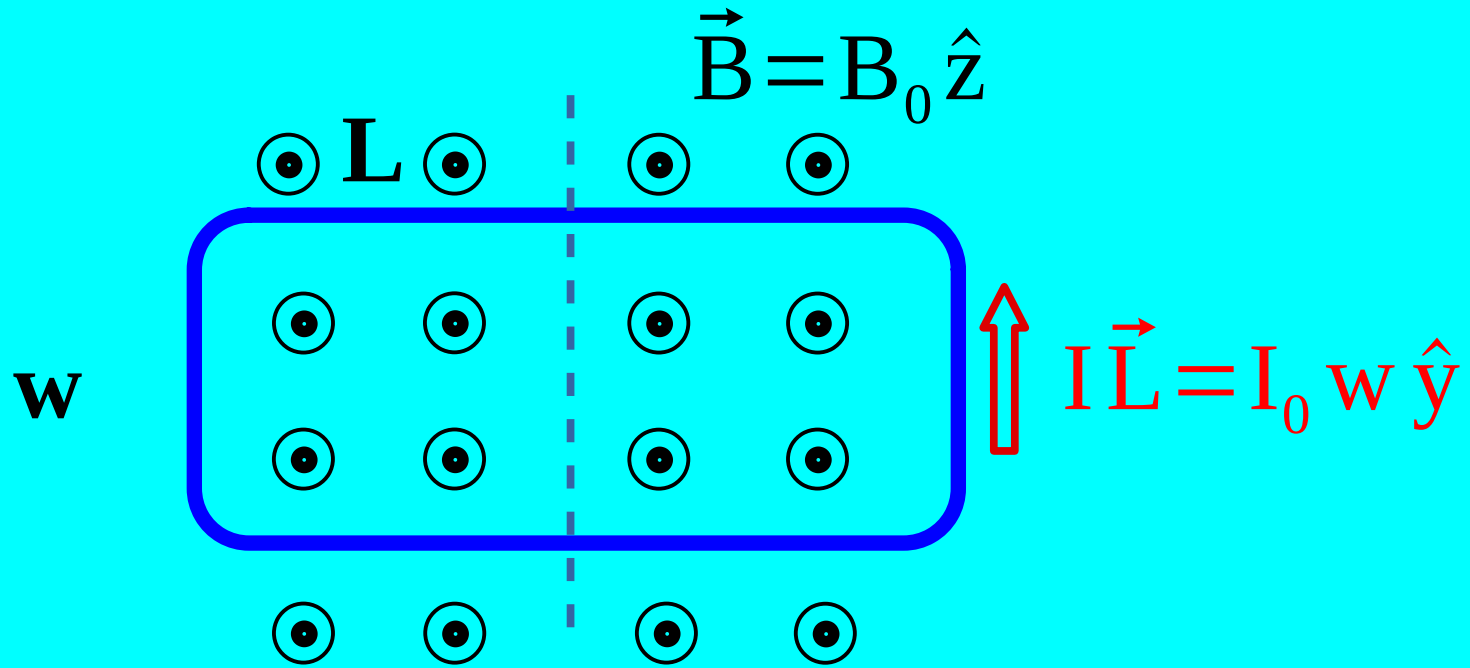
$$\vec{F} = I \vec{L} \times \vec{B} \quad \text{Force on current } I$$

