Lecture 14: 02/29/2024

- Announcements
  - Leap Day
  - Eclipse
  - Written HW4 due Tuesday

- Last Time
  - Exam Review

- Today
  - Online HW Review
  - Written HW explanation
  - Potential from field
  - Field from potential
  - Electric potential of point charges

Electric Potential:
What’s a volt anyway?

Electric Potential:

Is also called voltage. They are the **same thing**.

Electric potential is measured in Volts. (SI unit)

The term potential ties back to potential energy because potential IS the potential energy per unit charge.

$$U = Q V \qquad V = \frac{U}{Q}$$

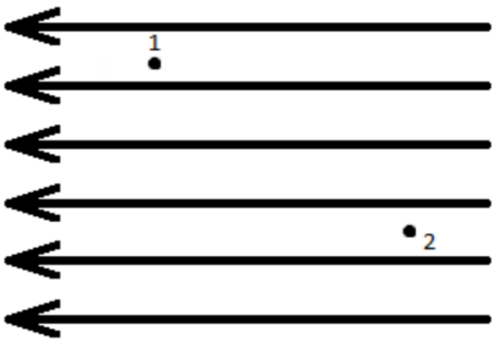
Every problem on Online HW5 required either this formula or

$$\Delta V = -E \Delta x$$

HW5 Review – Problem 1:

Problem 1: Two points, point 1 and point 2, are located inside a region with an electric field pointing to the left, as shown in the figure.

richard.sonnenfeld@nmt.edu



Part (a) If a proton is moved from point 1 to point 2, how will the potential energy of the charge-field system change? How will the potential change?
MultipleChoice :

1) Increases / Increases
2) Decreases / Increases
3) Decreases / Decreases
4) Increases / Decreases

Positive charges always move to lower potential.
Negative charges “go uphill”

HW5 Problem 3:

Problem 3: A charge q is accelerated from rest up to a speed v through a potential difference V.
richard.sonnenfeld@nmt.edu

What speed will the charge have after accelerating through a potential difference equal to $4V$?
MultipleChoice :

1) $4v$
2) $v/2$
3) v
4) $v/4$
5) $2v$

$$U_{electric} = Q V$$

Energy is conserved $K_i + U_i = K_f + U_f$

$$\frac{1}{2} m v_i^2 + U_i = \frac{1}{2} m v_f^2 + U_f \rightarrow \quad \frac{1}{2} m 0^2 + qV = \frac{1}{2} m v^2 + 0$$

$$\frac{1}{2} m 0^2 + q 4 V = \frac{1}{2} m (2 v)^2 + 0$$

HW5 Problem 4:

Problem 4: A pair of parallel conducting plates have a potential difference of 3.2×10^4 V and a separation of 1.85 cm.
richard.sonnenfeld@nmt.edu

What is the magnitude, in volts per meter, of the electric field strength between the parallel conducting plates?
Numeric : A numeric value is expected and not an expression.
$|E|$ = ____ V/m

$$\Delta V = -E \Delta x$$

HW5 Problem 7:

Problem 7: A 12.0 V battery-operated bottle warmer heats *48* g of glass, *2.7* × 10^2 g of baby formula, and *1.9* × 10^2 g of aluminum from 20.0 °C to 35.0 °C.

richard.sonnenfeld@nmt.edu

Part (a) How much charge is moved by the battery in C? You may assume the specific heat of baby formula is approximately the same as for water, and the specific heat of glass is 0.840 J/(g·°C), and the specific heat of aluminium is 0.900 J/(g·°C).

Numeric : A numeric value is expected and not an expression.

q = ____ C

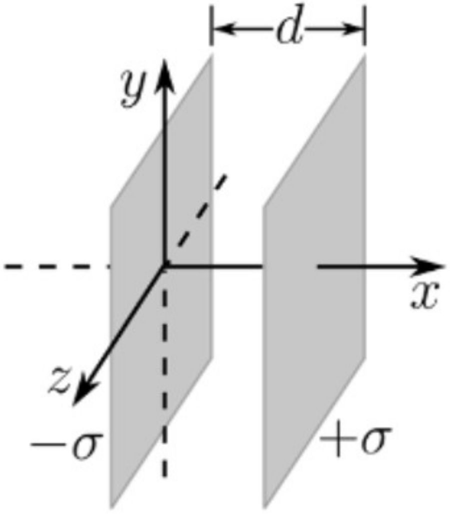
$$\Delta U_{chemical} = \Delta U_{electrical}$$

$$\Delta U_{chemical} = m_{glass} C_{glass} \Delta T + m_{Al} C_{Al} \Delta T + m_{milk} C_{milk} \Delta T$$

$$\Delta U_{electrical} = Q V$$

HW5 Problem 13:

Problem 13: Two very large parallel plates are perpendicular to the x axis and have a small separation, d. (The dimensions are distorted for purposes of visualization.) The first plate, located at $x = 0$, has a negative uniform charge density, $-\sigma$, and is designated as the zero of electric potential. The second plate, located at $x = d$, has a positive uniform charge density, $+\sigma$.
richard.sonnenfeld@nmt.edu



$$\Delta V = -E \Delta x$$

$$E = \frac{\sigma}{\epsilon_0}$$

a) What's "E" in the gap?

