# Lecture 12: 02/21/2024

- Announcements
  HW5 is up
  "Lecture 11" does not exist (it was the test)
  I made the ones go away from Canvas …
  Will publish solutions to exam

- Last Time
  - Gauss law tricks
  - Introduction to electric potential

- Today
  Potential from field
  Field from potential
  Work, Potential Energy, Conservative Forces
  Field between two plates
  Electric potential of point charges

# **Electric Potential:**
What's a volt anyway?

**Electric Potential:**

Is also called voltage. They are the same thing.

Electric potential is measured in Volts. (SI unit)

The term potential ties back to potential energy because potential IS the potential energy per unit charge.

$$U = Q V \qquad V = \frac{U}{Q}$$

# Voltage from Electric Field:

If the electric field is constant, then: $V = -E\,\Delta x$

If it depends on x: $V = -\int_a^b E\,dx$

If it's constant but not along $\Delta\vec{r}$: $V = -\vec{E}\cdot\Delta\vec{r}$

If it depends on x, y, and z: $V = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{r}$

# Electric Field from Voltage:

If the electric field is constant, then: $E = -\frac{V}{\Delta x}$

If it depends on x: $E = -\frac{dV}{dx}$

.

If it depends on x, y, and z: $\vec{E} = -\nabla V$

# Electric Field from Voltage:

**Problem 4:** A pair of parallel conducting plates have a potential difference of *3.2* $\times 10^4$ V and a separation of *1.85* cm.

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What is the magnitude, in volts per meter, of the electric field strength between the parallel conducting plates?

**Numeric** : A numeric value is expected and not an expression.

$|E|$ = ________________________________________ V/m

# For the bio majors…

The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick.

What is the average electric field magnitude?

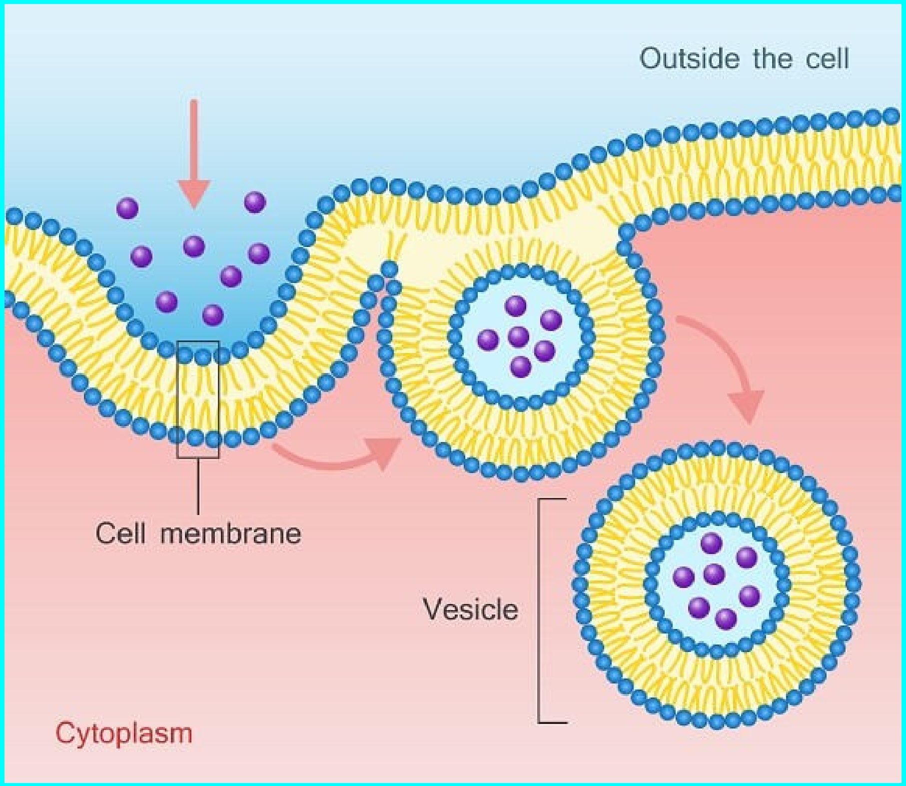
(**A**) $7.2\times10^{-10}\,\mathrm{V/m}$