Try B-field out of page

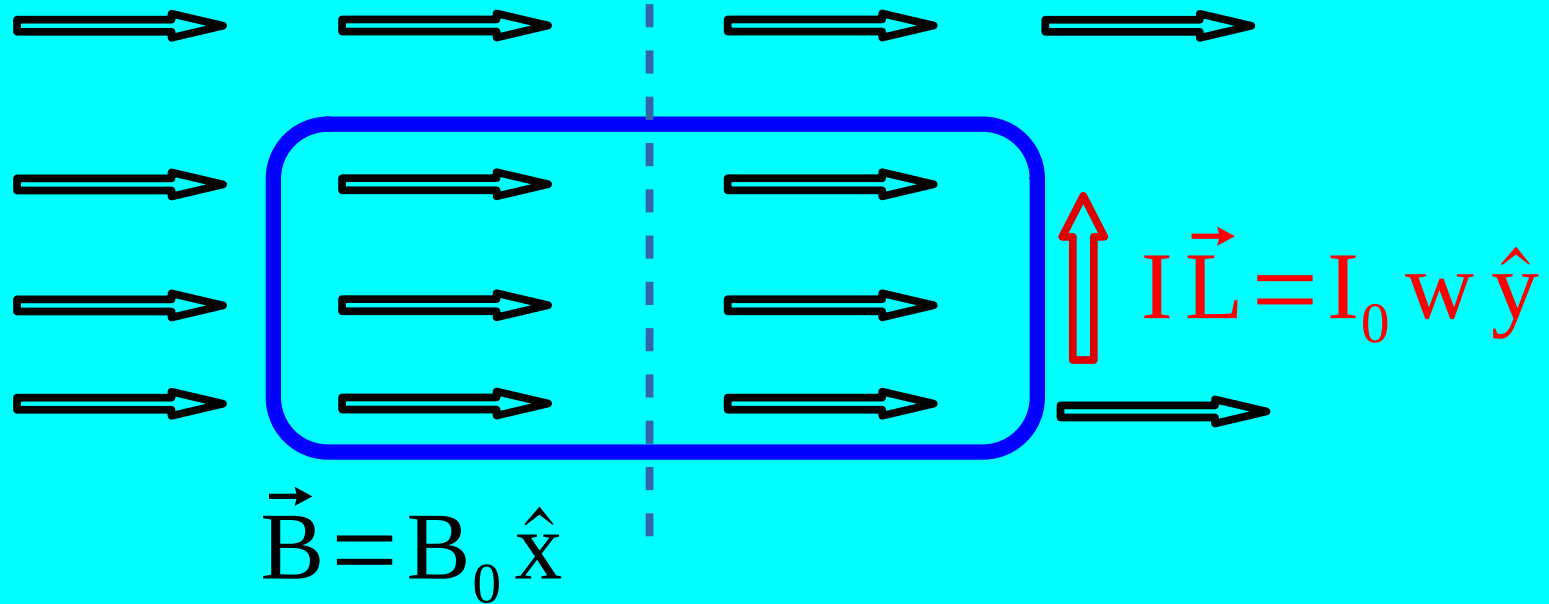


$$\vec{F} = I \vec{L} \times \vec{B} \quad \text{Force on current } I$$

Try B-field out of page



Try B-field to the right



$$\vec{F} = I\vec{L} \times \vec{B} \quad \text{Force on current } I$$

Equations for motors

$$\vec{F} = I \vec{L} \times \vec{B} \quad \text{Force on current } I$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{Torque definition}$$

$$\vec{m} = N I \vec{A} \quad \text{Def. of Magnetic Moment}$$

$$\vec{\tau} = \vec{m} \times \vec{B} \quad \text{Torque on a Magnetic Moment}$$

Torque

- A wrench is 30 cm long and a person exerts 10 N at right angles to it. The torque is?
 - (A) 30 N·m
 - (B) 10 N·m
 - (C) 300 N·m
 - (D) 3 N·m
 - (E) 0 N·m

“Give me a moment”

“Moment” is an old physics/engineering word that has **NOTHING TO DO** with momentum. It is a general way of referring to quantities multiplied by lengths or area.

