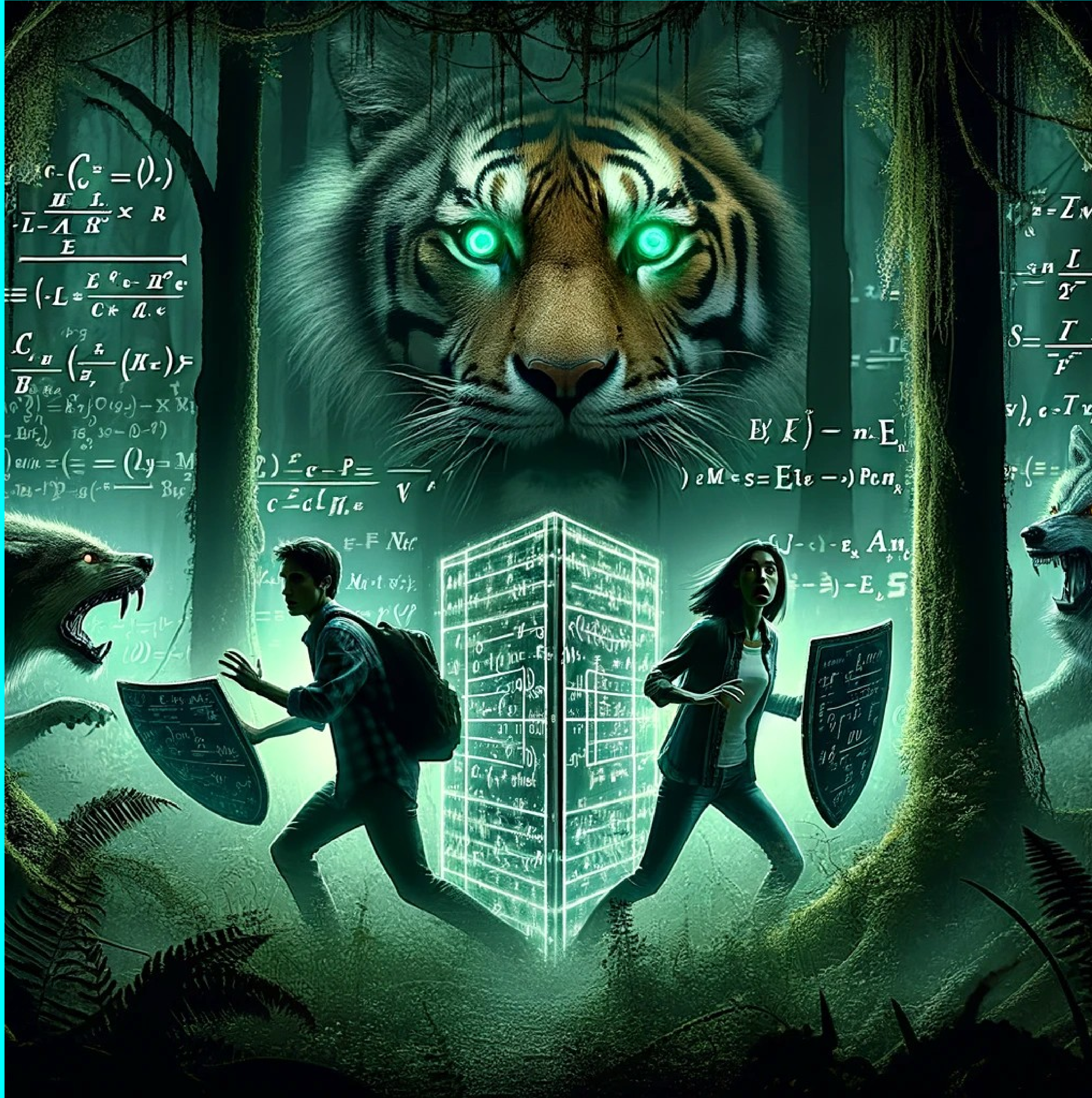
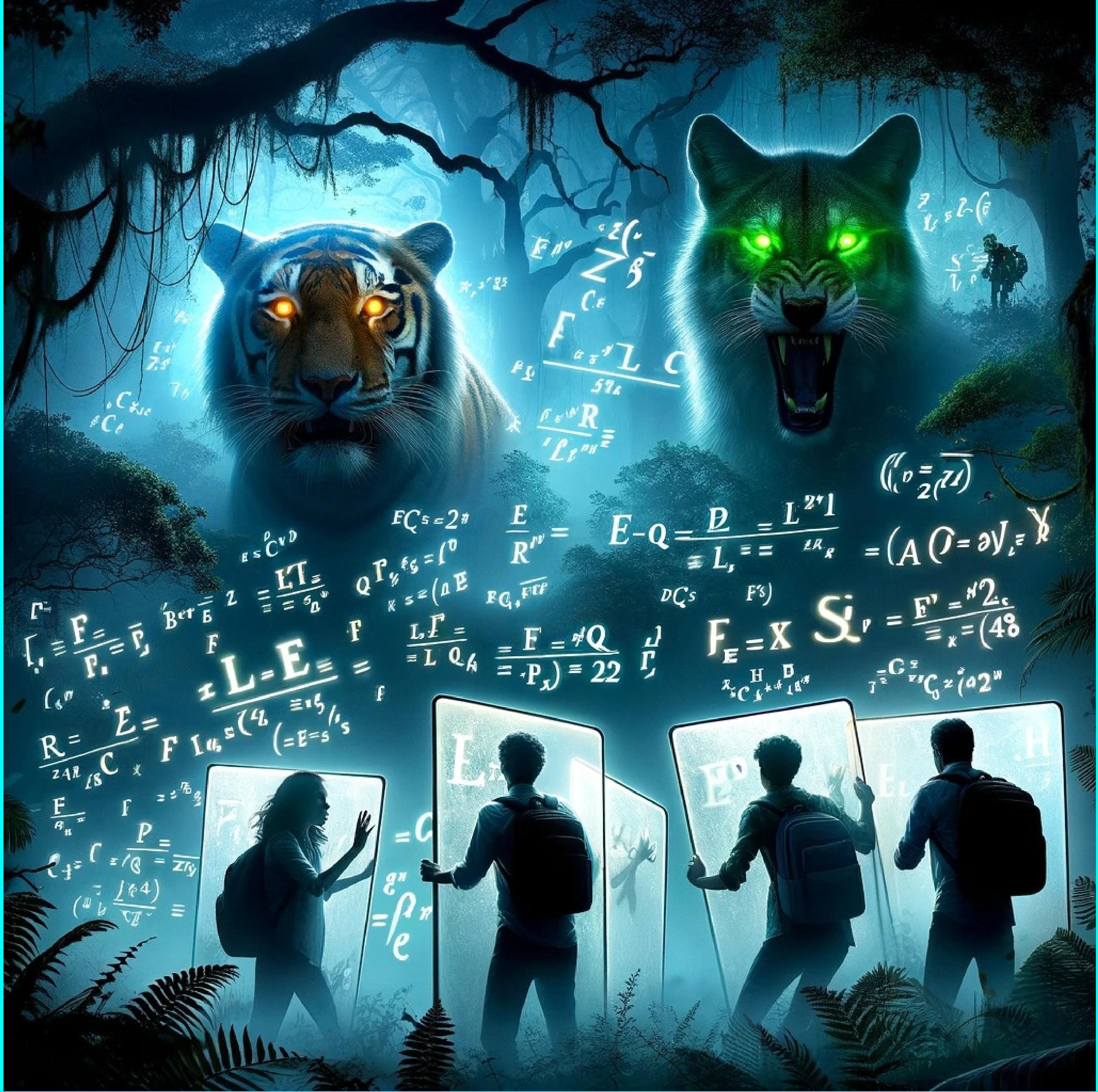
Lecture 15: 03/04/2024

- Announcements
  Written HW4 due today, We will do problem 6 (which is like 11)
  Written HW5 due next Monday. No online.

- Last Time
  - Electric potential of point charges
  - Potential from field
  - Field from potential

- Today
  The textbook can help you
  Problem 4-6 – Potential of a wire, and a sphere
  What is capacitance
  Parallel plate capacitor
  Current and Ohm's Law





The Story to come:

8.1: Capacitors and Capacitance
8.2: "Caps" in series and parallel
8.3: Energy in a capacitor

9.1: Electric Current
9.2: Origin of resistance
9.3: Resistivity and resistance
9.4: Ohm's Law

10.1: EMF
10.2: Resistors in Series/Parallel
10.3: Kirchoff's Rules

11.1: Magnetism
11.2: Magnetic Fields
11.3: Magnetic Forces

The Story so far:

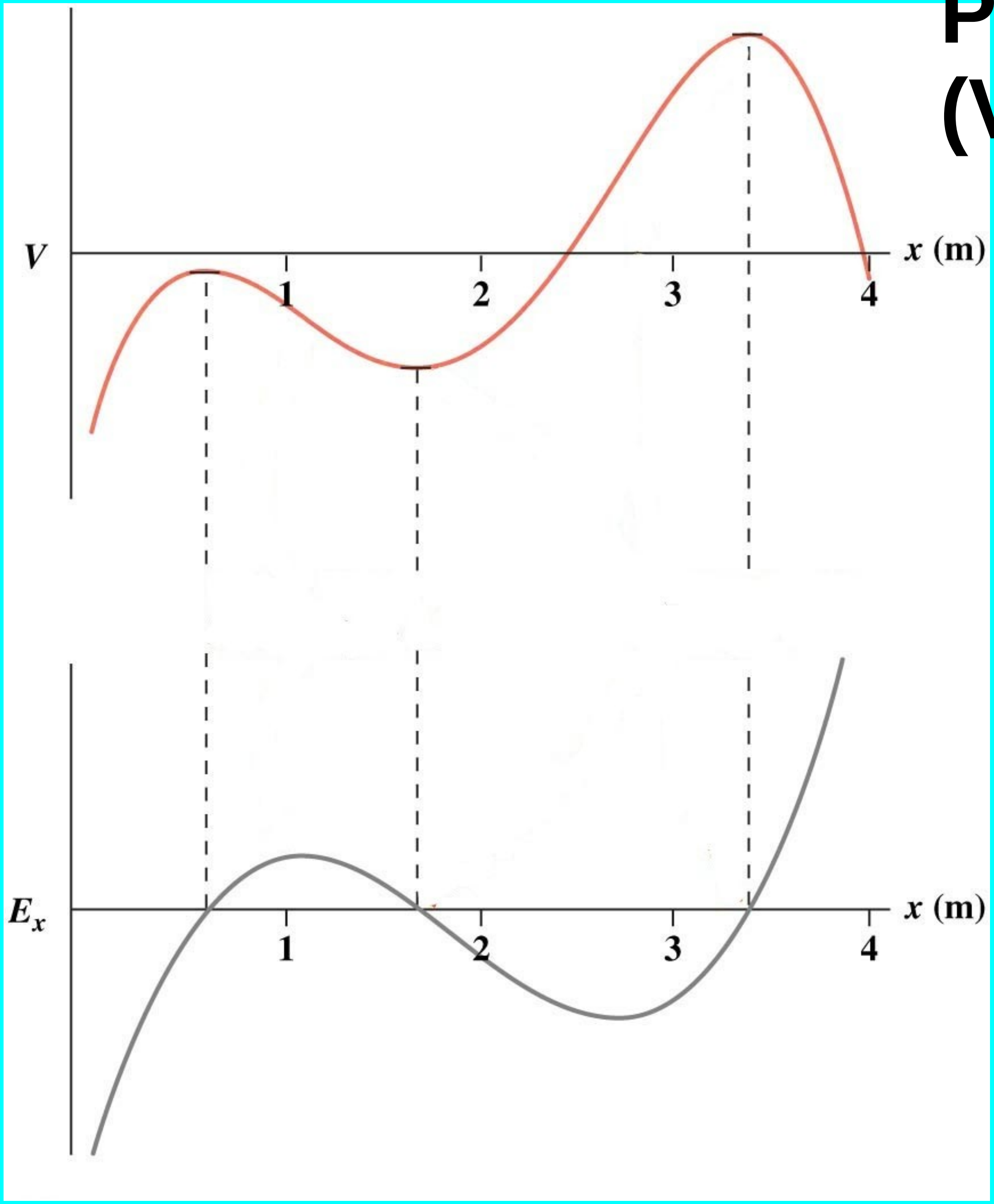
5.1: Electric Charge
5.2: Conductors and Insulators
5.3: Coulomb's Law
5.4: Electric Field

6.1: Electric Flux
6.2: Explaining Gauss' Law
6.3: Applying Gauss' Law

7.1: Electric Potential Energy (U, or "P.E.")
7.2: Electric Potential (V)
7.3: Calculating V
7.4: Determining Field from V

$\Delta V =$

Potential (Voltage) in 1-D

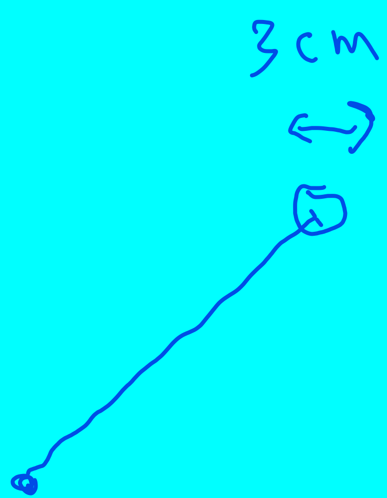


$$V(x) = -\int E\,dx$$

$$E_x = -\frac{dV}{dx}$$

Written HW4 Problem 11:

11. A power line with diameter 3.0 cm is at a voltage of 4.0 kV relative to a point one meter away. What is the line charge density (λ) of the power line?



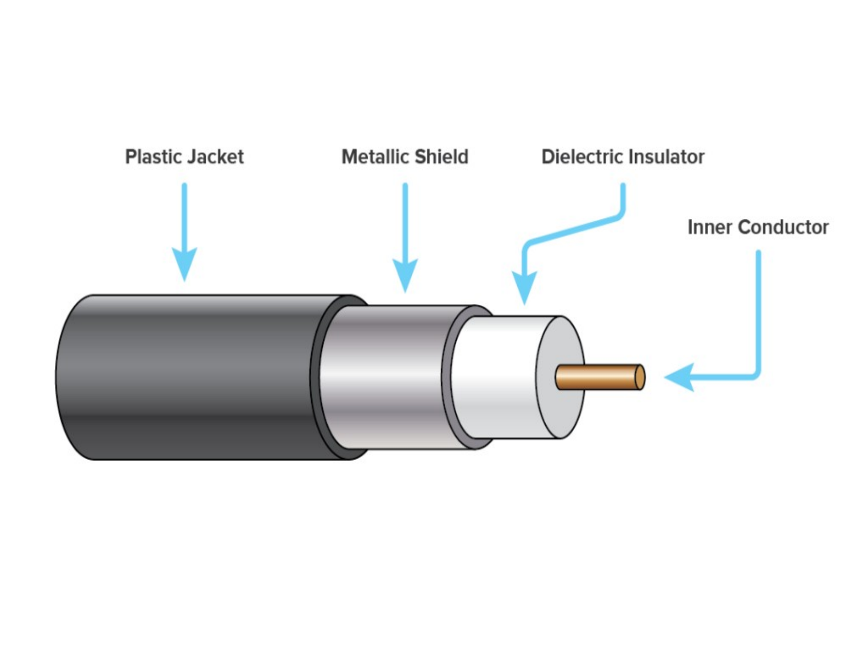
$r_2 = 1$ m
$r_1 = 1.5$ cm

Written HW4 Problem 11:

11. A power line with diameter 3.0 cm is at a voltage of 4.0 kV relative to a point one meter away. What is the line charge density (λ) of the power line?

Written HW4 Problem 6:

6. A coaxial cable consists of a 2.0-mm diameter inner conductor and an outer conductor of diameter 1.6 cm. If the conductors carry line charge densities of $\pm 0.67 nC/m$, what is the magnitude of the potential difference between them?



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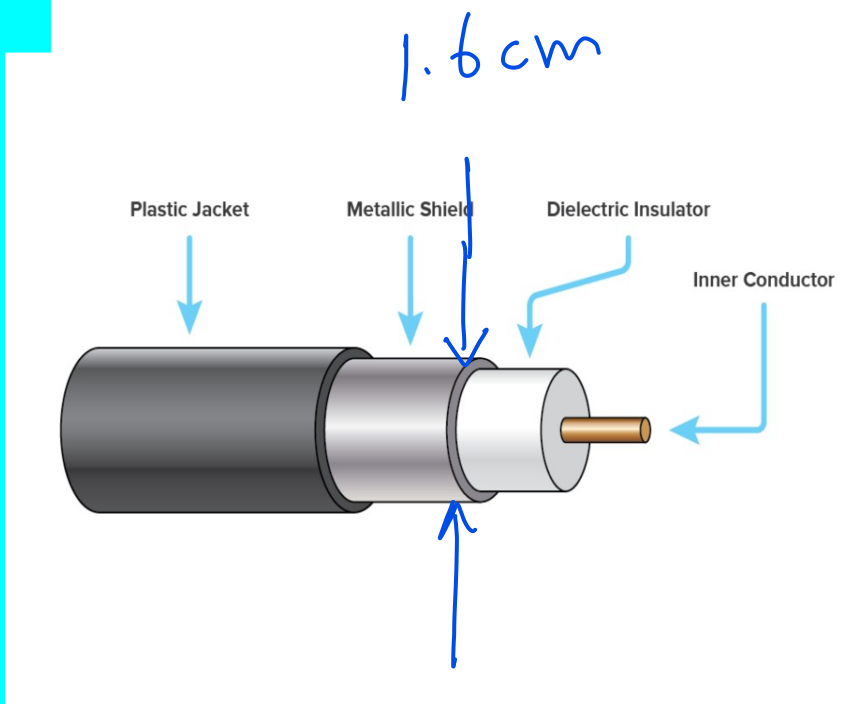
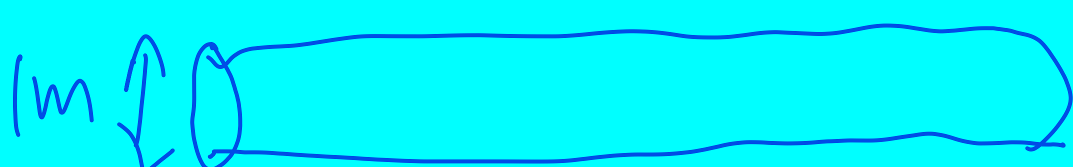


