Lecture 16: 03/07/2024

- Announcements
  Written HW5 due next Monday. No online.

- Last Time
  The textbook can help you
  Problem 4-6 – Potential of a wire, and a sphere
  What is capacitance
  Parallel plate capacitor
  Current and Ohm's Law

- Today
  Parallel plate capacitor
  Series and parallel connections
  Effective Capacitance (and resistance)
  Dielectrics

Lecture 15 Recap

We reviewed how to use the text
We learned potential for a wire or “coax”
We learned why $V=kQ/r$ (for a point or sphere)
We learned what capacitance is,
and the formula for parallel plate.
Current is flowing charge. Resistance is like a skinny pipe.

Next … More on Capacitance …

Key Equations

Capacitance	$C = \frac{Q}{V}$
Capacitance of a parallel-plate capacitor	$\boxed{C = \varepsilon_0 \frac{A}{d}}$
Capacitance of a vacuum spherical capacitor	$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$
Capacitance of a vacuum cylindrical capacitor	$C = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$
Capacitance of a series combination	$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$
Capacitance of a parallel combination	$C_P = C_1 + C_2 + C_3 + \cdots$
Energy density	$u_E = \frac{1}{2}\varepsilon_0 E^2$
Energy stored in a capacitor	$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$
Capacitance of a capacitor with dielectric	$\boxed{C = \kappa C_0}$
Energy stored in an isolated capacitor with dielectric	$U = \frac{1}{\kappa}U_0$
Dielectric constant	$\cancel{\kappa = \frac{E_0}{E}}$
Induced electrical field in a dielectric	$\cancel{\vec{\mathbf{E}}_i = \left(\frac{1}{\kappa} - 1\right)\vec{\mathbf{E}}_0}$

$C = Q/V$

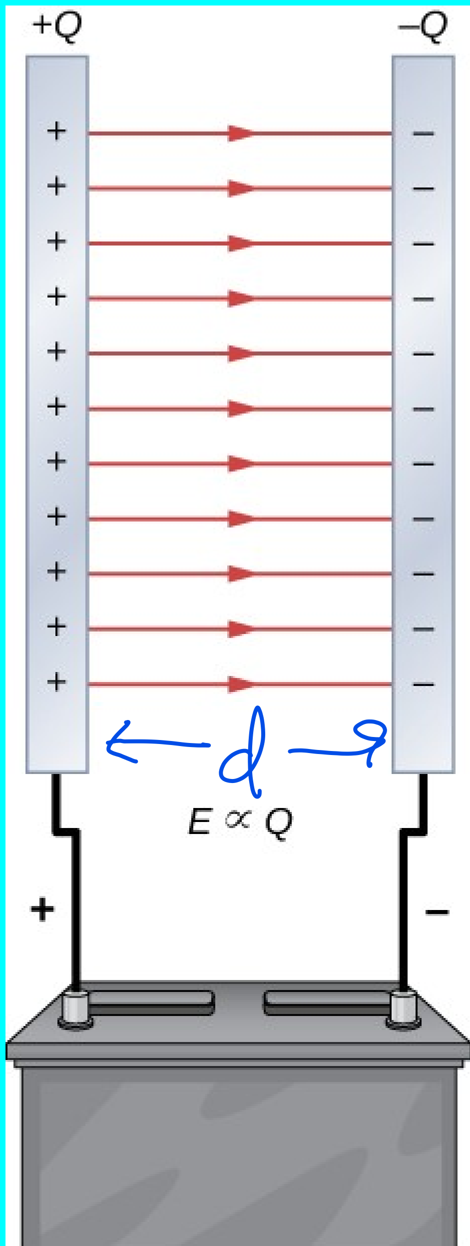
$C = \varepsilon_0 A/d$

$u_E = \frac{1}{2}\varepsilon_0 E^2$ (J/m^3)

$U = \frac{1}{2}CV^2$ (J)

The parallel plate capacitor (again)

$$\boxed{C \stackrel{def}{=} \frac{Q}{\Delta V}}$$



$Q = C\Delta V$

$$C_{parallel\_plate} = \epsilon_0 \frac{A}{d}$$

$\sigma = \frac{Q}{A}$

$V = E\Delta x$

$E = \frac{\sigma}{2\epsilon_0}$ (one plate)

$E = \frac{\sigma}{\epsilon_0}$

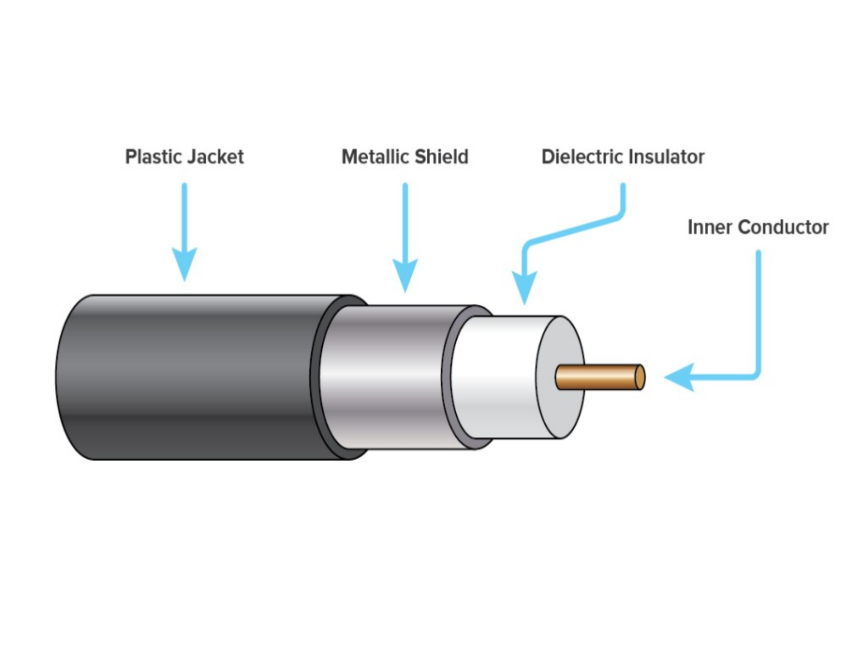
$V = \frac{\sigma}{\epsilon_0} d = \frac{Q}{A\epsilon_0} d$