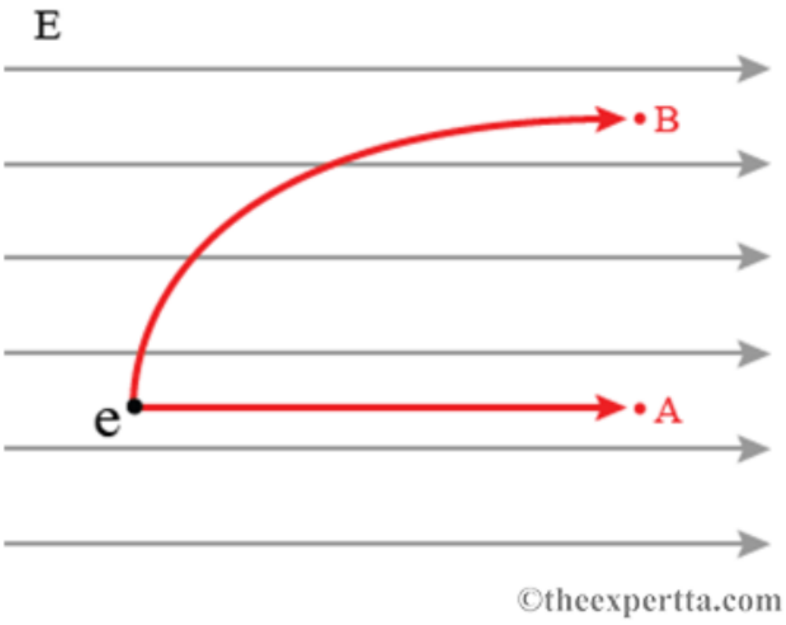
b) What's the potential?

c/d) Now what are these things with numbers?

e) An electron is released from the negative plate. At what speed does it strike the positive plate?

HW5 Problem 12

Problem 12: Consider a proton in a uniform electric field directed left to right, as shown in the figure. For both paths the initial speed of the proton is the same, but the direction of the initial velocity is different.
richard.sonnenfeld@nmt.edu



Part (a) Compare the change in electric potential energy along path A to the change in electric potential energy along path B.
MultipleChoice :

1) $\Delta U_A > \Delta U_B$
2) $\Delta U_A < \Delta U_B$
3) $\Delta U_A = \Delta U_B$
4) There is not enough information given - we need either the initial speed or the size of the electric field.

HW5 Problem 12

$$V = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{r}$$

Voltage from Electric Field:

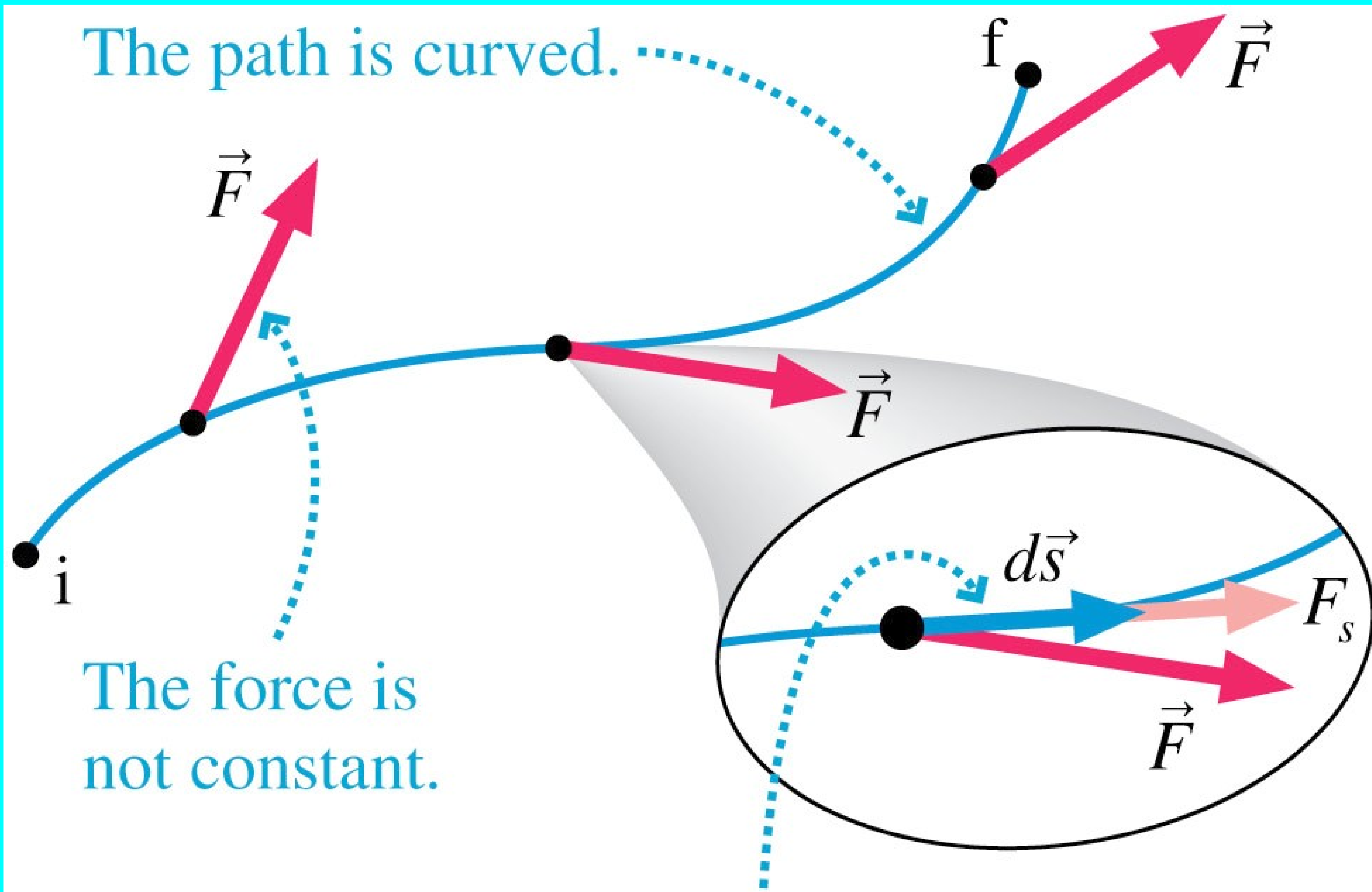
If the electric field is constant, then: $\Delta V = -E\,\Delta x$