2mm

$r_1 = 1 mm$

$r_2 = 8 mm$

$$V = -\int_{r_1}^{r_2} \vec{E}\, dx$$



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$$|V| = \int_{r_1}^{r_2} E\,dr = \int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r}\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_1}^{r_2} r^{-1}\,dr$$

$$E_{line} = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r} \qquad = \frac{\lambda}{2\pi\epsilon_0}\,\frac{r^0}{0}$$

$$|V| = \frac{\lambda}{2\pi\epsilon_0}\ln r\Big|_{r_1}^{r_2} = \frac{\lambda}{2\pi\epsilon_0}(\ln r_2 - \ln r_1) = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_2}{r_1}$$

$$|V| = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{.8}{.1} = \frac{\lambda}{2\pi\epsilon_0}\ln 8$$

Voltage from Electric Field:

If the electric field is constant, then: $\Delta V = -E \Delta x$

If it depends on x: $\Delta V = -\int_a^b E \, dx$

If it's constant but not along $\Delta \vec{r}$: $\Delta V = -\vec{E} \cdot \Delta \vec{r}$

If it depends on x, y, and z: $\Delta V = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{r}$

Potential of a wire:

Potential is a scalar!! Superposition is EASY!!

$$\Delta V_{r\infty} = k\frac{Q}{r}$$

$$E = k\frac{Q}{r^2}\hat{r}$$

Potential of a point or sphere:

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/potsph.html

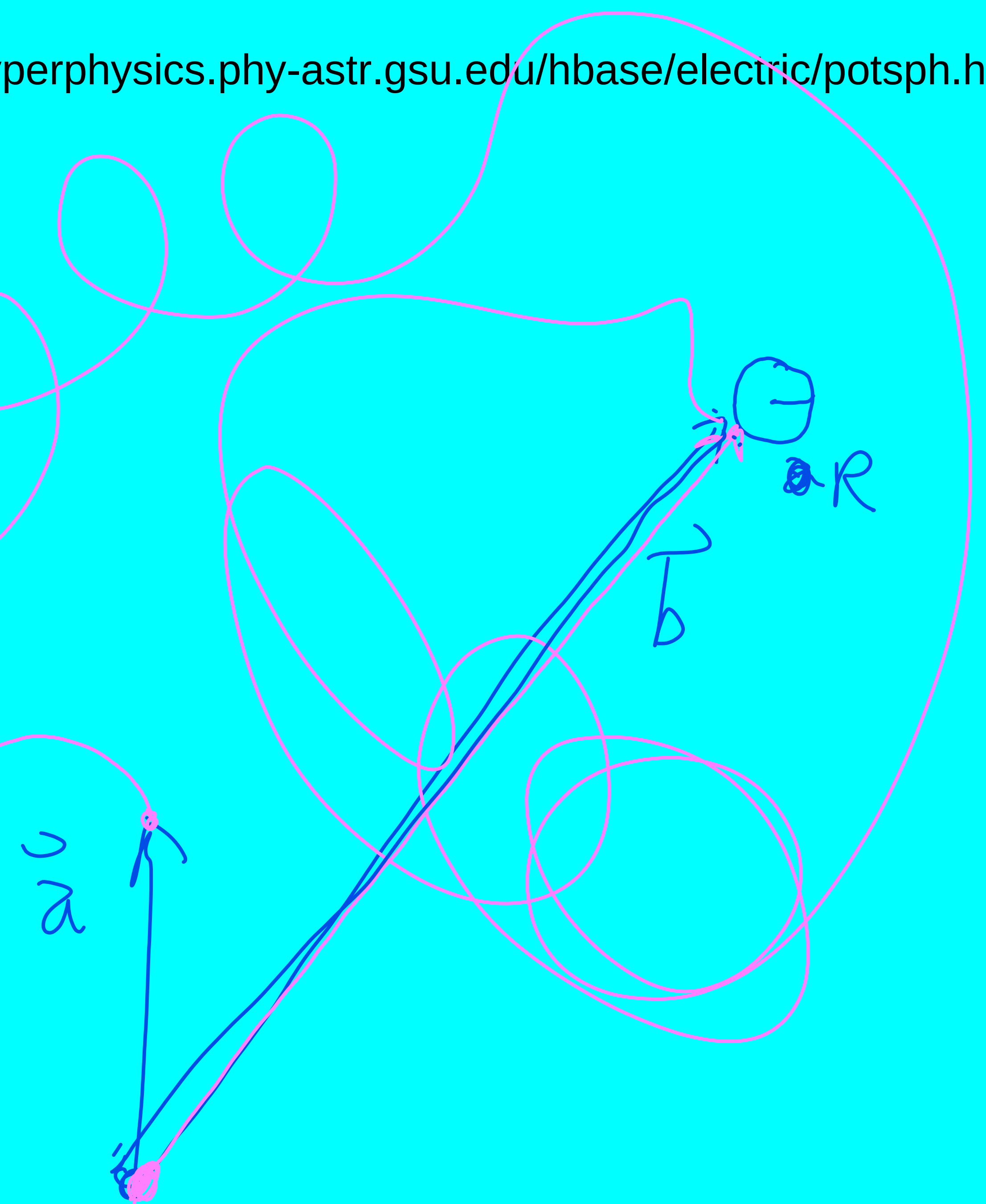
$$V = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{r}$$

$$V = -\int_{a}^{R} \frac{kq}{r^2}\,dr$$

$$V = -\int_{\infty}^{R} \frac{kq}{r^2}\,dr$$

$$V = \left.\frac{kq}{r}\right|_{\infty}^{R}$$

$$\boxed{V = \frac{kq}{R}} - \frac{kq}{\infty}$$



Potential of point charges … Why?

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

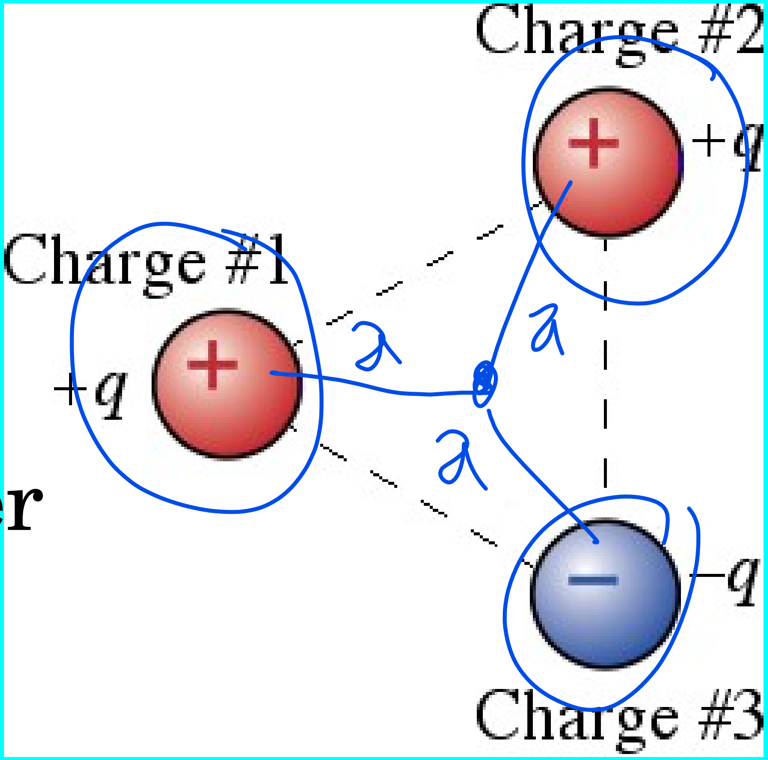
for this system, we have

$$V_r = -\int_\infty^r \frac{kq}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} dr,$$

which simplifies to

$$V_r = -\int_\infty^r \frac{kq}{r^2}\, dr = \frac{kq}{r} - \frac{kq}{\infty} = \frac{kq}{r}.$$