$Q = \frac{A\epsilon_0}{d} V$

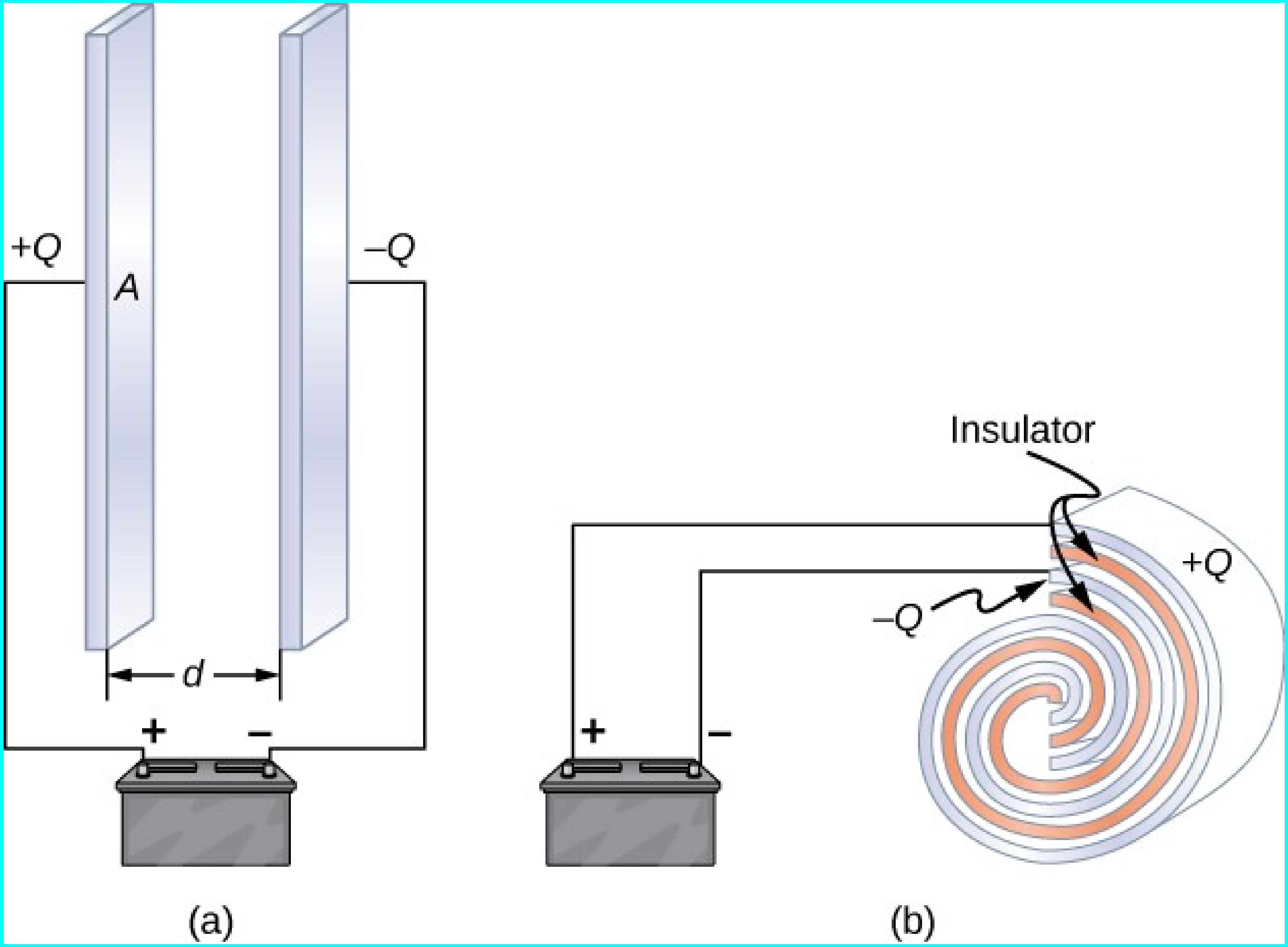
$C = \epsilon_0 \frac{A}{d}$

Not all capacitors are “parallel plate” capacitors

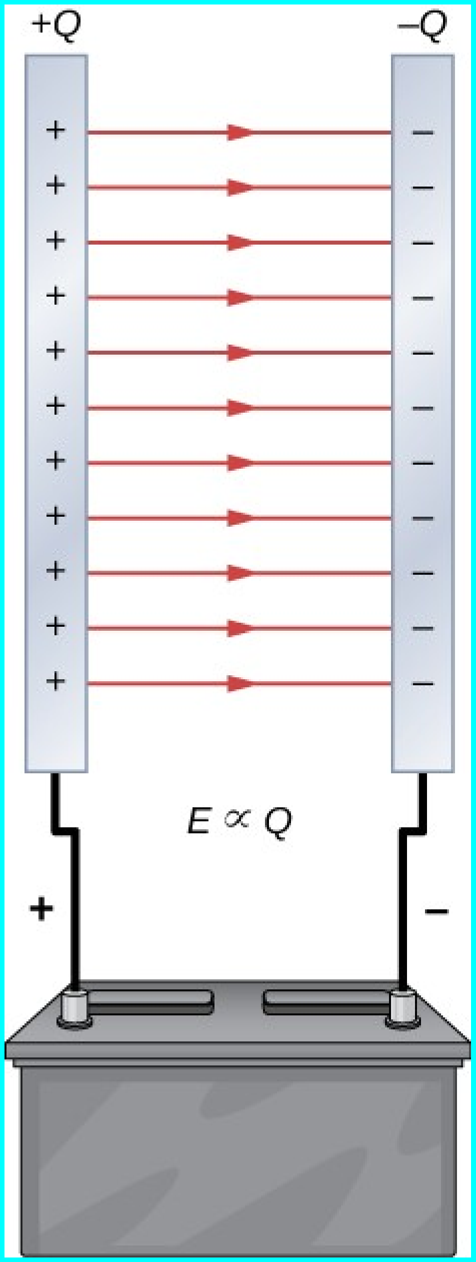
$$C \stackrel{\text{def}}{=} \frac{Q}{\Delta V}$$