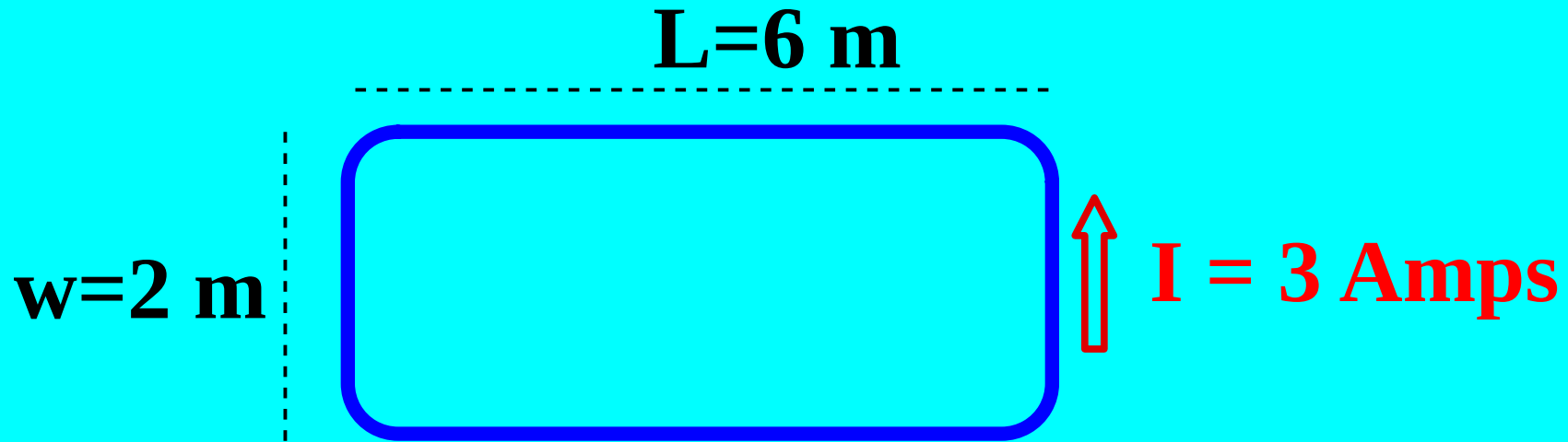
Thus “Moment of Inertia”

Or “Moment arm”

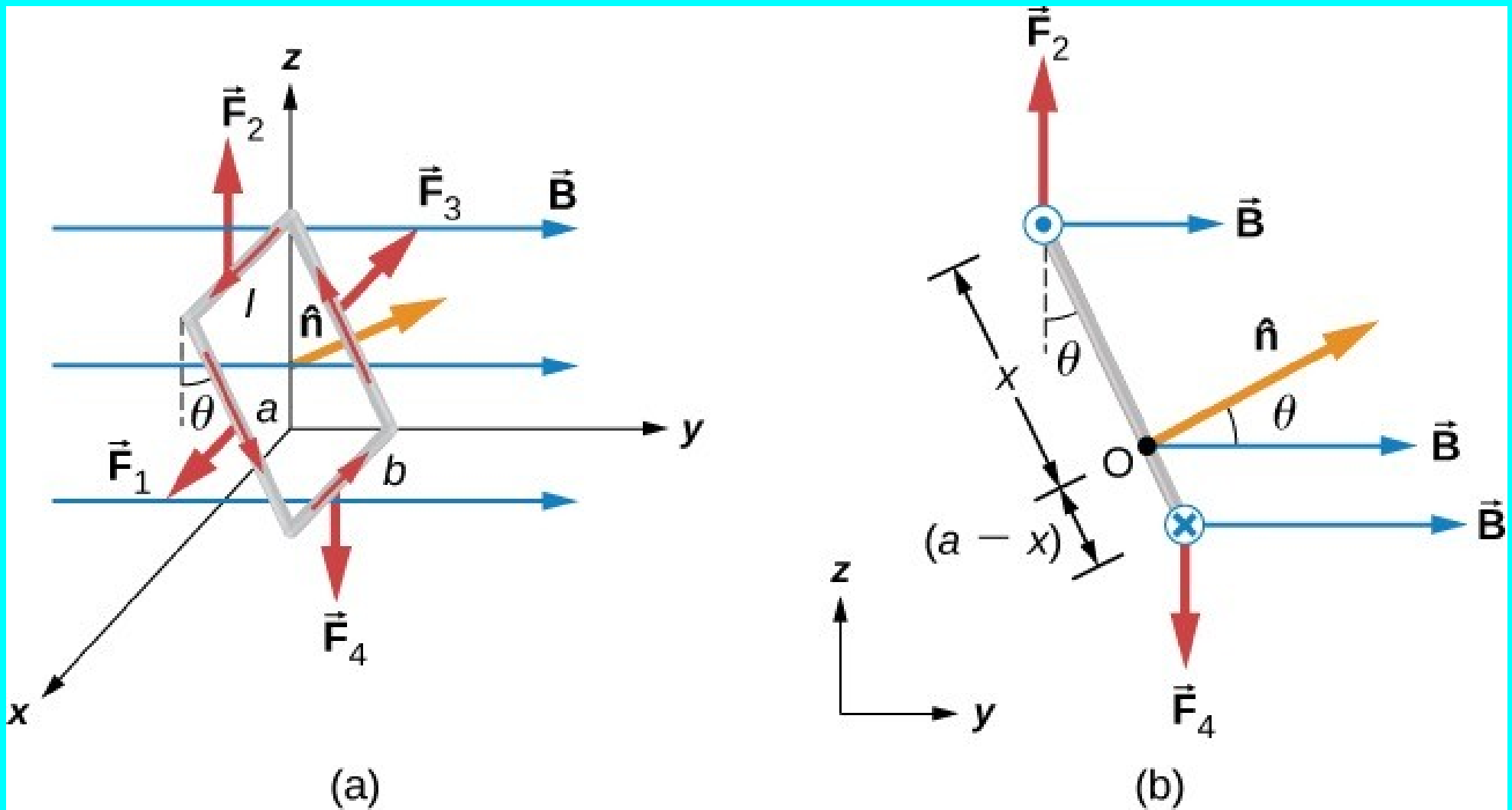
Or “Magnetic Moment”