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is



A. positive

B. negative

C. zero

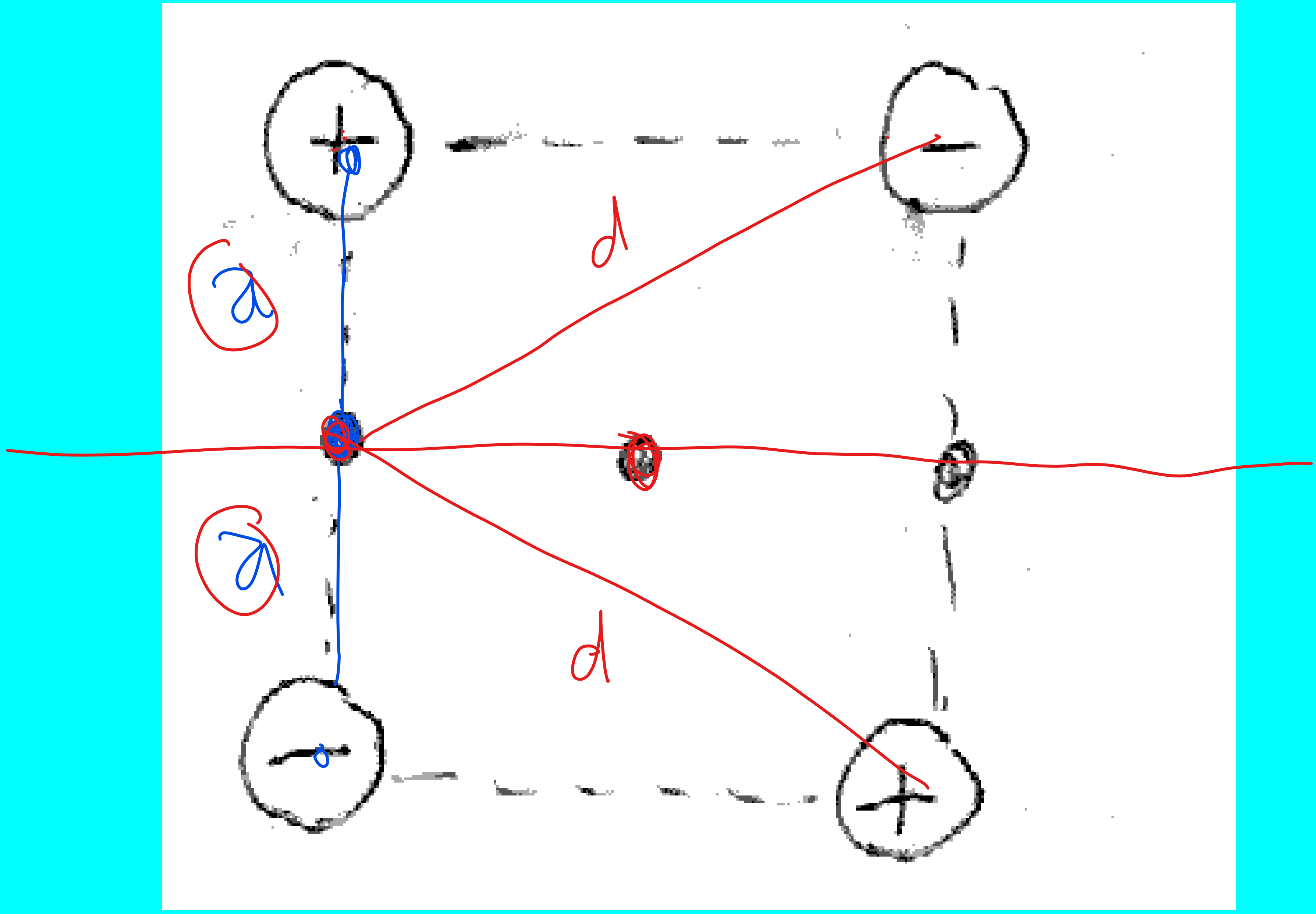
D. not enough information given to decide

$$V = \frac{kq}{a} + \frac{kq}{a} - \frac{kq}{a}$$

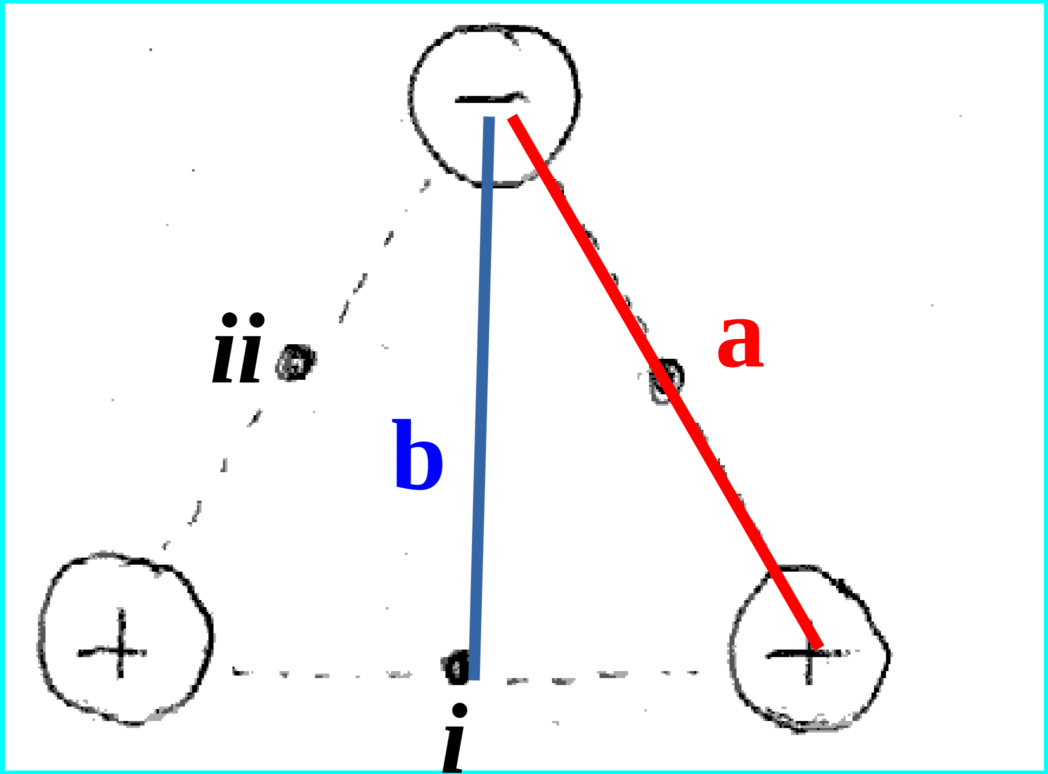
$$= \frac{kq_1}{a} + \frac{kq_2}{a} + \frac{kq_3}{a}$$

1μC 1μC −1μC

Superposition (Again):



What is V at point i?



(A) $4\frac{kq}{a} - \frac{kq}{b}$

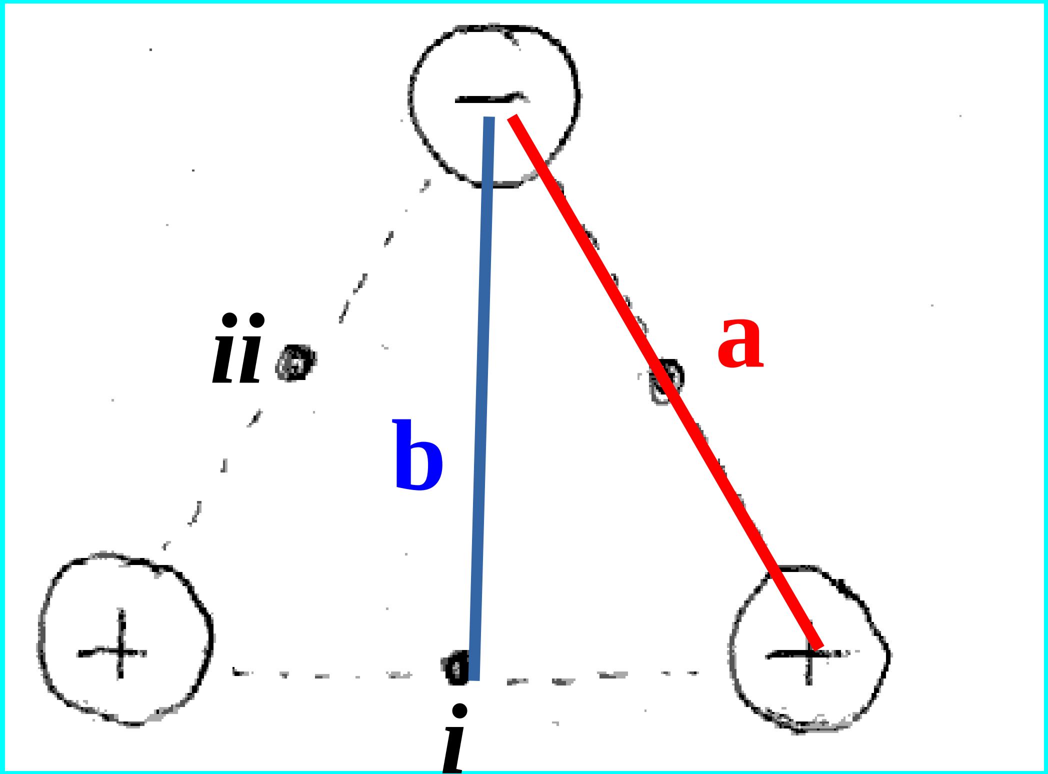
(B) $2\frac{kq}{a} - \frac{kq}{b}$

(C) $2\frac{kq}{a}\hat{i} + \frac{kq}{b}\hat{j}$

(D) $-\frac{kq}{b}$

(E) $\frac{kq}{b}$

What is V at point ii?