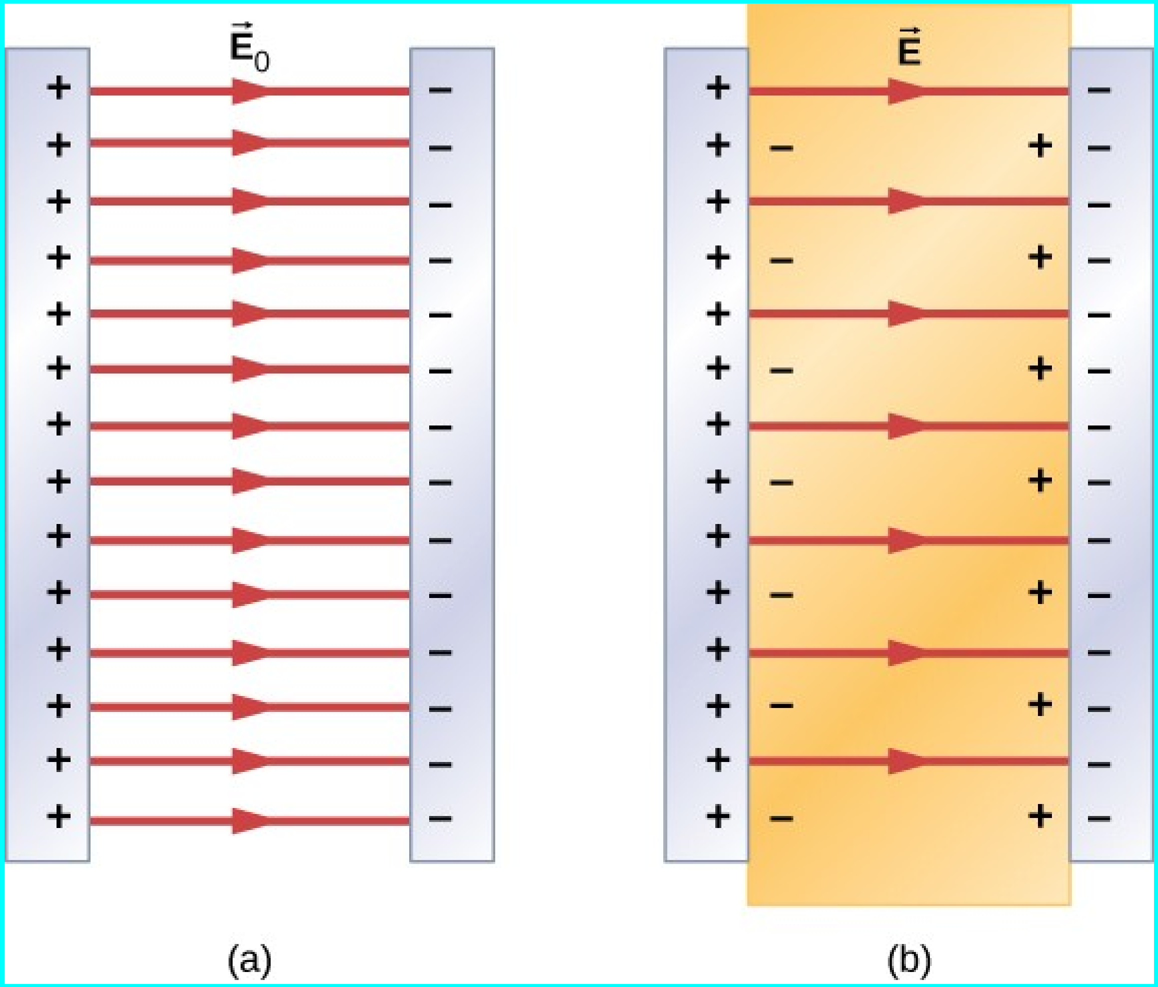
A commercial capacitor



The parallel plate capacitor with a dielectric

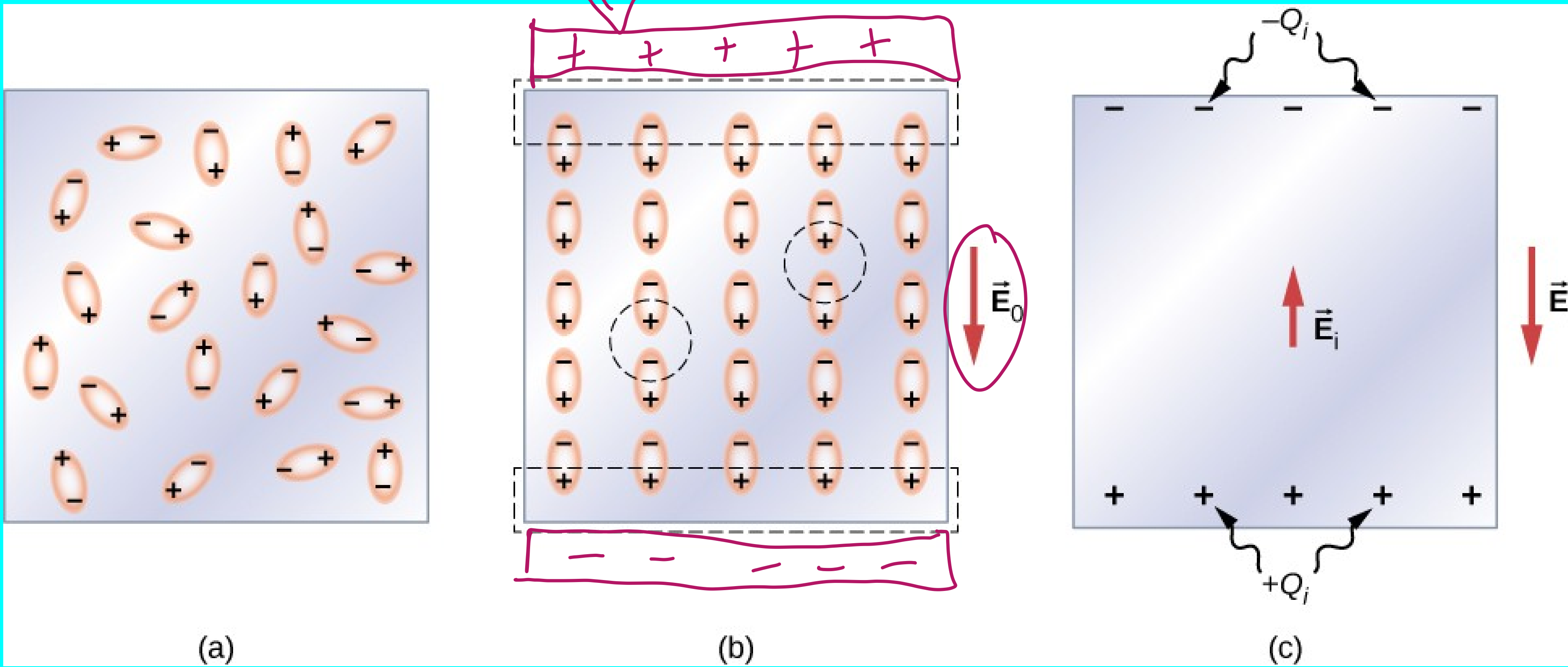


$$C_{parallel\_plate\_with\_dielectric} = \kappa \epsilon_0 \frac{A}{d}$$



Induced electric field (polarization) in a dielectric

$Q = CV$ $\quad E_{Net} = \frac{E_0}{K}$ $\quad \Delta V = \frac{\Delta V_0}{K}$



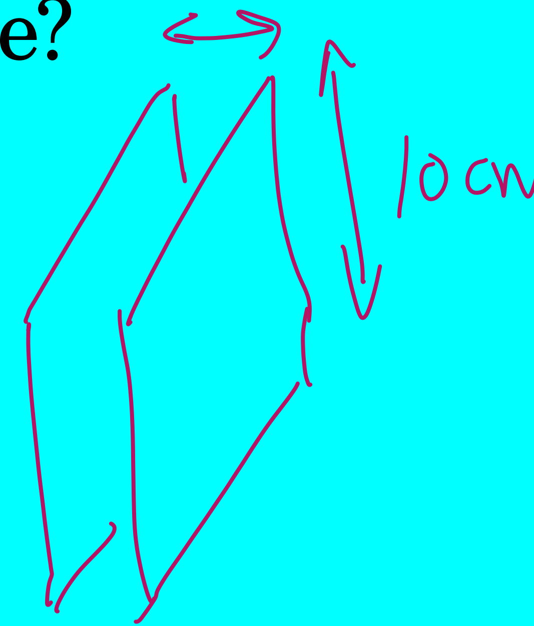
$Q = CV$ $\quad V = E_{net}\, d = \frac{E_0}{K}$

Example

10mm

Two 10 cm x 10 cm plates are separated by ~~0.1 mm~~. What is the capacitance?

$d = 0.1\,mm$



$$C = \epsilon_0 \frac{A}{d} = (8.85\,pf/m)\frac{(10^{-1})^2\,m^2}{10^{-4}\,m}$$

$$= 885\,pf = 885\,pF$$

$$C = 8.85\,pf/m\,\frac{(10^{-1})^2}{10^{-2}} = 8.85\,pF$$

Material	Dielectric constant κ	Dielectric strength $E_c\,[\times 10^6\,\mathrm{V/m}]$
Vacuum	1	∞
Dry air (1 atm)	1.00059	3.0
Teflon™	2.1	60 to 173
Paraffin	2.3	11
Silicon oil	2.5	10 to 15
Polystyrene	2.56	19.7
Nylon	3.4	14
Paper	3.7	16
Fused quartz	3.78	8
Glass	4 to 6	9.8 to 13.8

$$C_{parallel\_plate\_with\_dielectric} = \kappa\,\epsilon_0\,\frac{A}{d}$$

Example

Two 10 cm x 10 cm plates are separated by ~~0.1~~ mm. 10mm What is the capacitance? A dielectric material with constant “2” fills the gap. What is the capacitance now?

(A) 8.85 pf

$\boxed{8,85\,pF}$

(B) 4.425 pF

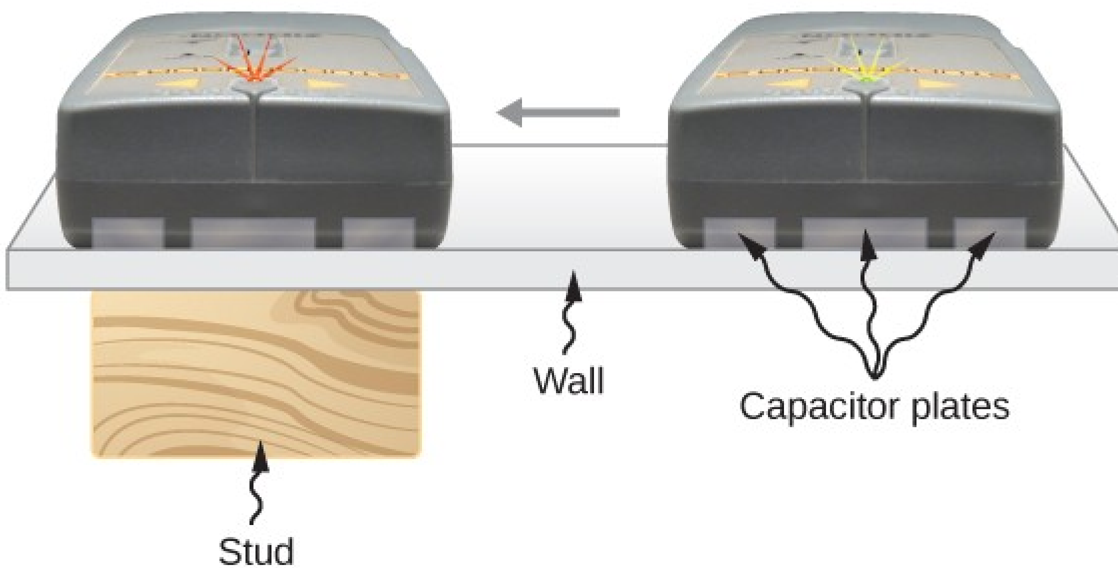
~~8.85 pF~~

(C) 0.0177 nF

(D) 2 pF

(E) 17.7 μF


STUDSENSOR 2
ZIRCON

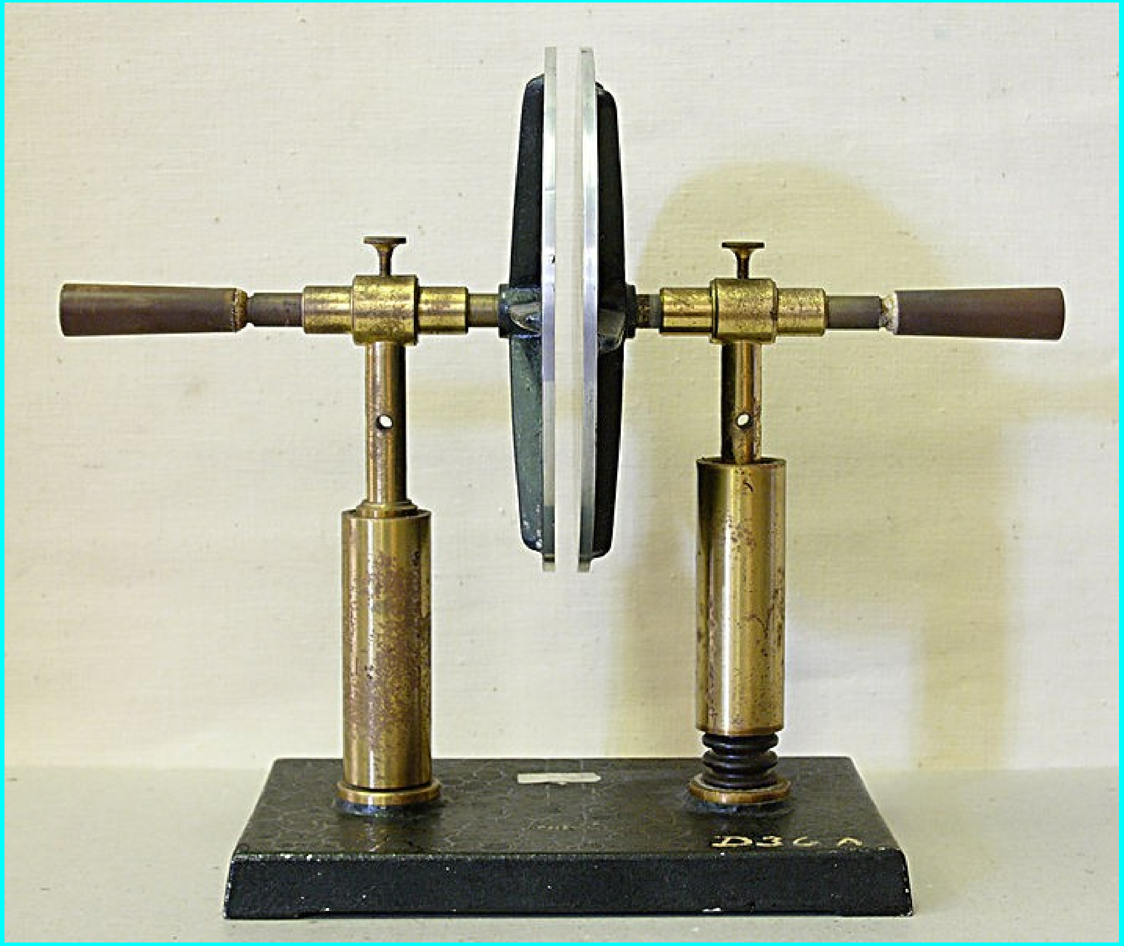

Wall
Capacitor plates
Stud

It takes ever higher pressure to force more water into a full tank, and ever higher voltage to force more charge onto a charged capacitor.

$\boxed{Q = CV}$ $\boxed{C = \epsilon_0 \frac{A}{d}}$

$$\underline{Q = \frac{\epsilon_0}{d} A V}$$

$$M_{water} = \frac{1}{g} A P$$



A water tank gets deeper as you
Increase pressure, but the plates
Of a capacitor don't get further apart
As you increase voltage.