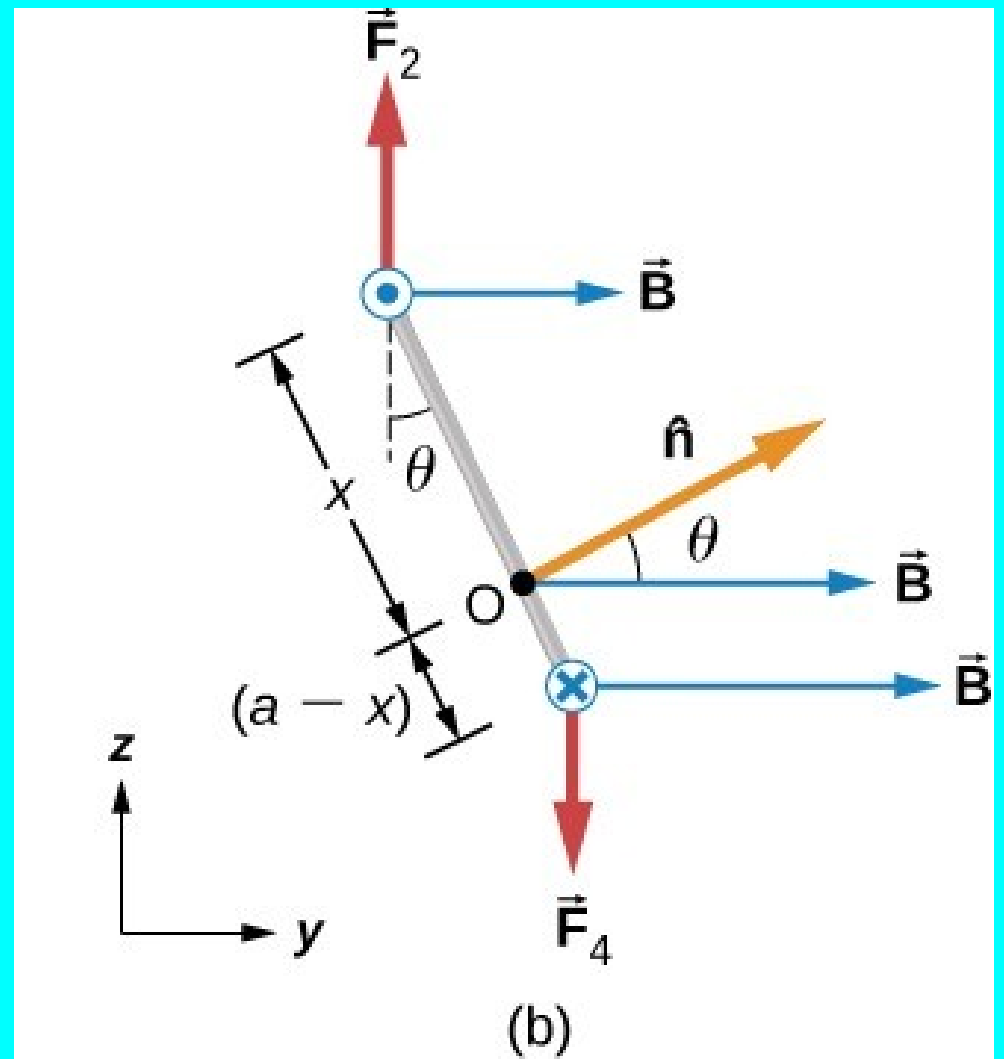
Using magnetic moment makes it much easier to calculate motor torques.