(A) $4\dfrac{kq}{a} - \dfrac{kq}{b}$

(B) $2\dfrac{kq}{a} - \dfrac{kq}{b}$

(C) $2\dfrac{kq}{a}\hat{i} + \dfrac{kq}{b}\hat{j}$

(D) $-\dfrac{kq}{b}$

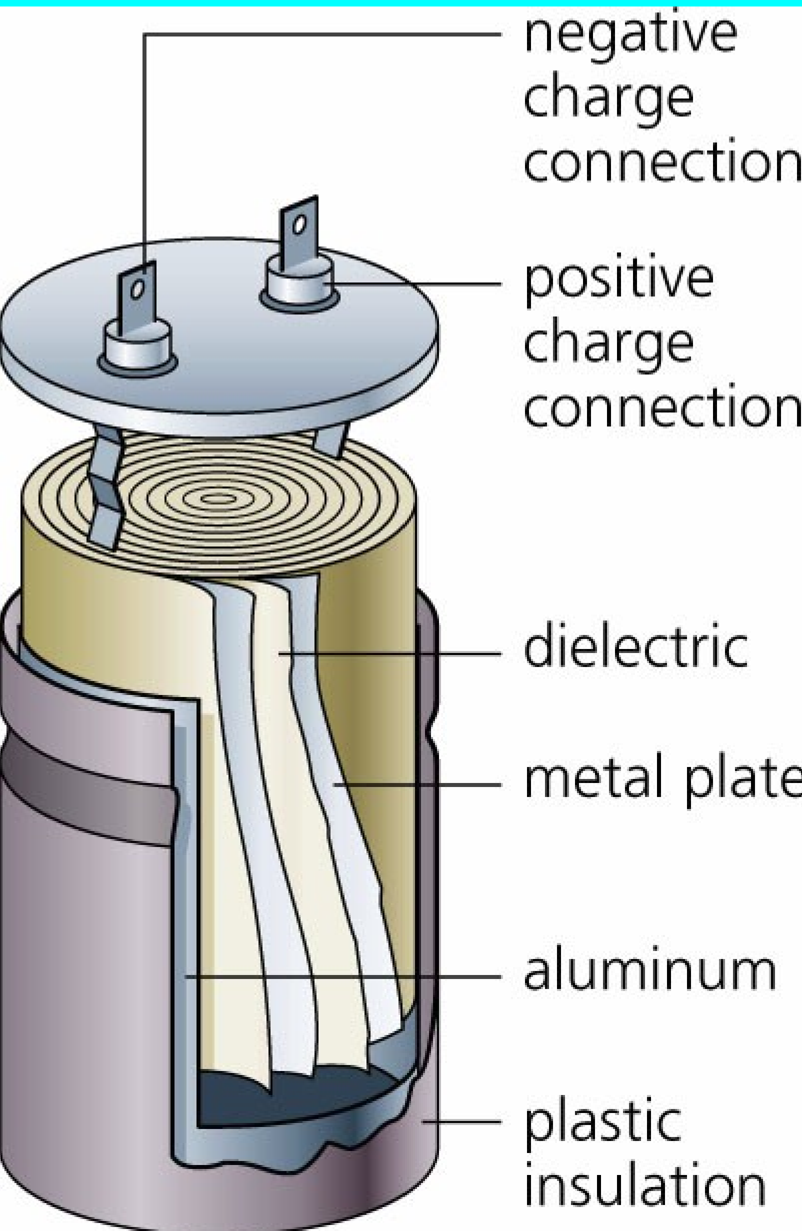
(E) $\dfrac{kq}{b}$

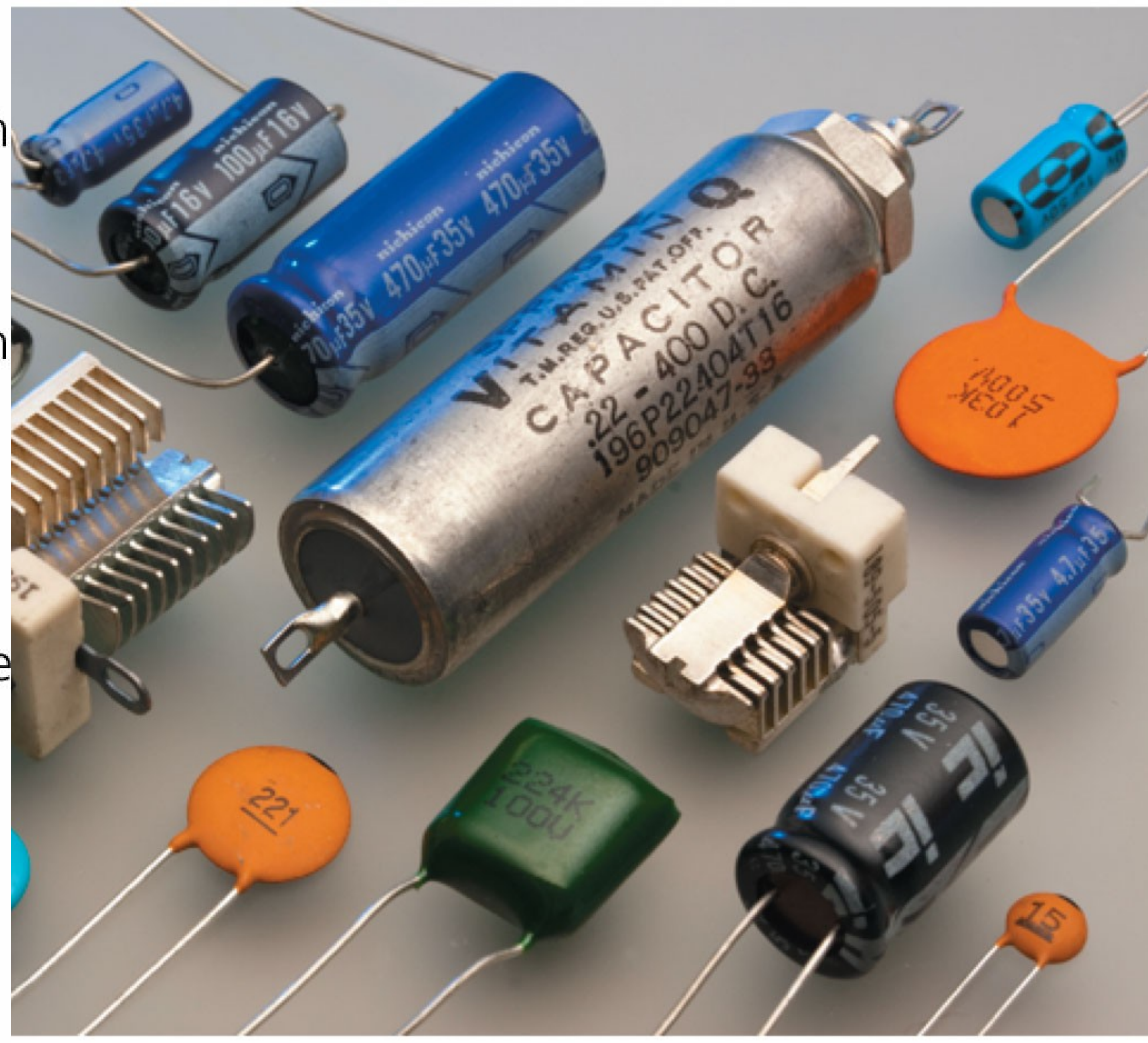
Appearance of commercial capacitors

$Q = CV$

CAPICATANCE

C = FARAD





Chapter 8

$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$

Big Idea: All electric fields represent stored energy. And capacitors are convenient containers for electric fields.

$\epsilon_0 = \frac{C^2}{m^2 \cdot N}$

Definition: Capacitance relates Voltage to stored charge. Generally voltage is easier to measure than charge.

$\epsilon_0 = F/m$

$= 8.85 \times 10^{-12} F/m$

$= 8.85\ pF/m$

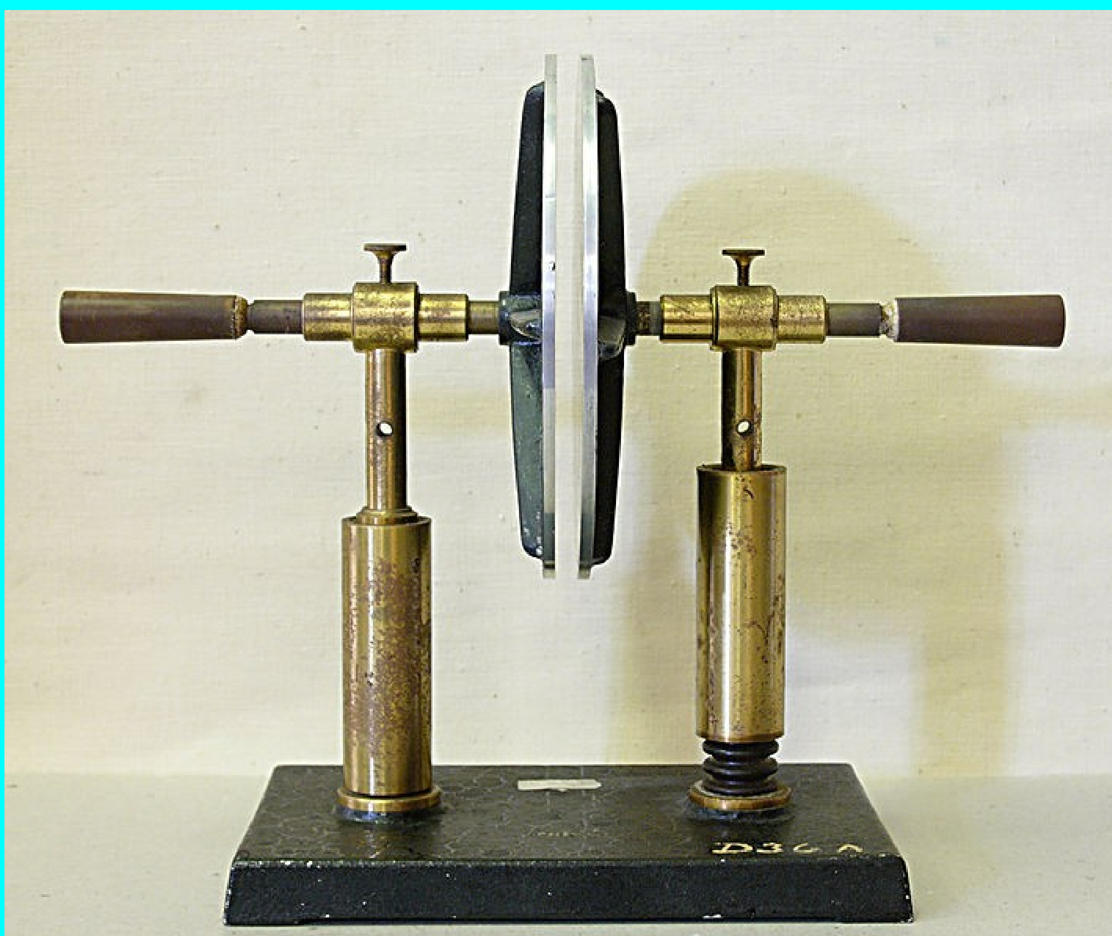
$$C = \frac{Q}{V} \qquad Q = C\,V$$

New SI Unit: "Farad". [C]=[F][V]
(Coulomb=Farad x Volt)

"puff"

It takes ever higher pressure to force more water into a full tank, and ever higher voltage to force more charge onto a charged capacitor.

$$Q = \frac{\epsilon_0}{d} A V \qquad M_{water} = \frac{1}{g} A P$$



A water tank gets deeper as you
Increase pressure, but the plates
Of a capacitor don't get further apart
As you increase voltage.