If it depends on x: $\Delta V = -\int_a^b E\,dx$

If it's constant but not along $\Delta \vec{r}$: $\Delta V = -\vec{E}\cdot\Delta\vec{r}$

If it depends on x, y, and z: $\Delta V = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{r}$

Work



$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_i^f \vec{F} \cdot d\vec{s}$$

9. The spark plug in an automobile has a center electrode made from 2.0 mm-diameter wire. The electrode is roughly a hemisphere and may be treated as a sphere. What voltage on the spark plug is needed to assure that it sparks? (The breakdown field for air is $E_B = 3MV/m$)

10. Three 2 nC positive charges make an equilateral triangle with side 'a'=10 cm.

    (a) What is the potential in the center of the triangle?

    (b) What is the potential in the middle of the base of the triangle?

11. A power line with diameter 3.0 cm is at a voltage of 4.0 kV relative to a point one meter away. What is the line charge density (λ) of the power line?

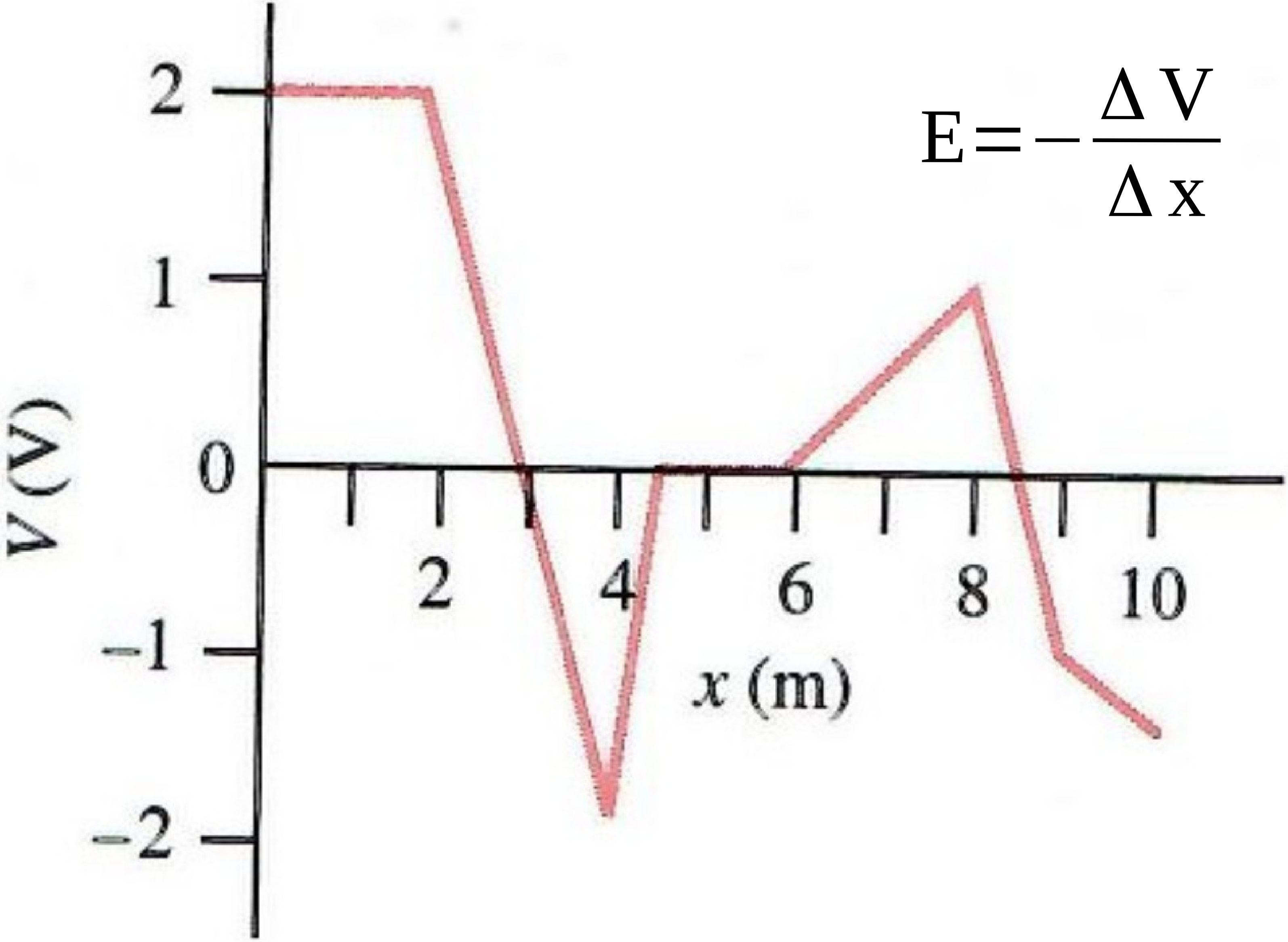
5. Four 3 nC positive charges make a square with side 'a'=10 cm.
   (a) What is the potential in the center of the square?
   (b) What is the potential in the middle of the base of the square?