(**B**) $1.125\times10^{-7}\,\mathrm{V/m}$

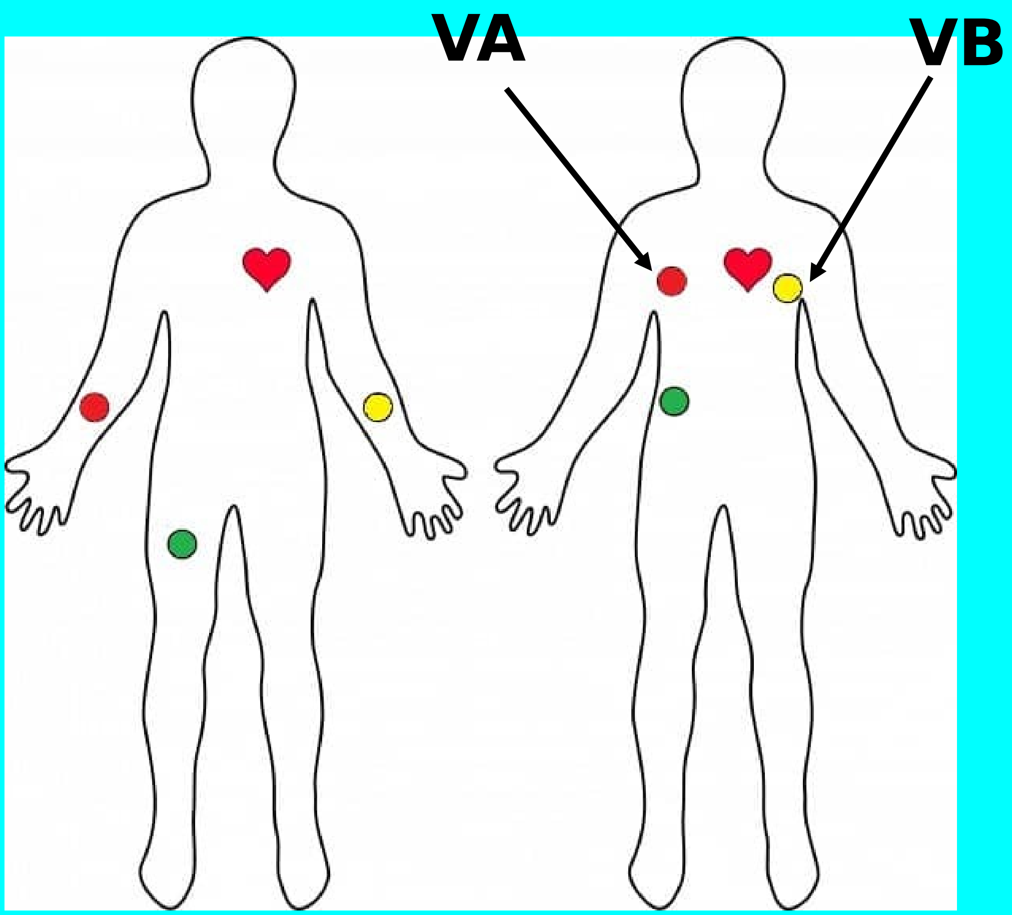
(**C**) $1.125\times10^{-4}\,\mathrm{V/m}$

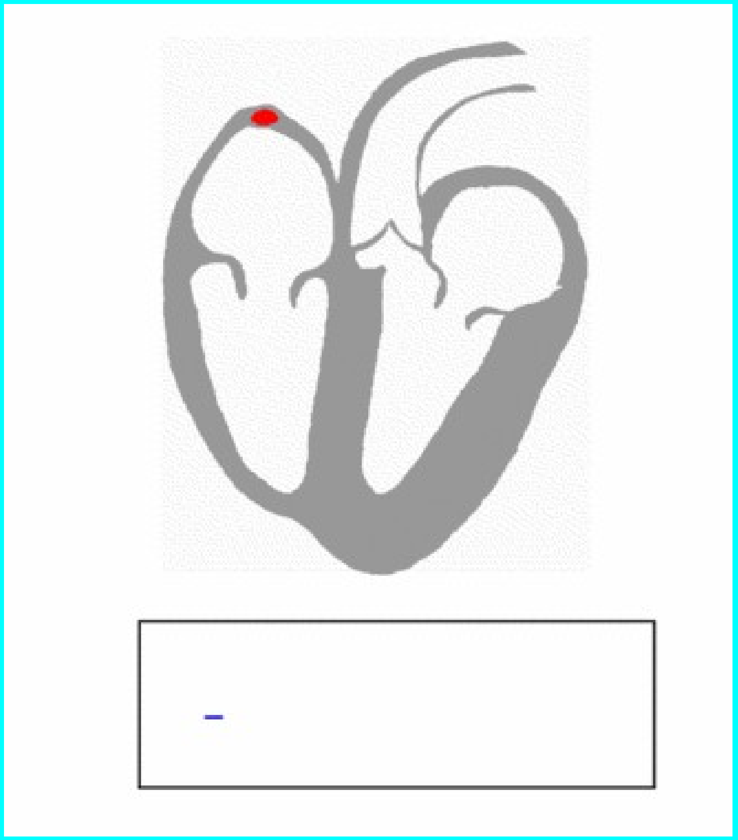
(**D**) $8.89\times10^{3}\,\mathrm{V/m}$