"Capacitance", sounds like "Capacity", and it is related. A large bottomed tank has a larger fluid capacity at given pressure than a small tank. "Voltage" is like "Pressure"

A large area capacitor has a larger charge capacity at given voltage than a small area capacitor.

$$Q=\frac{\epsilon_0}{d}\, A\, V \qquad \text{Mass}=\frac{1}{g}\, A\, P$$

$$\epsilon_0=8.86\times10^{-12}\,F/m,\ (C/V\,m),\ (J/V^2)$$

Capacitance and Resistance

From a math point of view,

$$C = \frac{Q}{V}$$

$$I = \frac{\Delta Q}{\Delta t}$$

Amps

$$R = \frac{V}{I}$$

Water current is total mass that passes an observer per second.

Electrical current is charge flow rate past a fixed point.
Units (C/s)

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Hydraulic Analogy

http://en.wikipedia.org/wiki/Hydraulic_analogy

Mass of water (M) Charge (Q)

Water current (dM/dt) Current (I = dQ/dt)

$V = IR$

$P = \text{Flow}\,(R)$

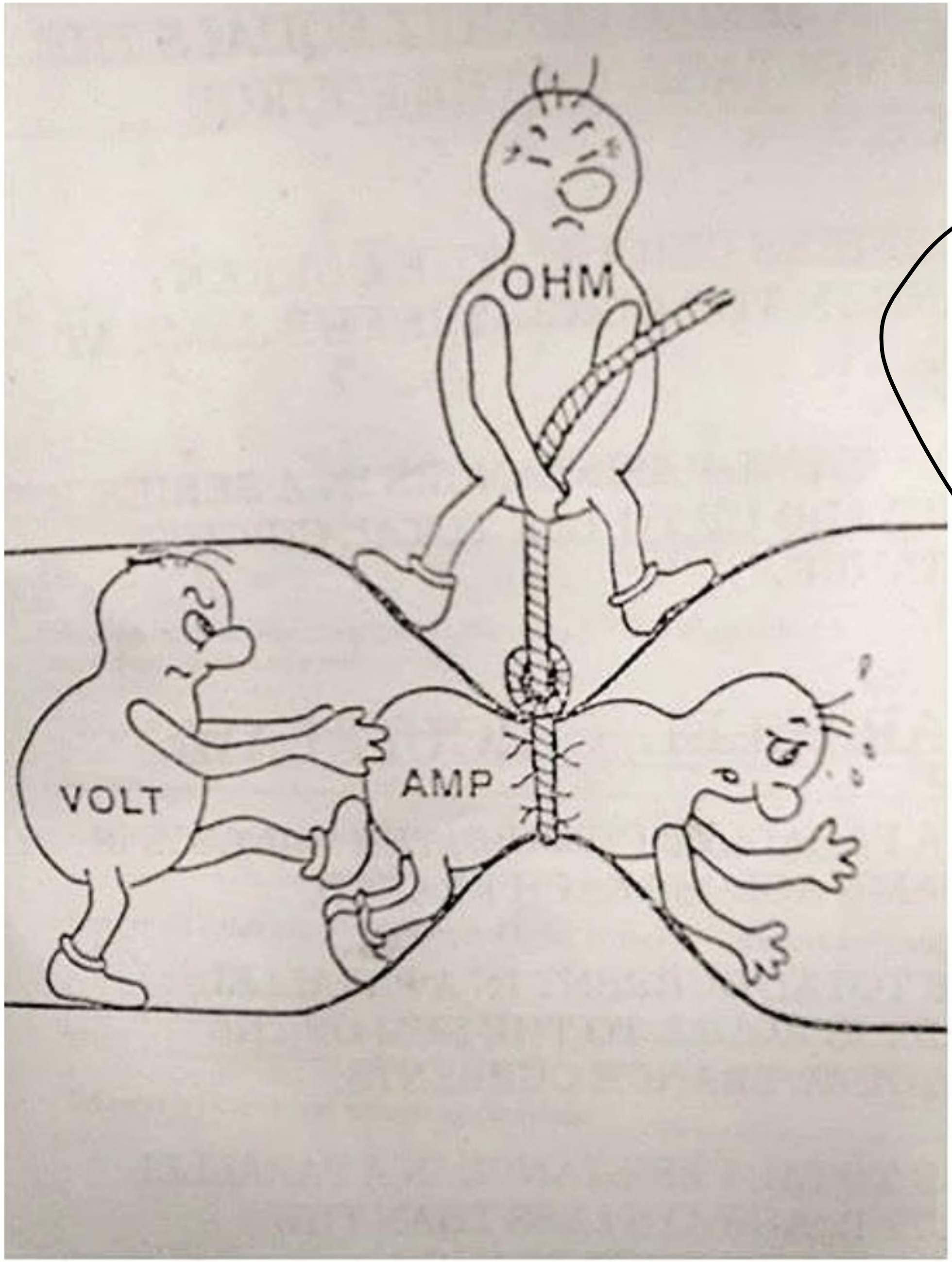
Water pressure (P) Voltage (V)

Posted by u/arbili 7 years ago

1.7k

Electricity explained

i.imgur.com/HsUd5e...