“Capacitance”, sounds like “Capacity”, and it is related. A large bottomed tank has a larger fluid capacity at given pressure than a small tank. “Voltage” is like “Pressure”

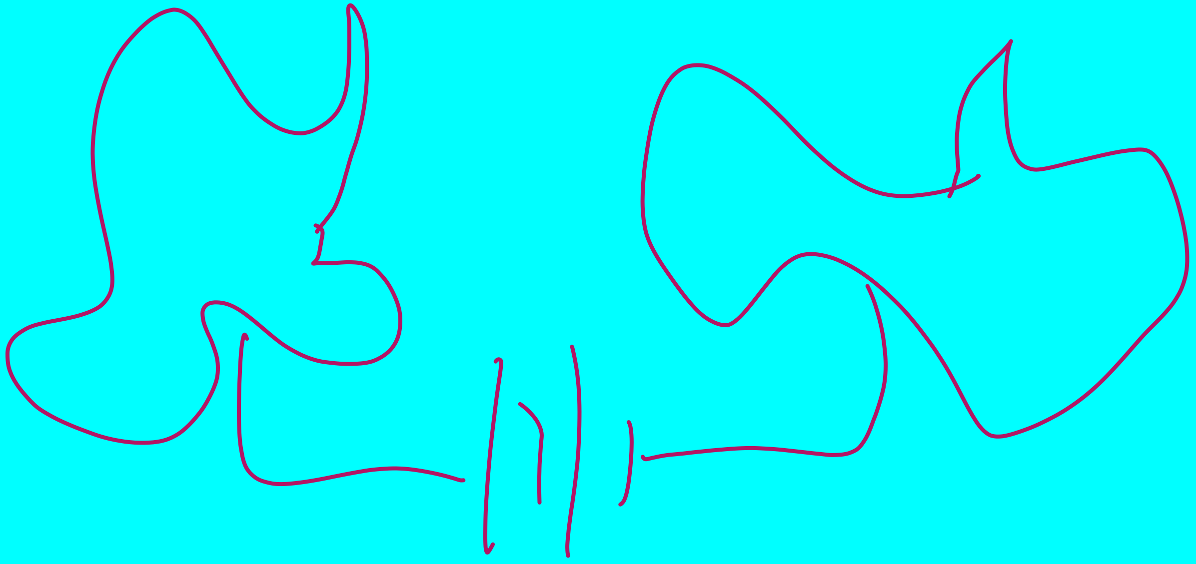
A large area capacitor has a larger charge capacity at given voltage than a small area capacitor.

$$Q = \frac{\epsilon_0}{d} A V \qquad \mathrm{Mass} = \frac{1}{g} A P$$

$$\epsilon_0 = 8.86 \times 10^{-12} \mathrm{F/m},\ (\mathrm{C/V\,m}),\ (\mathrm{J/V^2})$$

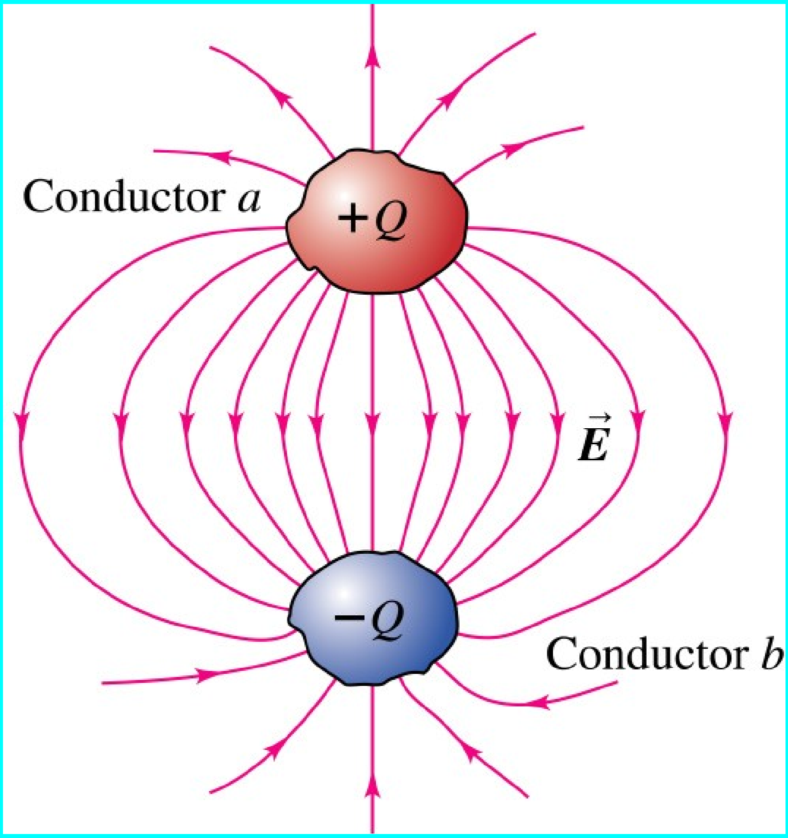
Note that the parallel plate formula only has "geometric" variables. Capacitance is a property of a set of conductors and does not depend on charge or voltage (or any other electric variables)

$$C = \frac{\epsilon_0 A}{d}$$



The two conductors *a* and *b* are insulated from each other, forming a capacitor. You increase the charge on *a* to $+2Q$ and increase the charge on *b* to $-2Q$, while keeping the conductors in the same positions.

What effect does this have on the capacitance C?



A. C is multiplied by a factor of 4

B. C is multiplied by a factor of 2

C. C is unchanged

D. C is multiplied by a factor of 1/2

E. C is multiplied by a factor of 1/4

$Q = CV$

$C = \frac{Q}{V}$

$R = \frac{V}{I}$

$C = \frac{Q}{V}$

A capacitor's plates hold 1.3 μC when it is charged to 80 V. What is its capacitance?

$$C = \epsilon_0 \frac{A}{d}$$

$$C = \frac{Q}{V}$$

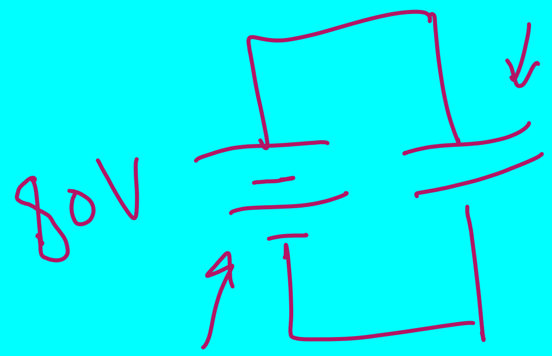
(A) 16.2 nF

(B) 16.2 μF

(C) 104 μF

(D) 80 μF

(E) 1.3 nF



$$\frac{1.3 \times 10^{-6}}{8 \times 10^{1}}$$

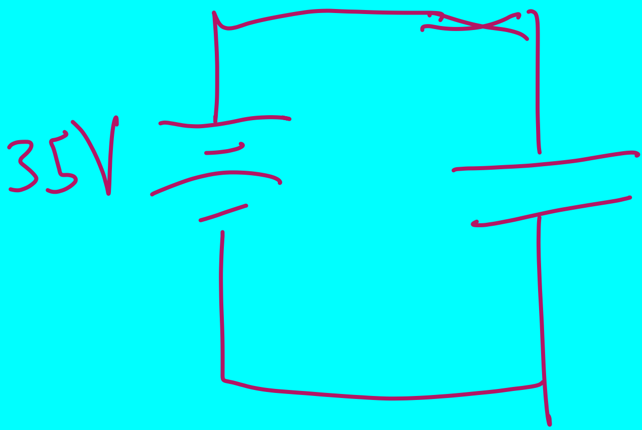
0.162×10^{-7}

1.62×10^{-8}

16.2×10^{-9}

[Ex. 3] A stereo receiver contains a 2500 μF capacitor charged to 35 V.
How much energy does it store?

$$C = \epsilon_0 \frac{A}{d}$$



$$U = \frac{1}{2}(2.5\times10^{-3}\,F)(35)^2$$

$$= \left(\frac{1}{2}\right)(2.5\times10^{-3})(10^3)$$

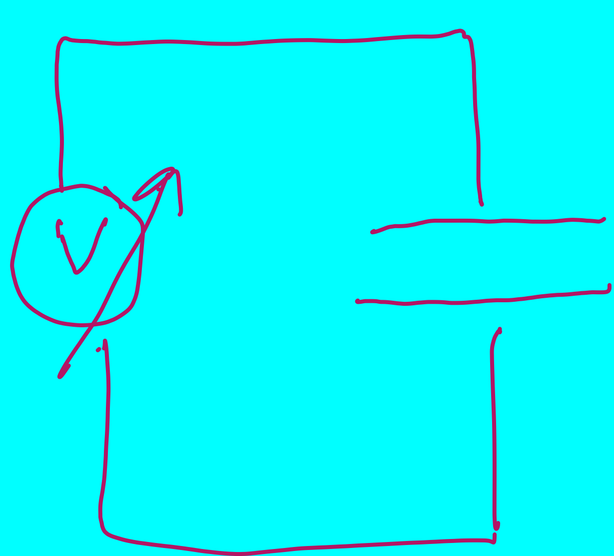
$$= 1.25\ J$$ ← In my head

$$C = \frac{Q}{V}$$

Correct 1.53 J

$$U = \frac{1}{2} C V^2$$

$$Q = CV$$



$$Q = CV$$

$$U = QV$$

$$dU = V\,dQ$$

$$\rightarrow Q = CV$$

$$\underline{dQ} = \cancel{V\,dC} + \underline{C\,dV}$$

$$dU = VC\,dV$$

$$\int_0^U dU' = C\int_0^V V'\,dV'$$

$$U = \frac{1}{2}CV^2$$

$$\frac{dU}{dV} = CV$$

You reposition the two plates of a capacitor so that the capacitance doubles.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, what happens to the potential difference V_{ab} between the two plates?