6. A coaxial cable consists of a 2.0-mm diameter inner conductor and an outer conductor of diameter 1.6 cm. If the conductors carry line charge densities of $\pm 0.67 nC/m$, what is the magnitude of the potential difference between them?

7. Figure 2 (next page) shows a plot of potential vs. position along the x-axis. Make a plot of the x-component of electric field for this situation.

8. The potential as a function of position in a region is given by $V(x) = 3x - 2x^2 - x^3$, with x in meters and V in volts.
   (a) Find all points on the x-axis where V=0.
   (b) Find all points where E=0.

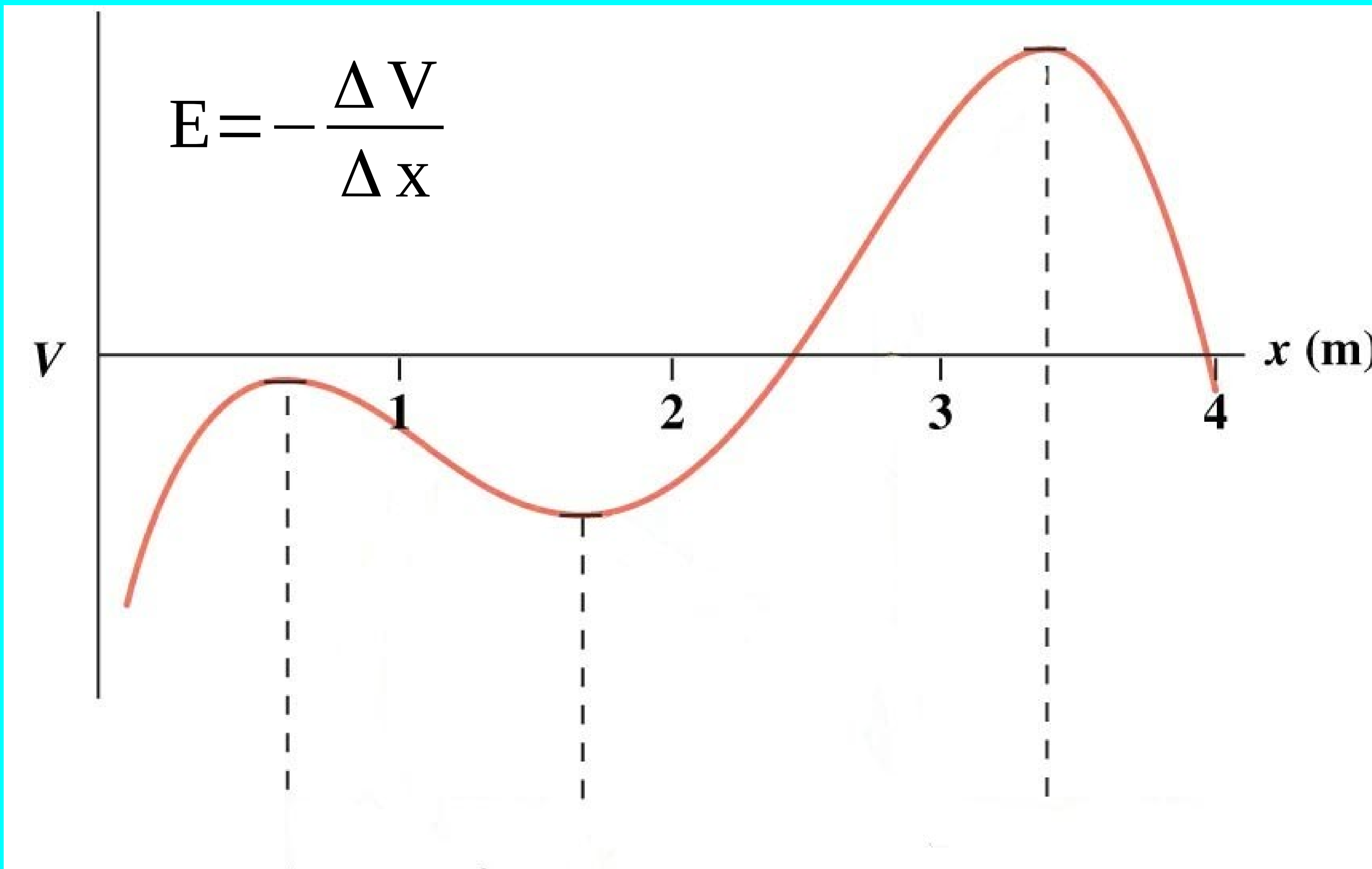
HW Written 4 Problem 7



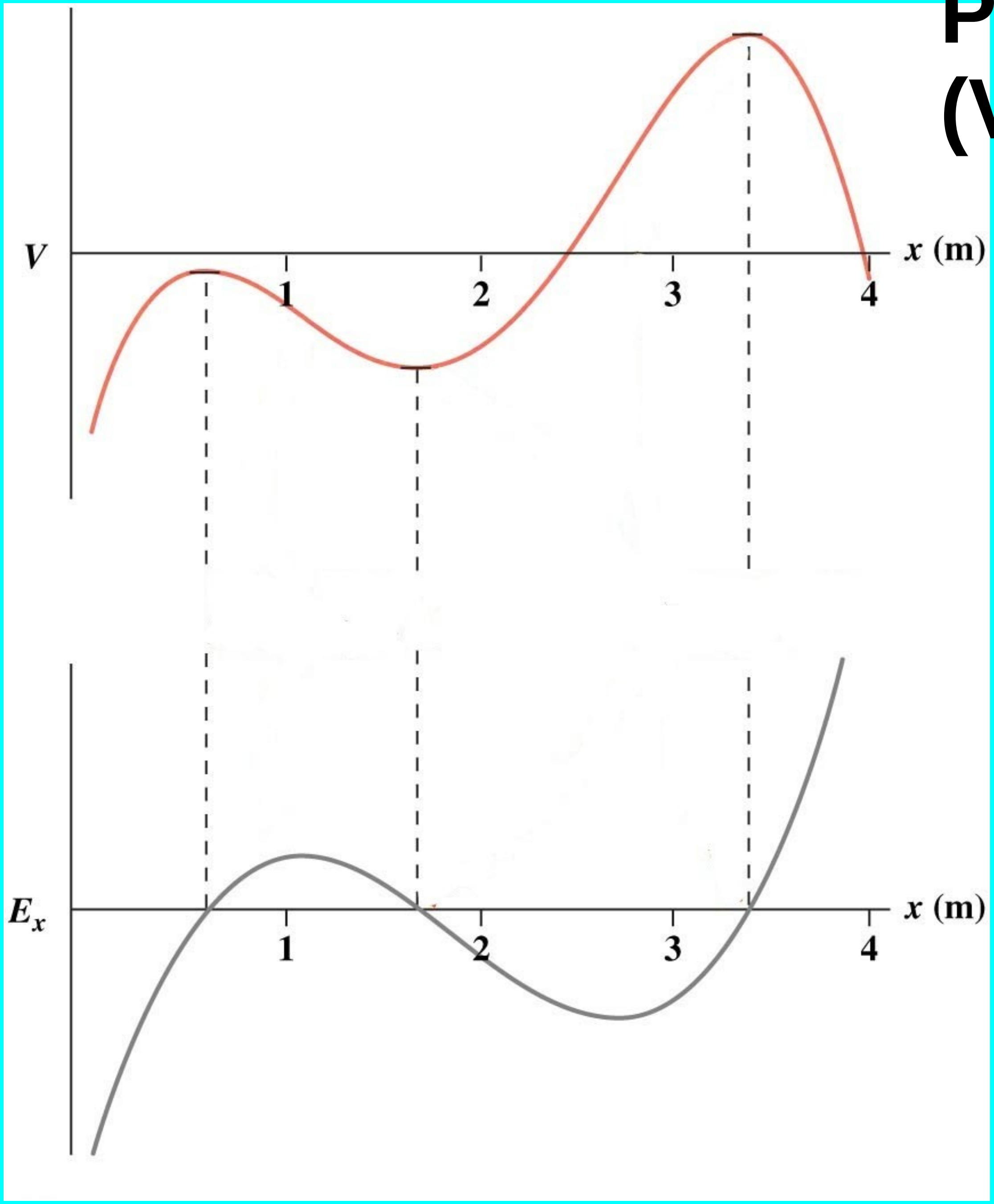
$$E = -\frac{\Delta V}{\Delta x}$$

Positive charges will move to lower voltage

Negative charges move to higher voltages.