What is the magnetic moment of this loop?

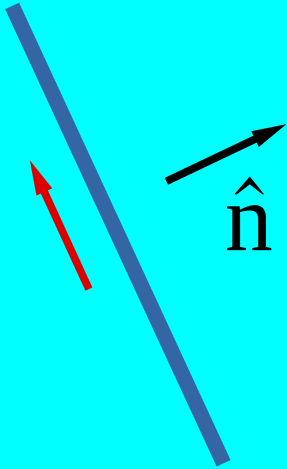


- (A) $\vec{m} = 3 \hat{x} \text{ Ampere} \cdot \text{meter}^2$
- (B) $\vec{m} = 3 \hat{z} \text{ Ampere} \cdot \text{meter}^2$
- (C) $\vec{m} = 36 \hat{z} \text{ Ampere} \cdot \text{meter}^2$
- (D) $\vec{m} = 3 \hat{y} \text{ Ampere} \cdot \text{meter}^2$



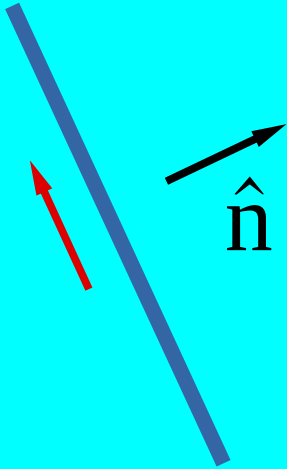


This is a side view of a wire loop. Which direction should \mathbf{B} point for maximum torque?