$$I = \frac{V}{R}$$

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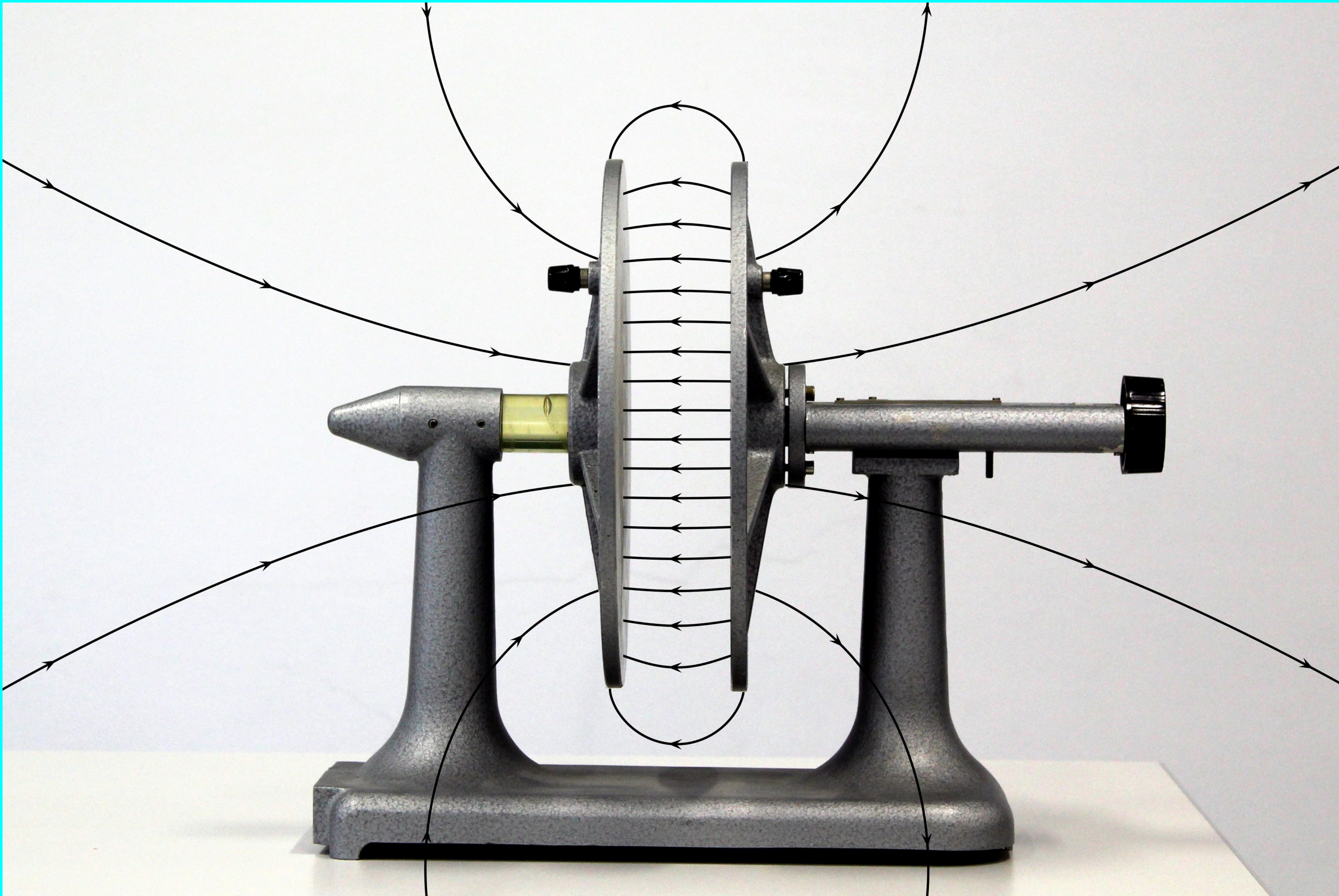
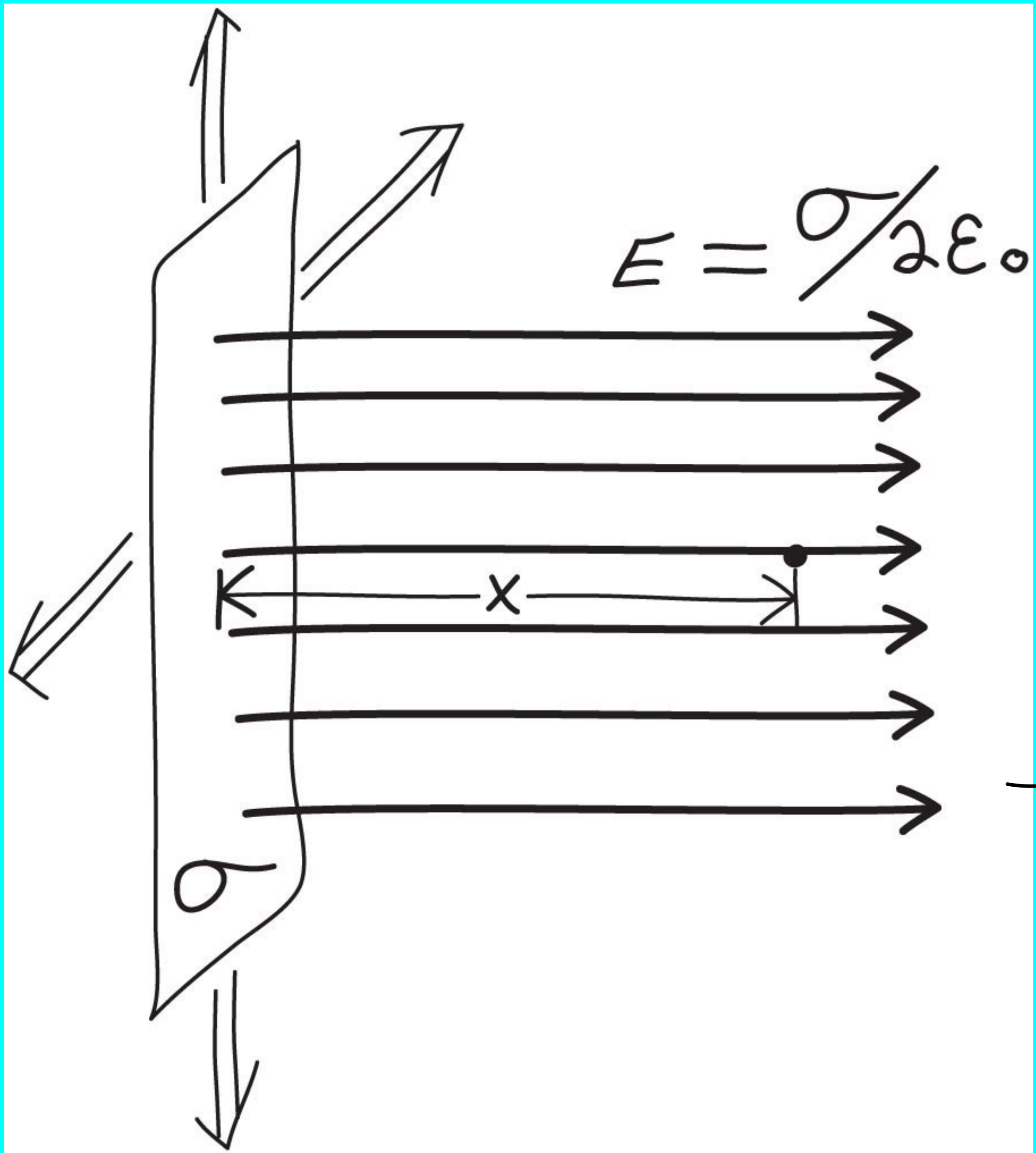
… and now Back to Capacitance



E-Field of a "large" charged plate:



$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{i}$$

$$\sigma = \frac{Q}{A}$$

$$V = -\int_0^x E \cdot dx'$$

E-Field of an infinite charged plate:

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{i} \qquad V = \frac{U}{q} = -\int_0^x E \cdot dx'$$

$\int c\, dx$

$\int x^0\, dx$

Electric Potential of an infinite plate:

$\int_1^{x'} dx$

[A] $V = \frac{\sigma^2}{4\epsilon_0}$

[B] $V = -\frac{\sigma^2}{4\epsilon_0}$

[C] $V = \frac{\sigma}{2\epsilon_0}x$

[D] $V = -\frac{\sigma}{2\epsilon_0}x$ ✓

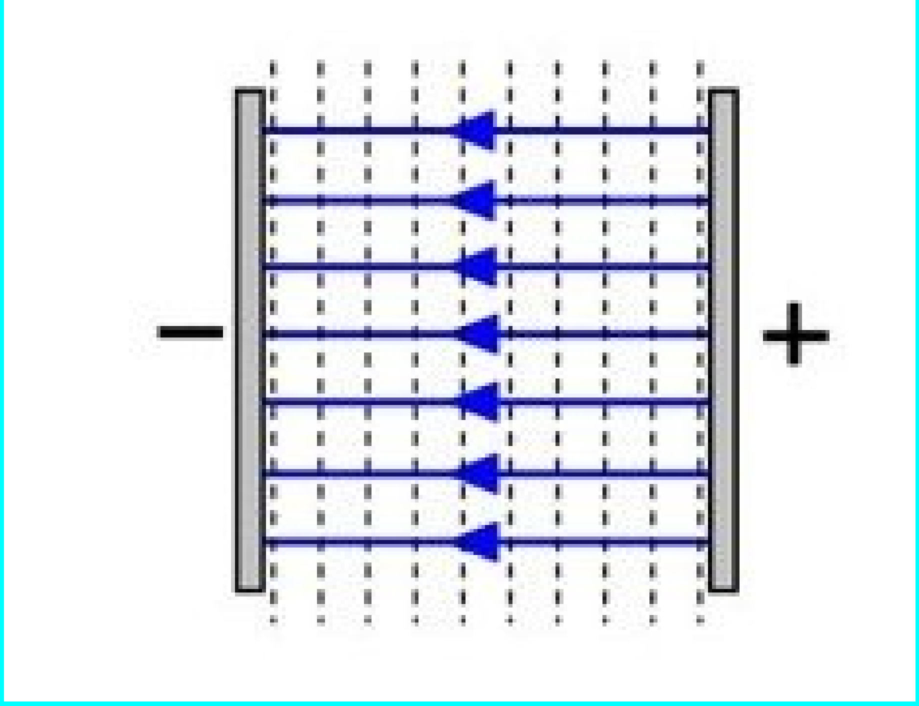
Electric potential of TWO large plates (Area "A" separation "d") …
Electric field is doubled because two charged plates of opposite sign.

$$\vec{E} = \frac{-\sigma}{\epsilon_0}\hat{i}$$

$$\Delta V = -E \cdot \Delta x$$

$$|\Delta V| = E\, d$$

$$|\Delta V| = \frac{\sigma}{\epsilon_0} d$$



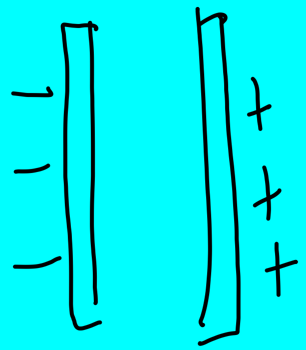
Deriving Parallel Plate Formula

$$[1]\ Q = CV$$

Definition of Capacitance

$$[2]\ V = \frac{\sigma}{\epsilon_0} d$$

Shown on last page



$$[3]\ \sigma = \frac{Q}{A}$$

Definition of surface charge density

$$[4]\ V = \frac{Q}{\epsilon_0 A} d$$

Plug 3 into 2

$$[5]\ Q = \frac{\epsilon_0 A}{d} V$$

Solve for Q

$$[6]\ C = \frac{\epsilon_0 A}{d}$$

How many capacitors are in your dorm room or house?

(A) None – I do not own a time machine or a DeLorean.

(B) About a dozen – Capacitors are energy storage and signal filtering devices used in power supplies for all electronics.

(C) Billions

How many capacitors are in your dorm room or house?

(A) None – I do not own a time machine or a Delorean.

(B) About a dozen – Capacitors are energy storage and signal filtering devices used in power supplies for all electronics.

(C) Billions – Capacitors store energy and are also elements of the memory cell in digital devices.