Potential (Voltage) in 1-D



$$V(x) = -\int E\,dx$$

$$E_x = -\frac{dV}{dx}$$

You are told that the electric potential is zero At some point “P”. Which statement is correct?

A. A charge placed at *P* would feel no electric force

B. The electric field at *P* is also zero.

C. The electric field at *P* is negative

D. Both A and B

E. not enough information given to decide

HW Written 4 Problem 8

$$V(x) = 3x - 2x^2 - x^3$$

Where is V=0?

Where is E=0?

Potential is a scalar!! Superposition is EASY!!

$$\Delta V_{r\infty} = k\frac{Q}{r}$$

HW Written 4 Problem 10a

What is the potential in the center of an equilateral triangle with side “a” and equal charges q?

$$\Delta V_{r\infty} = k\frac{Q}{r}$$

HW Written 4 Problem 10b

What is the potential in the center of the base of an equilateral triangle with side “a” and equal charges q?

$$\Delta V_{r\infty} = k\frac{Q}{r}$$

Potential of point charges … Why?

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

for this system, we have

$$V_r = -\int_\infty^r \frac{kq}{r^2} \hat{\mathbf{r}} \cdot \hat{\mathbf{r}} dr,$$

which simplifies to

$$V_r = -\int_\infty^r \frac{kq}{r^2}\, dr = \frac{kq}{r} - \frac{kq}{\infty} = \frac{kq}{r}.$$

Voltage from Electric Field:

If the electric field is constant, then: $\Delta V = -E\Delta x$

If it depends on x: $$\Delta V = -\int_a^b E\,dx$$

If it's constant but not along $\Delta\vec{r}$: $\Delta V = -\vec{E}\cdot\Delta\vec{r}$

If it depends on x, y, and z: $$\Delta V = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{r}$$

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11. A power line with diameter 3.0 cm is at a voltage of 4.0 kV relative to a point one meter away. What is the line charge density (λ) of the power line?

Electric Field from Voltage:

If the electric field is constant, then: $$E = -\frac{V}{\Delta x}$$

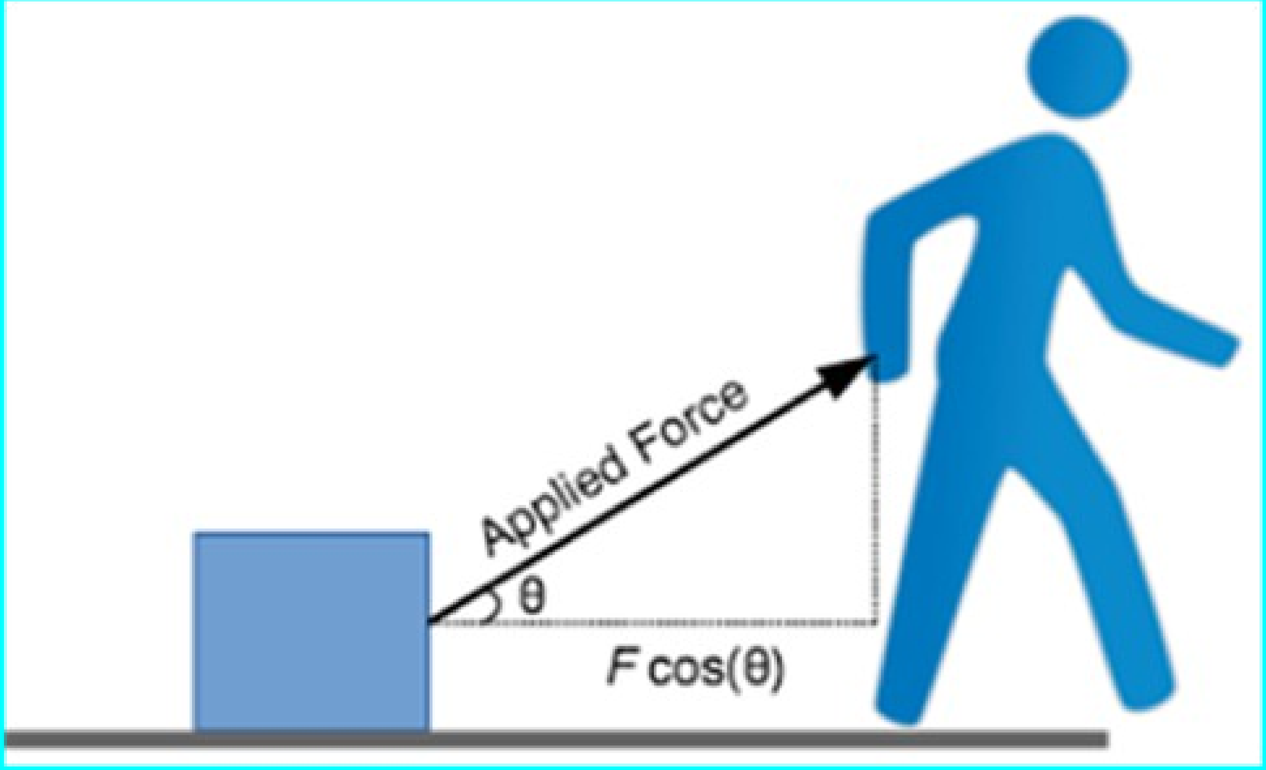
If it depends on x: $$E = -\frac{dV}{dx}$$