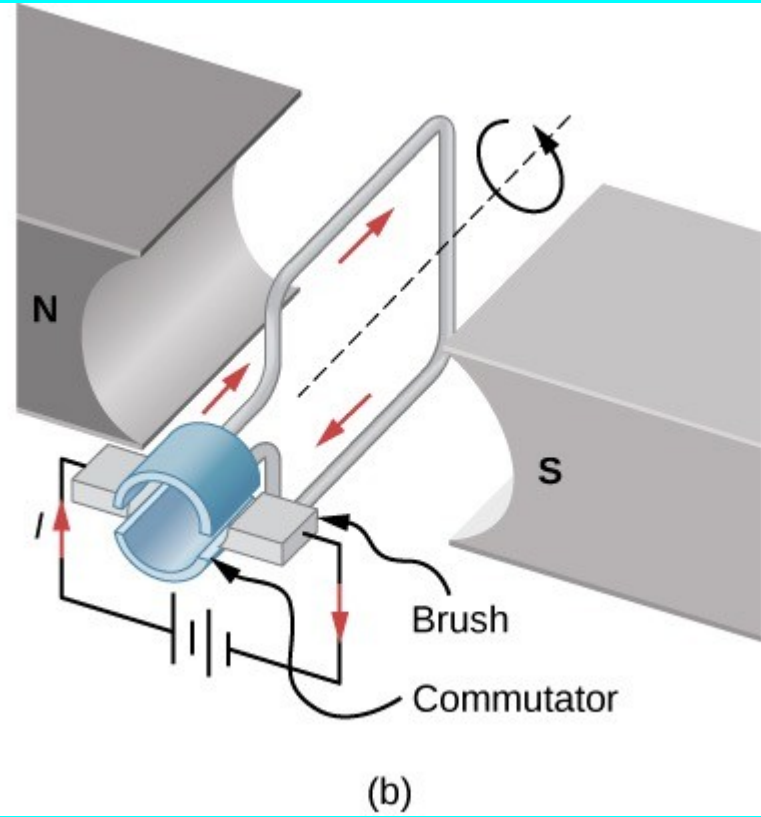
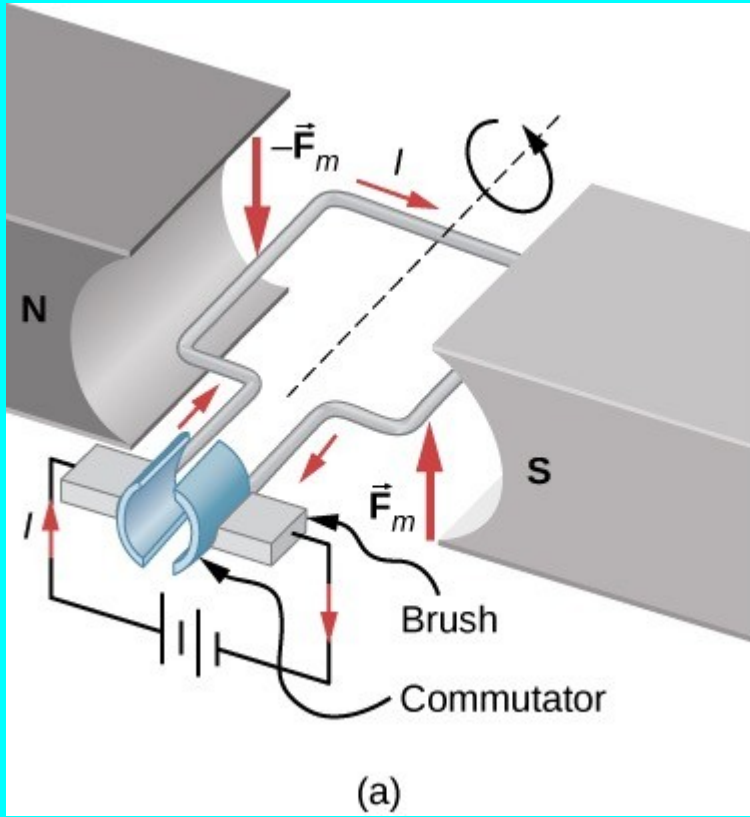


- (A) To the right
- (B) Up
- (C) Along \hat{n}
- (D) Perpendicular to \hat{n}

This is a side view of a wire loop. Which direction should \mathbf{B} point for zero torque?



- (A) To the right
- (B) Up
- (C) Along \hat{n}
- (D) Perpendicular to \hat{n}



Equations of Magnetism

$$\vec{F} = Q \vec{v} \times \vec{B} \quad \text{Force on charge } Q$$

$$\vec{F} = I \vec{L} \times \vec{B} \quad \text{Force on current } I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi} \quad \text{Field of Infinite wire}$$

$$\vec{B} = \frac{\mu_0 I}{2a} \hat{z} \quad \text{Field in center of wire loop}$$

$$\vec{B} = \mu_0 n I \hat{z} \quad \text{Field of an infinite coil (solenoid)}$$