A. V_{ab} is multiplied by a factor of 4

B. V_{ab} is multiplied by a factor of 2

C. V_{ab} is unchanged

D. V_{ab} is multiplied by a factor of 1/2

E. V_{ab} is multiplied by a factor of 1/4

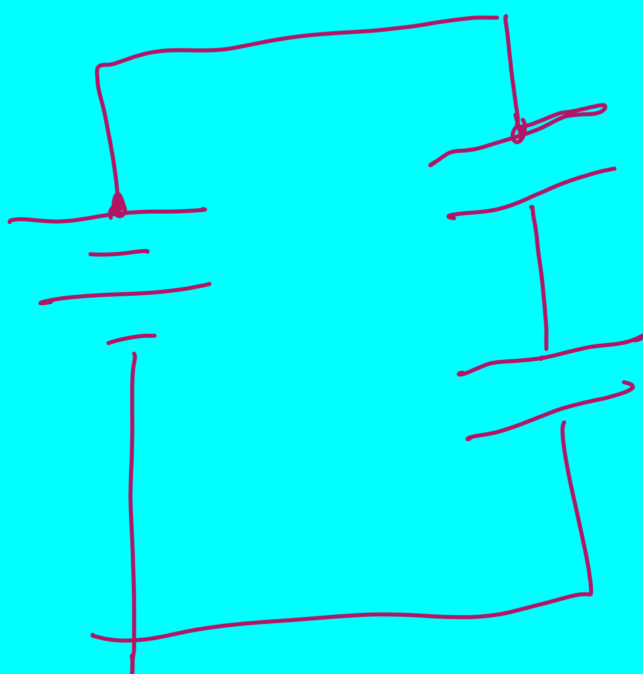
$$Q = CV$$

Equivalent or Effective Capacitance

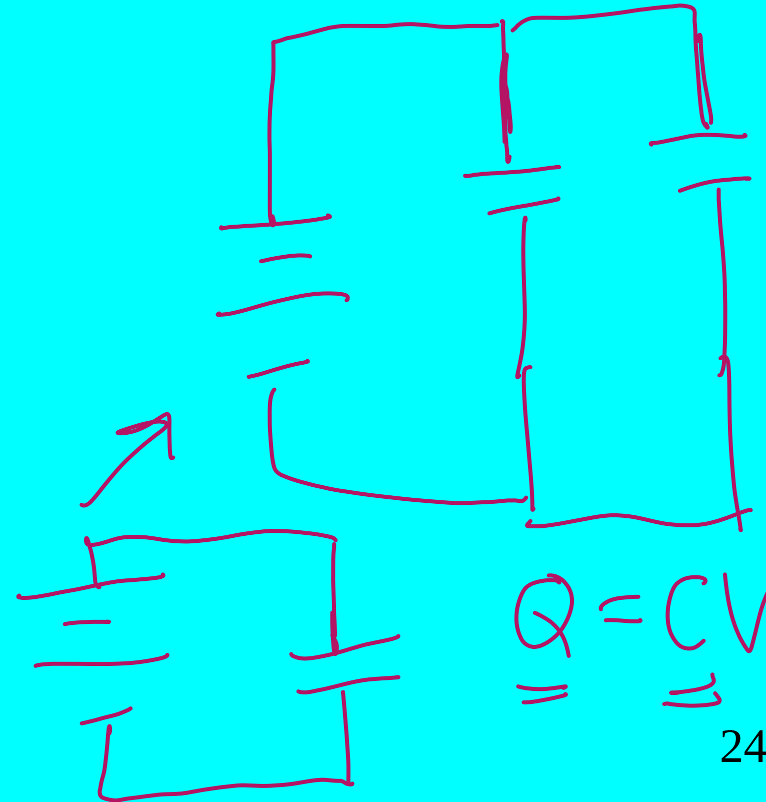
Combine all of the capacitors in a circuit into one capacitor.

$$Q = C_{equivalent} V_{total}$$

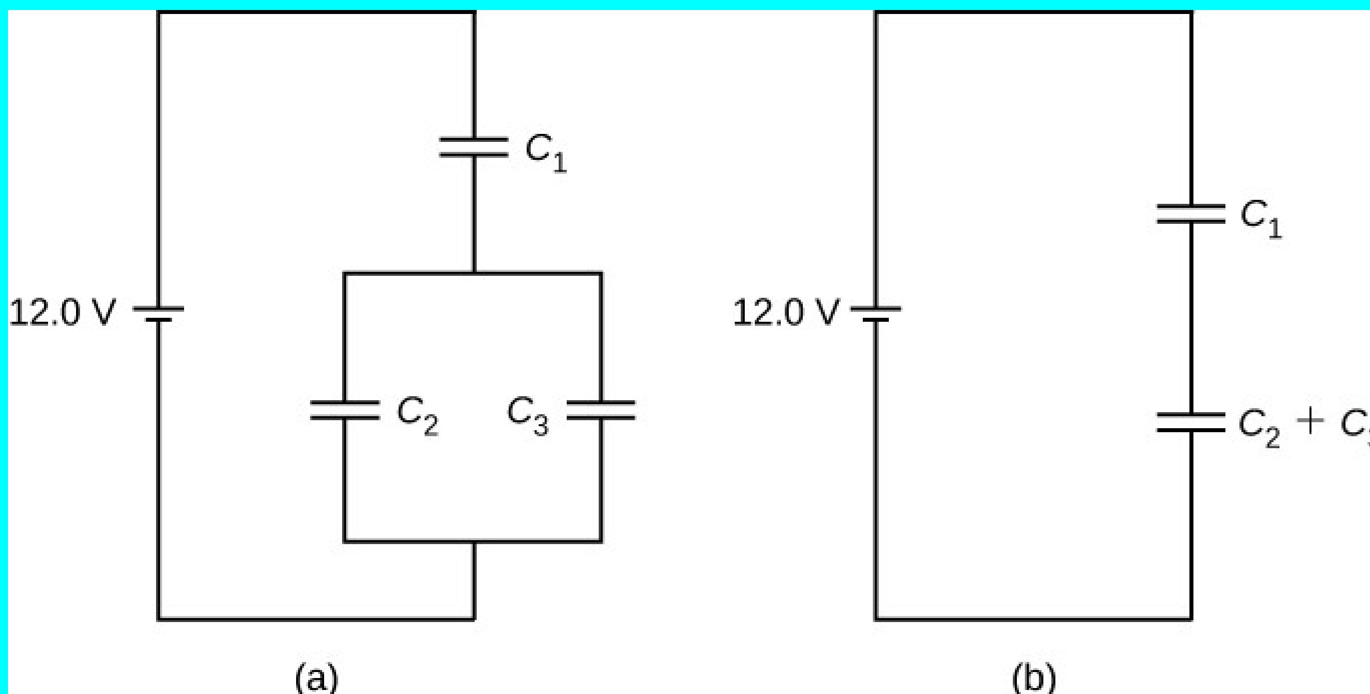
Parallel



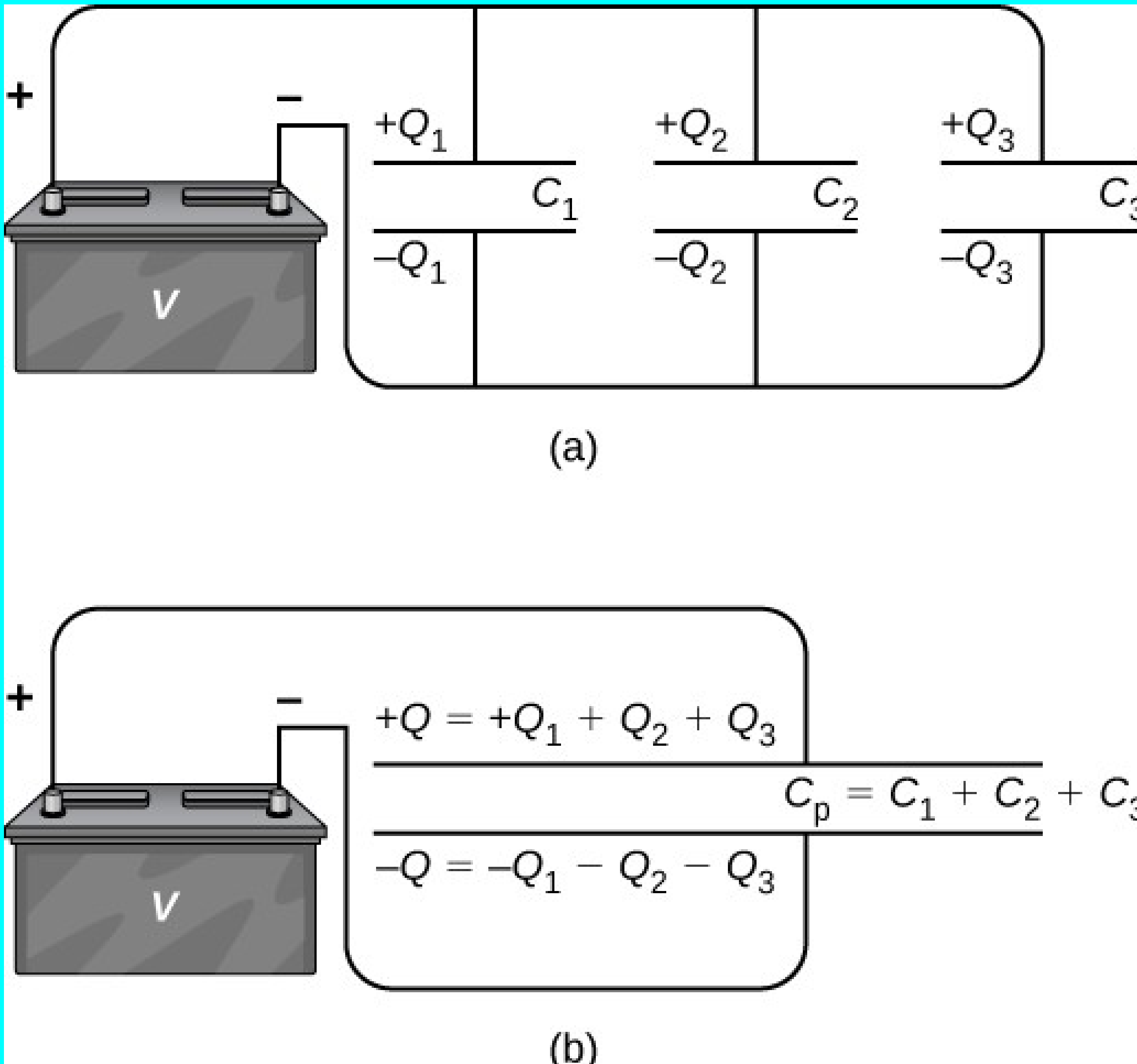
Series



Q = CV


12.0 V
C1
C2
C3
(a)
12.0 V
C1
C2 + C3
(b)

Equivalent or Effective Capacitance (Parallel Circuit)