Note that the parallel plate formula only has "geometric" variables. Capacitance is a property of a set of conductors and does not depend on charge or voltage (or any other electric variables)

$$C = \frac{\epsilon_0 A}{d}$$

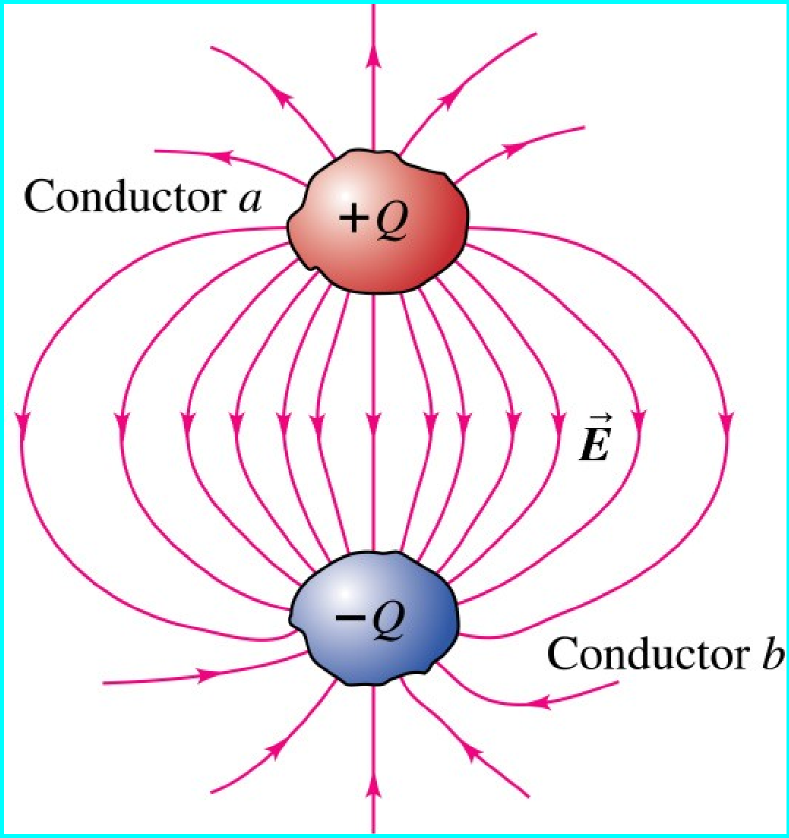
[Ex. 1] Find the capacitance of a parallel plate capacitor consisting of circular plates 10 cm in radius separated by 1.5 mm.

$$\epsilon_0 = 8.86 \times 10^{-12}\ \mathrm{F/m}$$

$$C = \epsilon_0 \frac{A}{d}$$

$$C = \frac{Q}{V}$$

The two conductors *a* and *b* are insulated from each other, forming a capacitor. You increase the charge on *a* to +2*Q* and increase the charge on *b* to –2*Q*, while keeping the conductors in the same positions.



What effect does this have on the capacitance *C*?

A. *C* is multiplied by a factor of 4

B. *C* is multiplied by a factor of 2

C. *C* is unchanged

D. *C* is multiplied by a factor of 1/2

E. *C* is multiplied by a factor of 1/4

[Ex. 2] A capacitor's plates hold 1.3 μC when it is charged to 80 V. What is its capacitance?

$$C = \epsilon_0 \frac{A}{d}$$

$$C = \frac{Q}{V}$$

[Ex. 3] A stereo receiver contains a 2500 μF capacitor charged to 35 V.
How much energy does it store?

$$C = \epsilon_0 \frac{A}{d}$$

$$C = \frac{Q}{V}$$

$$U = \frac{1}{2} C V^2$$

You reposition the two plates of a capacitor so that the capacitance doubles.

If the charges $+Q$ and $–Q$ on the two plates are kept constant in this process, what happens to the potential difference V_{ab} between the two plates?

A. V_{ab} is multiplied by a factor of 4

B. V_{ab} is multiplied by a factor of 2

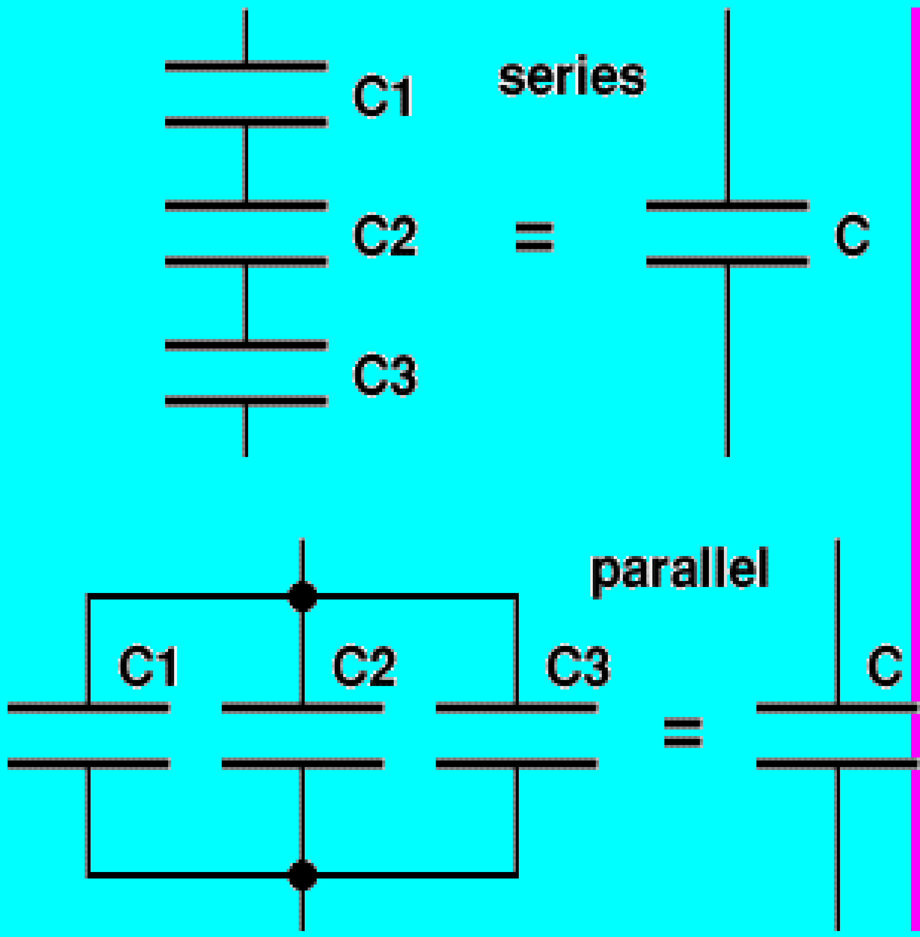
C. V_{ab} is unchanged

D. V_{ab} is multiplied by a factor of 1/2

E. V_{ab} is multiplied by a factor of 1/4

$$Q = CV$$

Capacitors in Series and Parallel



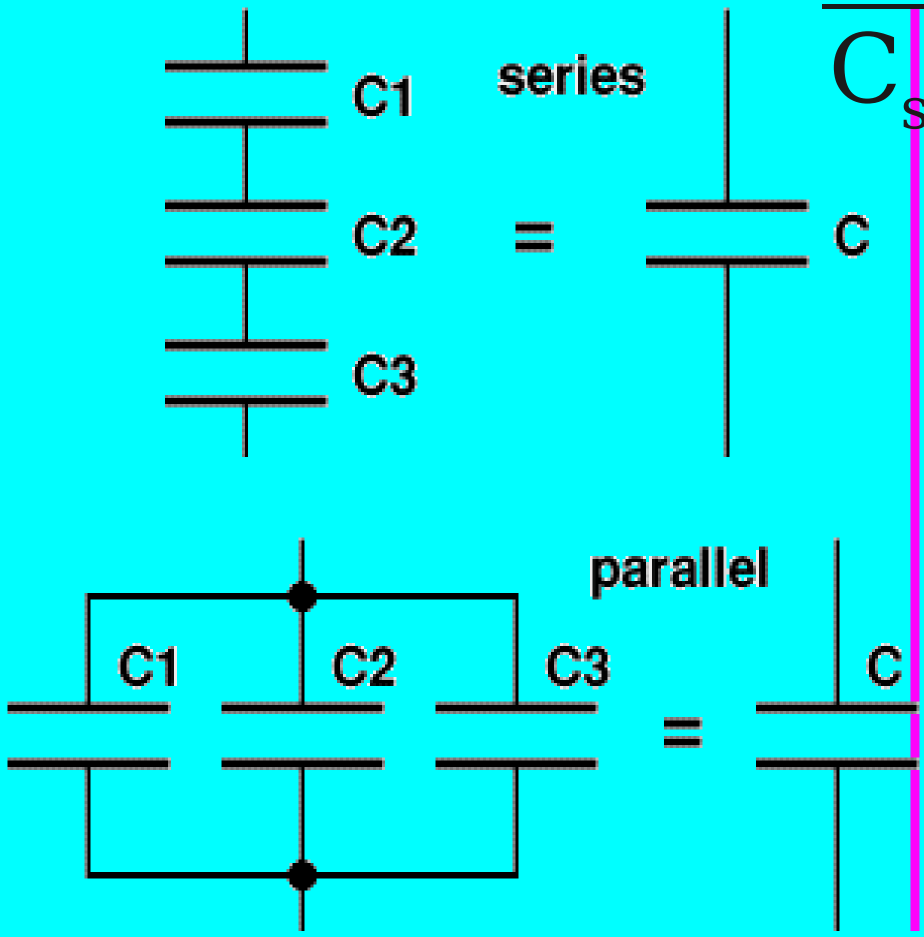
Caps in series have the
"bottom wire" or one
Connected to the "top wire"
Of the next.

Caps in parallel have all
Their tops connected to
One wire and all their
Bottoms also connected to
A second wire.

Caps in series have equal charges

Capacitors in Series and Parallel

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$



$$C_{parallel.} = C_1 + C_2 + C_3 + \ldots$$

Caps in parallel have equal voltages.

Caps in series have equal charges.

Recap

We reviewed how to use the text
We learned potential for a wire or "coax"
We learned why V=kQ/r (for a point or sphere)
We learned what capacitance is,
and the formula for parallel plate.
Current is flowing charge. Resistance is like a skinny pipe.

Next … More on Capacitance …