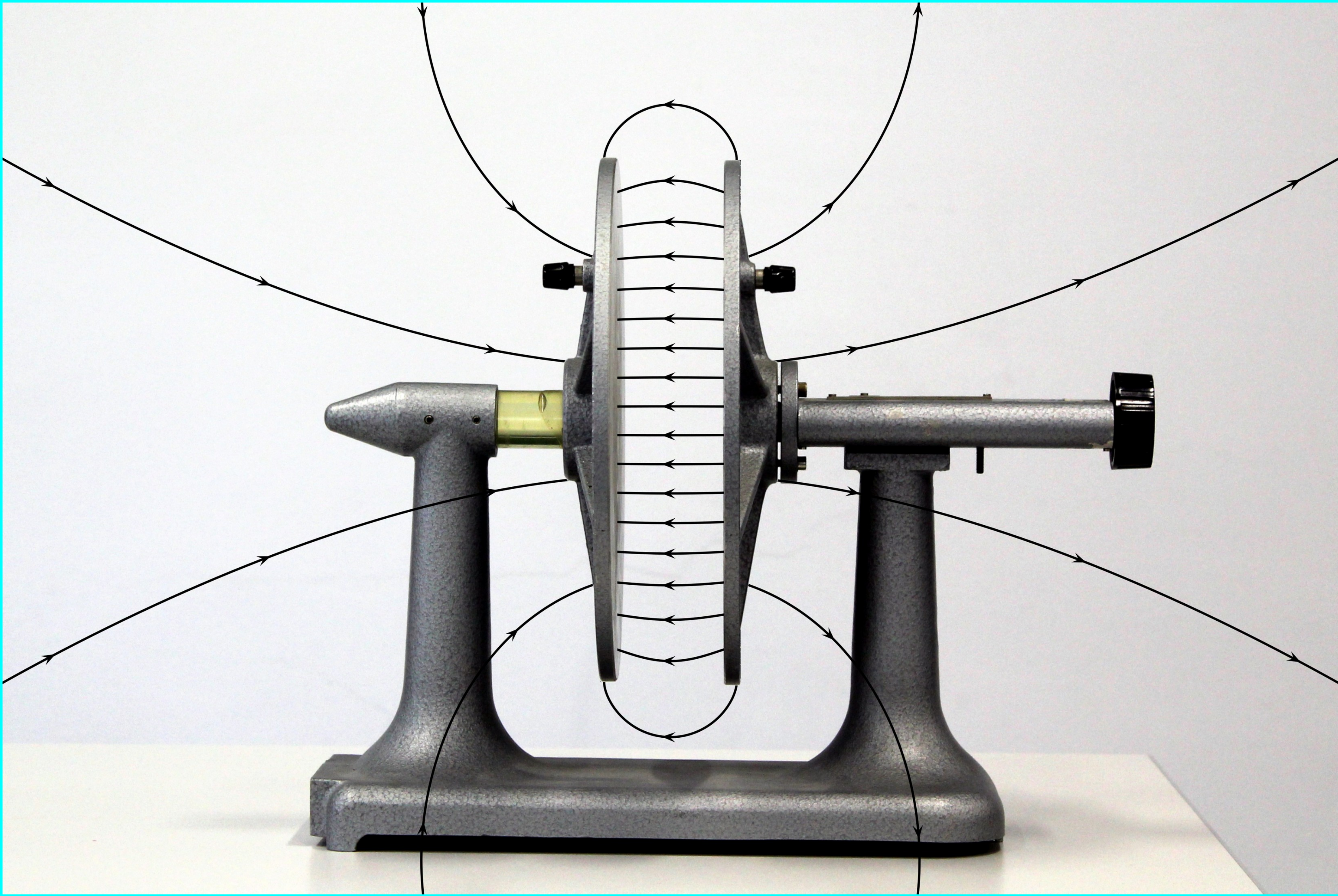
.