$$Q_{total} = Q_1 + Q_2 + Q_3$$

$$Q_{total} = C_{equiv} V$$

$$(Q_1 + Q_2 + Q_3) = C V$$

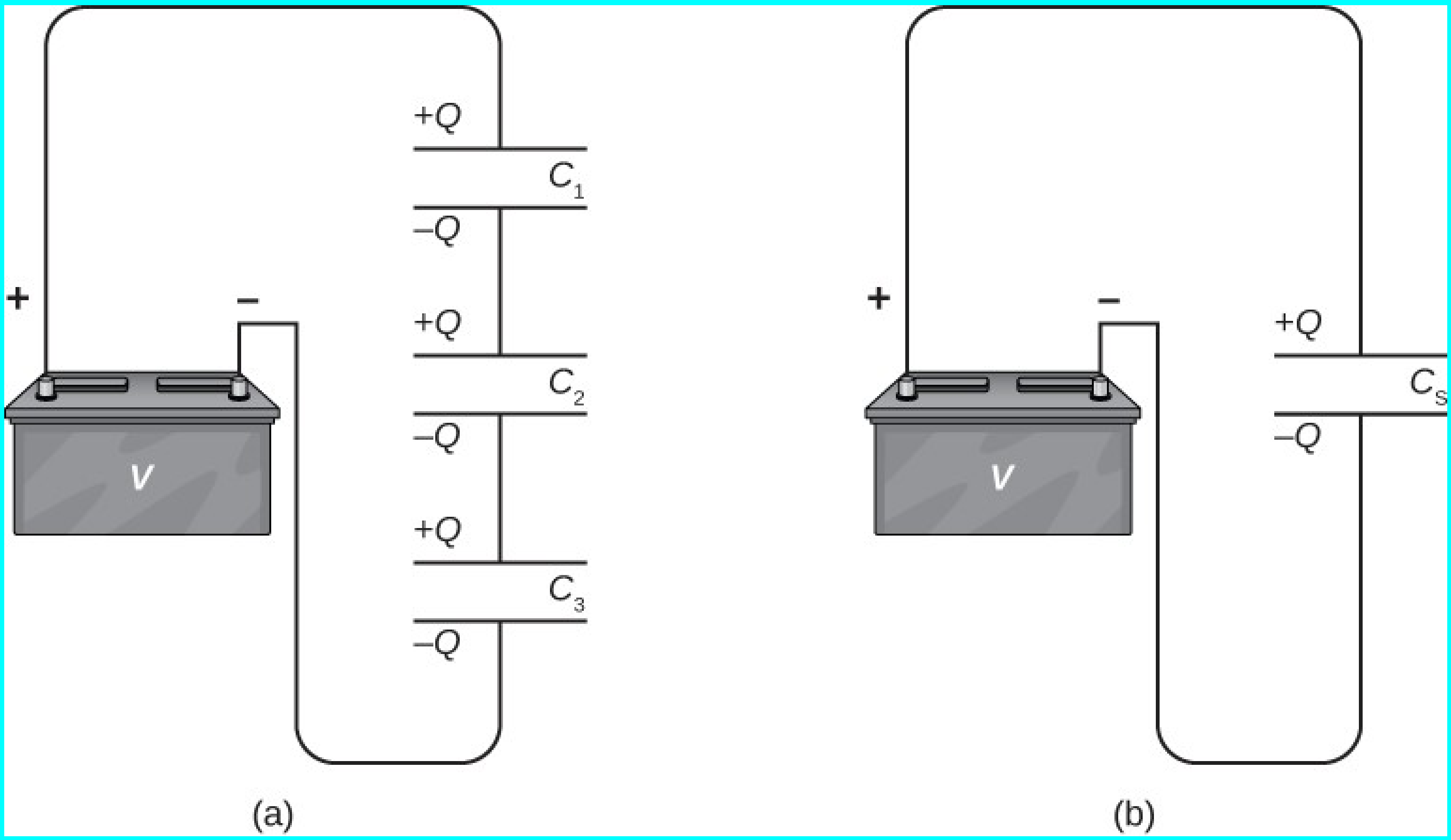
$$Q_1 = C_1 V$$

$$Q_2 = C_2 V \rightarrow (C_1 + C_2 + C_3) V$$

$$Q_3 = C_3 V$$

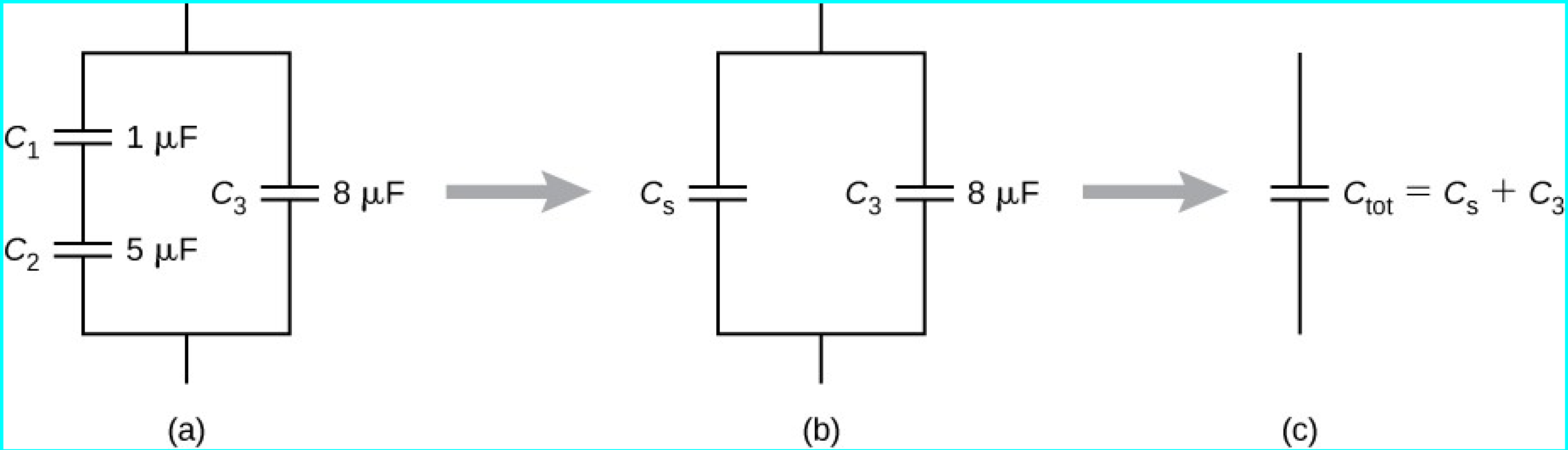
$$Q = C_{equivalent} V_{total}$$

Equivalent or Effective Capacitance (Series Circuit)



$$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

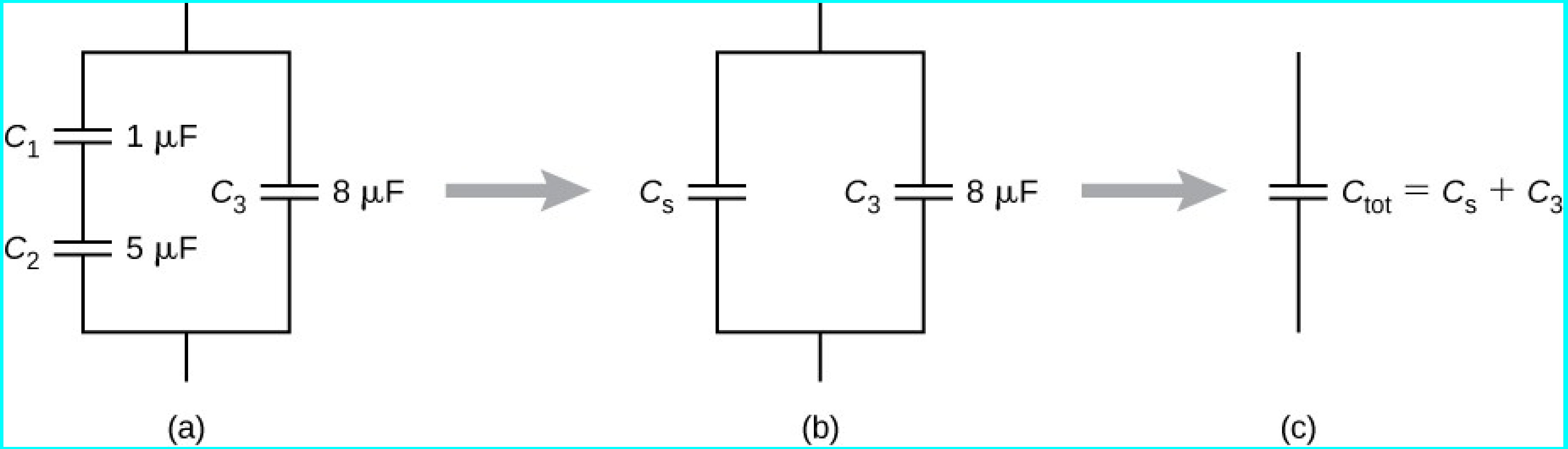
$$Q = C_{equivalent} V_{total}$$



C_1
1 μF
C_2
5 μF
C_3
8 μF
(a)
C_s
C_3
8 μF
(b)
$C_{tot} = C_s + C_3$
(c)

Example

What is C effective for this circuit?