Maxwell's Equations

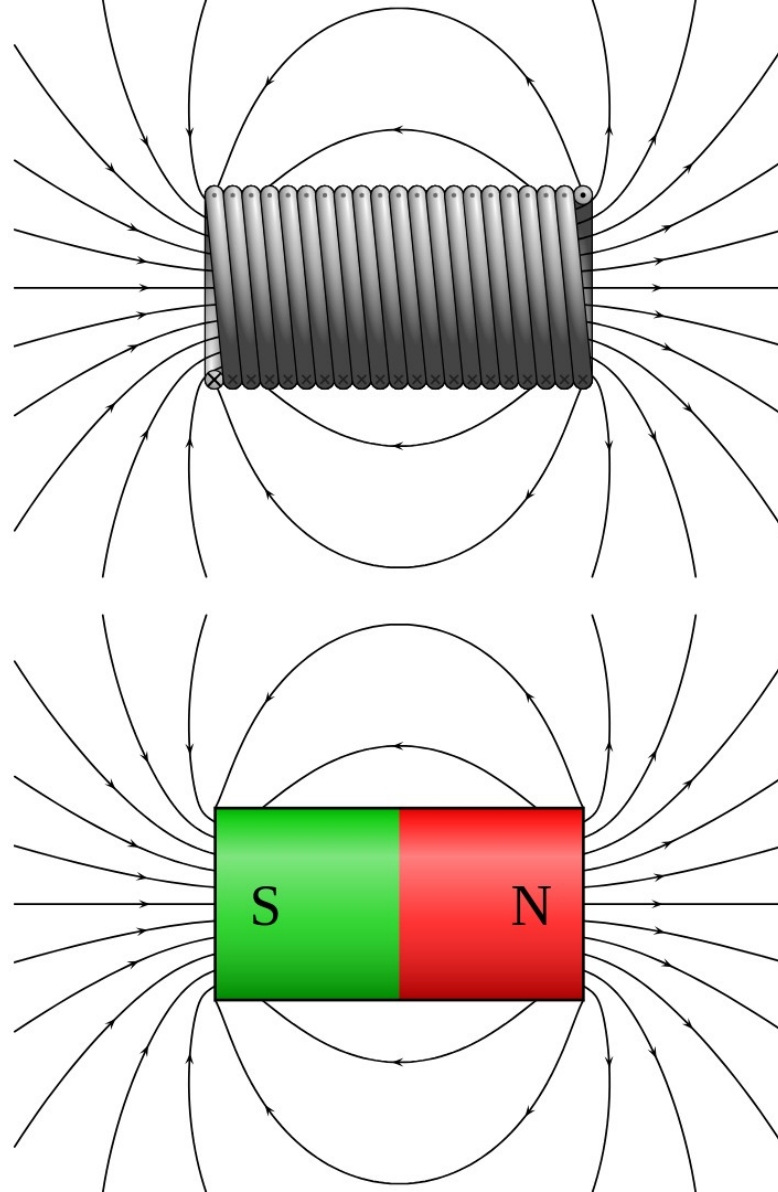
$$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \quad \text{To calculate E for symmetrical charges.}$$

$$\int \vec{B} \cdot d\vec{A} = 0 \quad \text{Cannot have North magnet w/o a South pole.}$$

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I \quad \text{To calculate B for symmetrical currents.}$$

$$\int \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} \quad \text{Magnetic induction! Generators! Light!}$$

A solenoid is like a bar magnet



Faraday's Law

A changing magnetic flux makes a voltage.

Moving a magnet in a loop of wire makes a voltage.

Spinning a wire in a magnetic field makes a Voltage

Turning on an electromagnet near a coil of wire makes a voltage.

Faraday's Law Lab

You will yank a loop of wire out of a magnetic field. This will allow you to measure the magnetic field.

You will run an AC current through a coil. This will cause a voltage to appear in a different coil.

**Electric fields are calculated with
Coulomb's law or Gauss's Law**

**Magnetic Fields are calculated with the
Biot-Savart Law or Ampere's Law**

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \quad \text{Biot Savart}$$

Recap Lecture 22

- Magnetic forces are at right angles to velocity and field
- Superposition works for magnetic fields just like electric fields
- Motors are clever combos of wire and B
- Solenoids are “simple” ... they have uniform B-fields inside.
- Charged particles go in circles in magnetic fields
- Torque is cross product of magnetic moment and B
- Ampere’s Law and Biot Savart law are used to calculate magnetic fields