If it depends on x, y, and z: $$\vec{E} = -\nabla V$$

Work and dot product and conservative force

$$W = \vec{F}\cos(\theta)\Delta r = \vec{F}\cdot\Delta r = -\Delta U$$



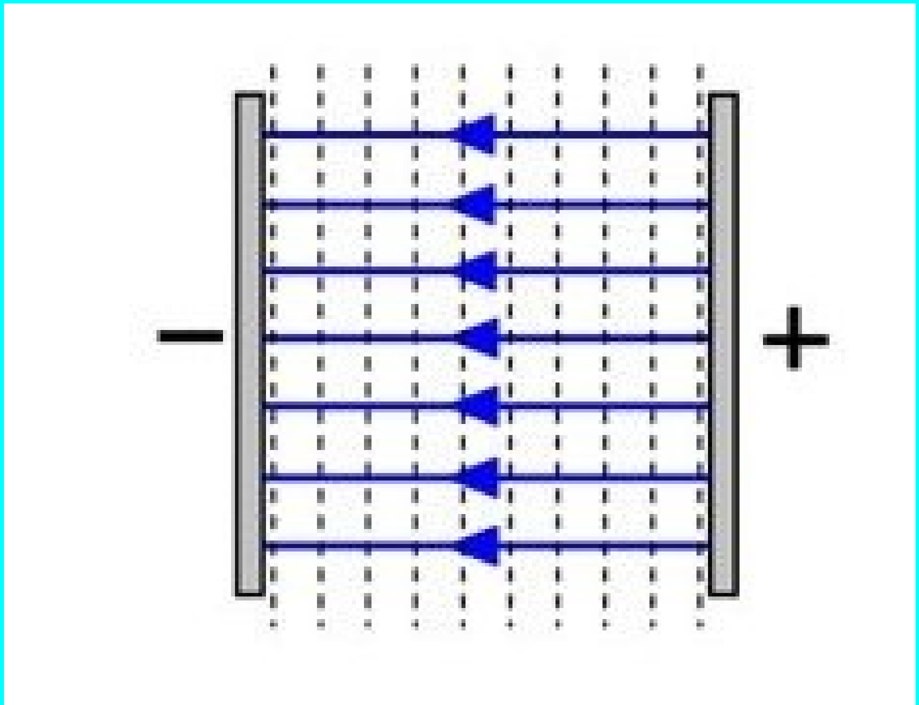


Electric potential of TWO large plates (Area "A" separation "d") …
Electric field is doubled because two charged plates of opposite sign.

$$\vec{E} = \frac{-\sigma}{\epsilon_0}\hat{i}$$

$$V = -\vec{E}\cdot\Delta\vec{r} = E\,d$$

$$V = \frac{\sigma}{\epsilon_0}x = \frac{\sigma}{\epsilon_0}d$$



What does work have to do with Electric Potential?

$$W = \vec{F} \cdot \Delta \vec{r} = Q \vec{E} \cdot \Delta \vec{r}$$

What if force or Electric field are not constant?

$$W = \int \vec{F} \cdot d\vec{r} = Q \int \vec{E} \cdot d\vec{r}$$

$$\Delta U = -W = QV = -Q \int \vec{E} \cdot d\vec{r}$$

$$\text{therefor } V \stackrel{\text{def}}{=} -\int \vec{E} \cdot d\vec{r}$$

Voltage of a point charge:

(Or a spherically symmetric charge)

$$\Delta V_{AB} = -\int_A^B \frac{kq}{r^2}\hat{r}\cdot d\vec{r}$$

$+$ $A \rightarrow B$

$$\Delta V_{AB} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

If A goes to infinity:

$$\Delta V_{\infty B} = kq\frac{1}{r_B} \qquad V = \frac{kq}{r}$$

It takes work to move a charge in from infinity toward another charge.

Recap

We learned that $E=-dV/dx$
We learned $V=kQ/r$ (for a point)
We learned the potential is a scalar!
Because of the dot product, only the field along the direction of motion contributes to potential (or kinetic energy change)

We learned that there are only a couple of formulae for potential and they will defeat many beasts in the fearsome jungle of physics!

Frightened creatures in the jungle at night ...

T = period of a wave (1/f)

T = tension in a string $\mathbf{v = \sqrt{\frac{T}{\mu}}}$

λ = wavelength $\mathbf{v = f\lambda}$

λ = charge/length $\mathbf{\lambda = \frac{Q}{L}}$

V = Voltage (potential)

v = velocity (e.g. $\mathbf{K = \frac{1}{2} m v^2}$)

k = Coulomb's constant $\mathbf{E = k\frac{q}{r^2}}$

k = The wave number $\mathbf{k = \frac{2\pi}{\lambda}}$

K = kinetic energy

Frightened creatures in the jungle at night (2)

E = Electric field $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ $E = \frac{\lambda}{2\pi\epsilon_0 r}$ $E = \frac{\sigma}{2\epsilon_0}$ $E = \frac{\sigma}{\epsilon_0}$

ϵ_0 = Permittivity of free space

e = charge of an electron (or a proton)

e = the base of natural logarithms