(A) 1.2 μF

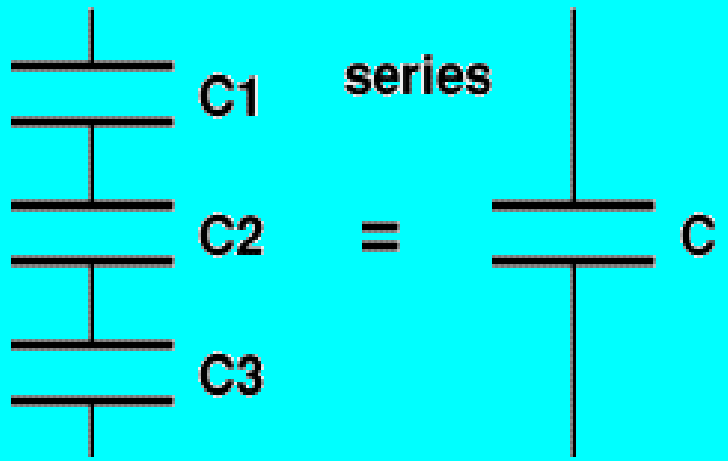
(B) 9.2 μF

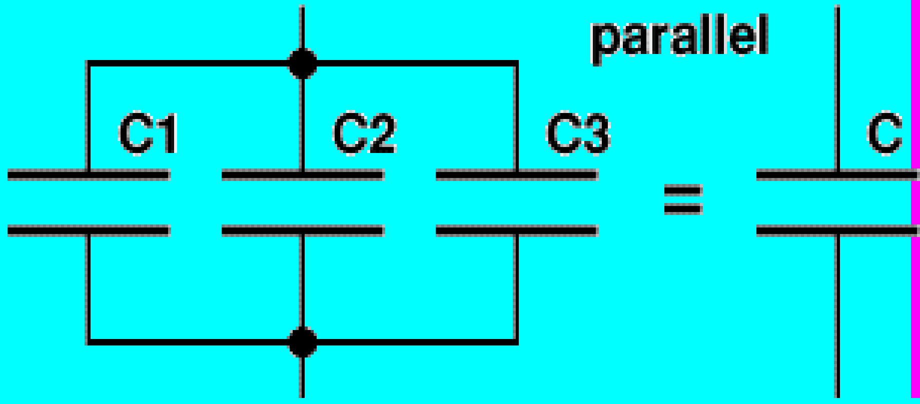
(C) 0.83 μF

(D) 14 μF

(E) 8.83 μF

Capacitors in Series and Parallel

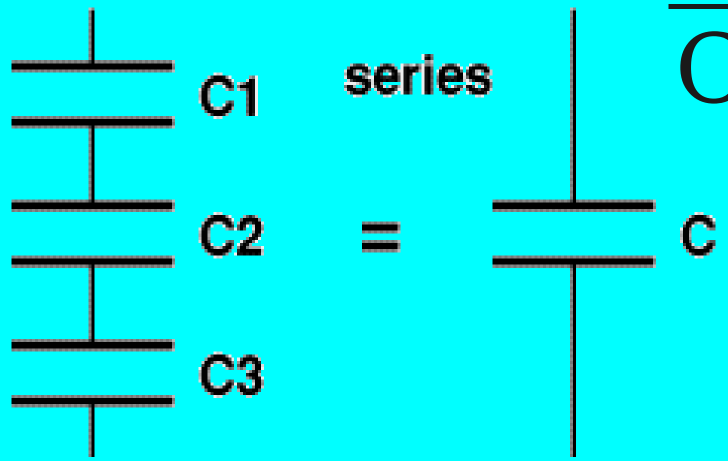




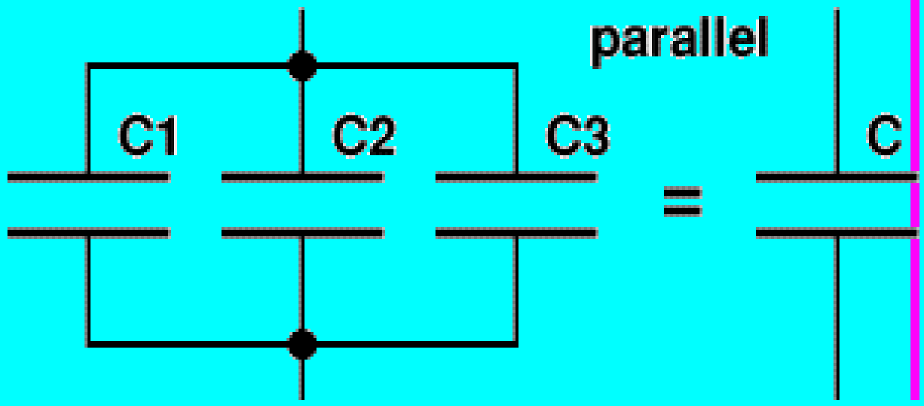
Caps in series have the
“bottom wire” of one
Connected to the “top wire”
Of the next.
Caps in series have equal Q’s

Caps in parallel have all
Their tops connected to
One wire and all their
Bottoms also connected to
A second wire.
Caps in parallel have equal
V’s.

Capacitors in Series and Parallel



$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

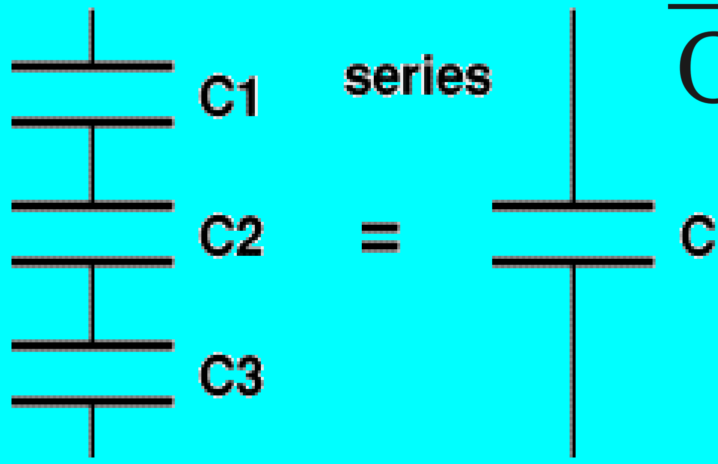


$$C_{parallel.} = C_1 + C_2 + C_3 + \dots$$

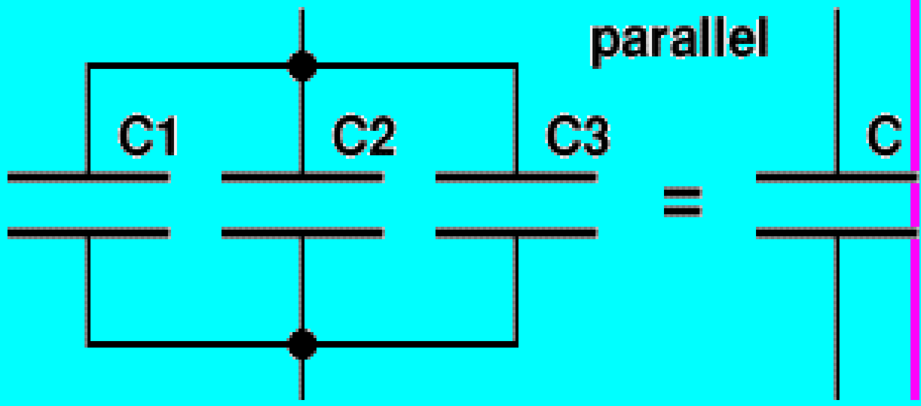
Caps in parallel have equal voltages.

Caps in series have equal charges.

Capacitors in Series and Parallel



$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$



$$C_{parallel.} = C_1 + C_2 + C_3 + \ldots$$

Caps in parallel have equal voltages.

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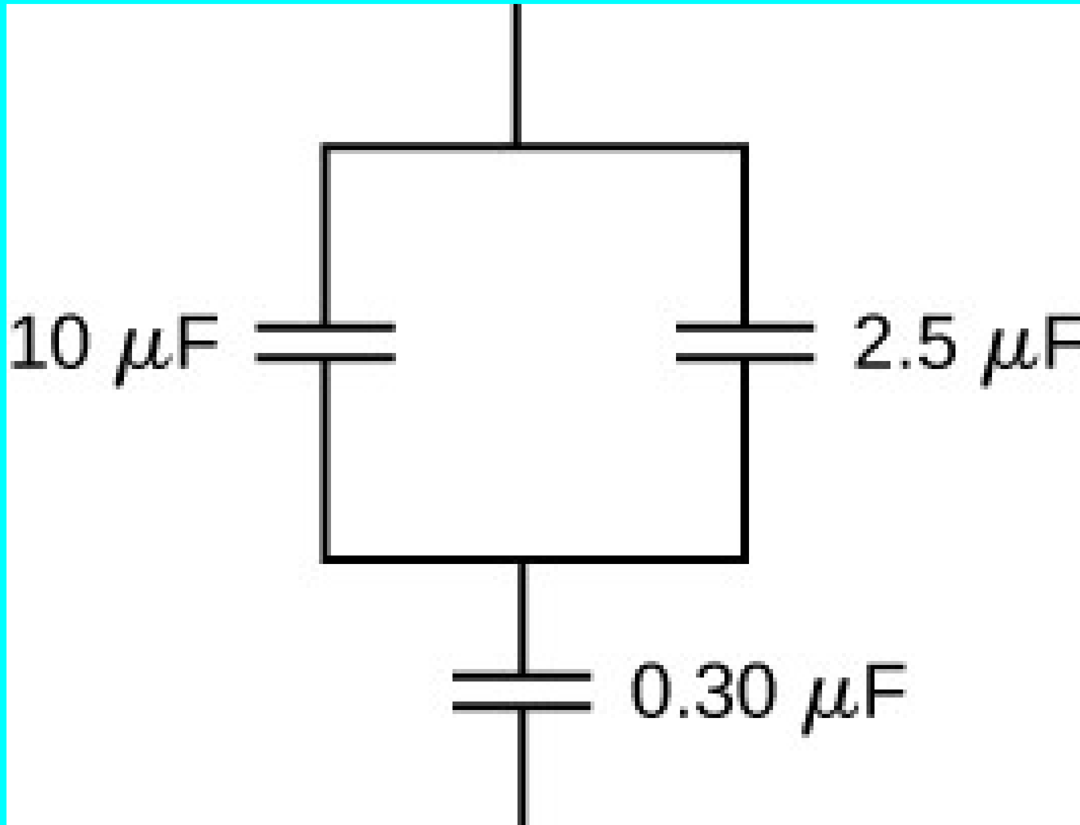
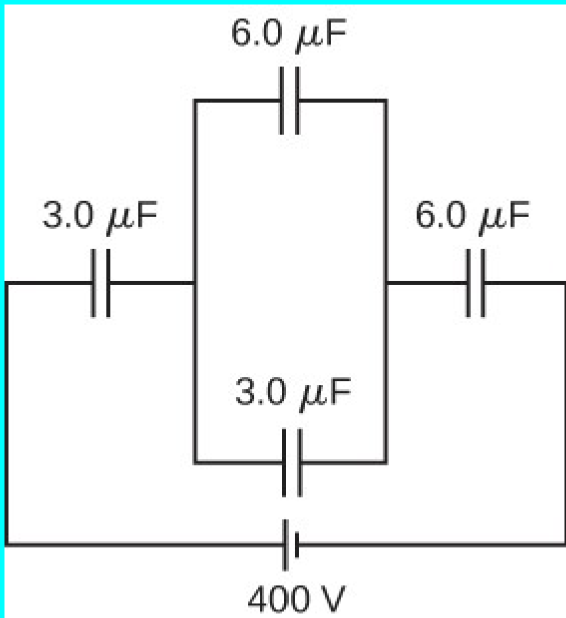
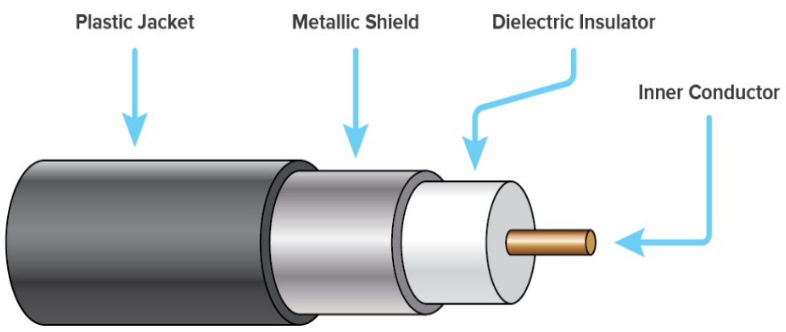


10 μF
2.5 μF
0.30 μF


6.0 μF
3.0 μF
6.0 μF
3.0 μF
400 V

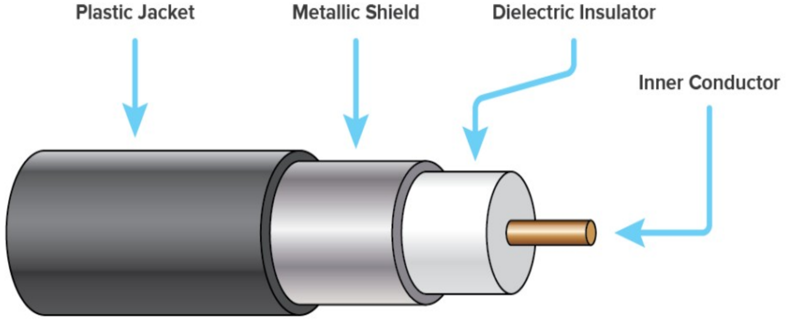
Written HW4 Problem 6:

6. A coaxial cable consists of a 2.0-mm diameter inner conductor and an outer conductor of diameter 1.6 cm. If the conductors carry line charge densities of $\pm 0.67 nC/m$, what is the magnitude of the potential difference between them?



What is the capacitance of this cable?

6. A coaxial cable consists of a 2.0-mm diameter inner conductor and an outer conductor of diameter 1.6 cm. If the conductors carry line charge densities of $\pm 0.67 nC/m$, what is the magnitude of the potential difference between them?



What is the capacitance of this cable?

Water current is total mass that passes an observer per second.

Electrical current is charge flow rate past a fixed point.
Units (C/s)

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Hydraulic Analogy

http://en.wikipedia.org/wiki/Hydraulic_analogy

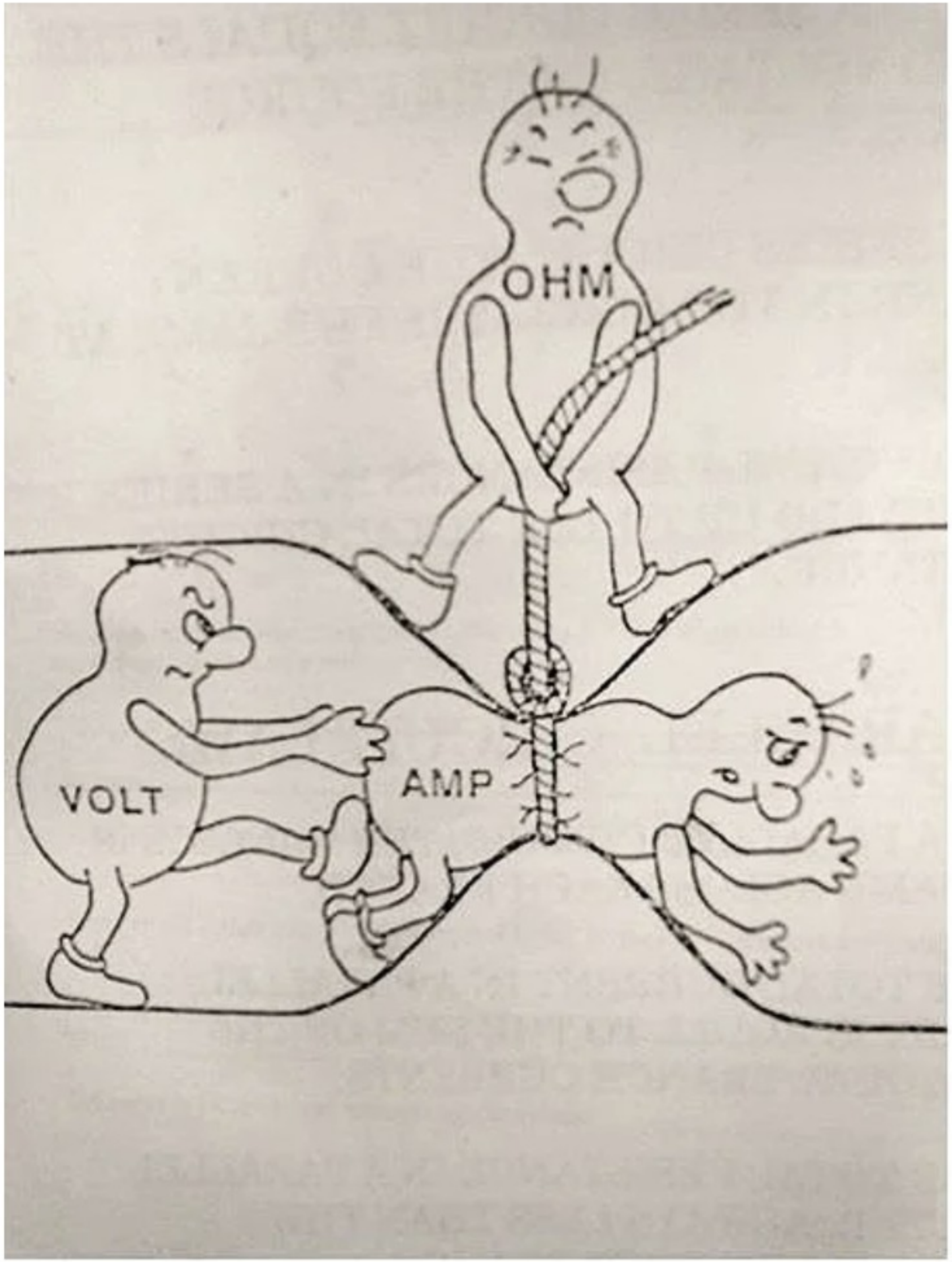
Mass of water (M)	Charge (Q)
Water current (dM/dt)	Current ($I = dQ/dt$)
Water pressure (P)	Voltage (V)

Posted by u/arbili 7 years ago

1.7k

Electricity explained

i.imgur.com/HsUd5e...



$$I = \frac{V}{R}$$

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Lecture 16 Recap

We learned about parallel plate capacitors with dielectrics
We learned how to combine multiple capacitors into an “effective” or “equivalent” capacitance.

Energy Formula $\quad U = \frac{1}{2} C V^2$