(**E**) $8.89\times10^{6}\,\mathrm{V/m}$



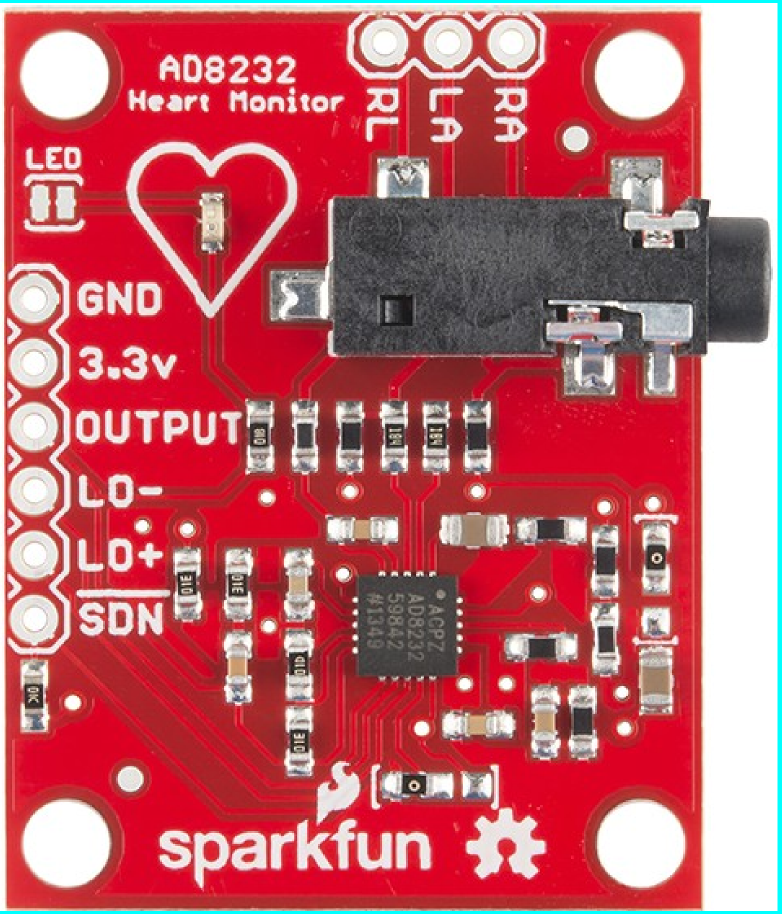
# ECG or Electrocardiogram

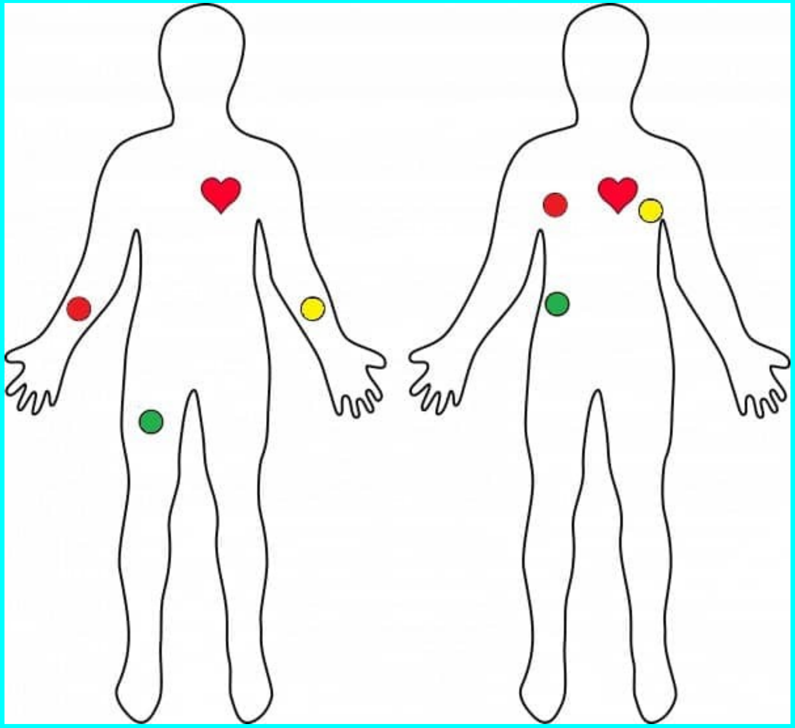


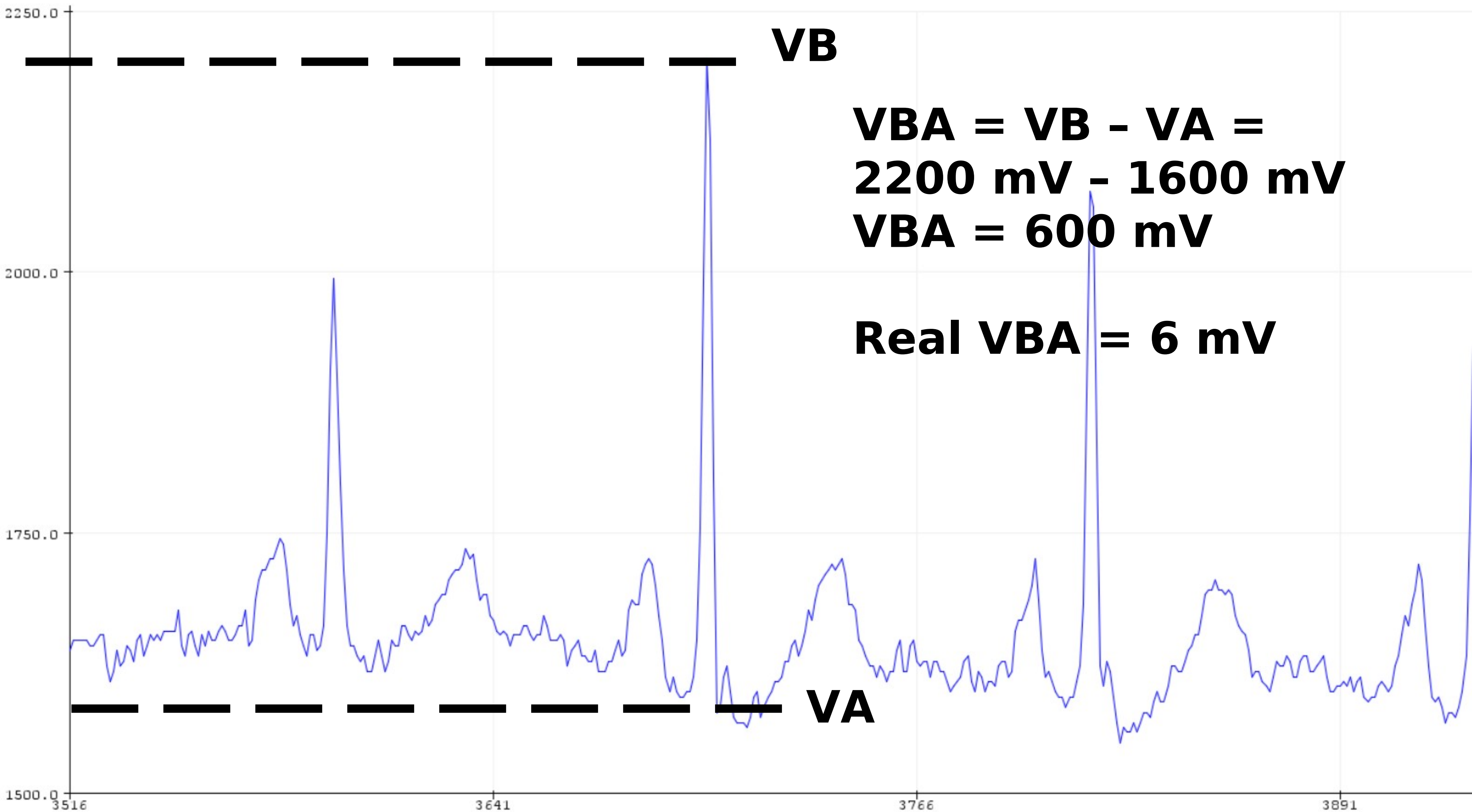



# ECG or Electrocardiogram

- Gain = 100




2250.0
2000.0
1750.0
1500.0
3516
3641
3766
3891
VB
VA
VBA = VB – VA =
2200 mV – 1600 mV
VBA = 600 mV
Real VBA = 6 mV

Electric field can be expressed in N/C, but it can be expressed in terms of other SI Units.

What other units are appropriate for electric field?

**(A)** N

**(B)** $\frac{V}{m}$

**(C)** $\frac{J}{C}$

**(D)** $N \cdot m$

**(E)** More than one correct answer

Electric potential is usually expressed in Volts, but it can be expressed in terms of other SI Units.

What other units are appropriate for electric potential?

**(A)** $\dfrac{\text{N}\cdot\text{m}}{\text{C}}$

**(B)** $\dfrac{\text{V}}{\text{m}}$

**(C)** $\dfrac{\text{J}}{\text{C}}$

**(D)** $\text{N}\cdot\text{m}$

**(E)** More than one correct answer

## Voltage from Electric Field:

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## Conservative Forces

$$\text{Work}: \quad W \stackrel{\text{def}}{=} \int \vec{F} \cdot d\,\vec{r}$$

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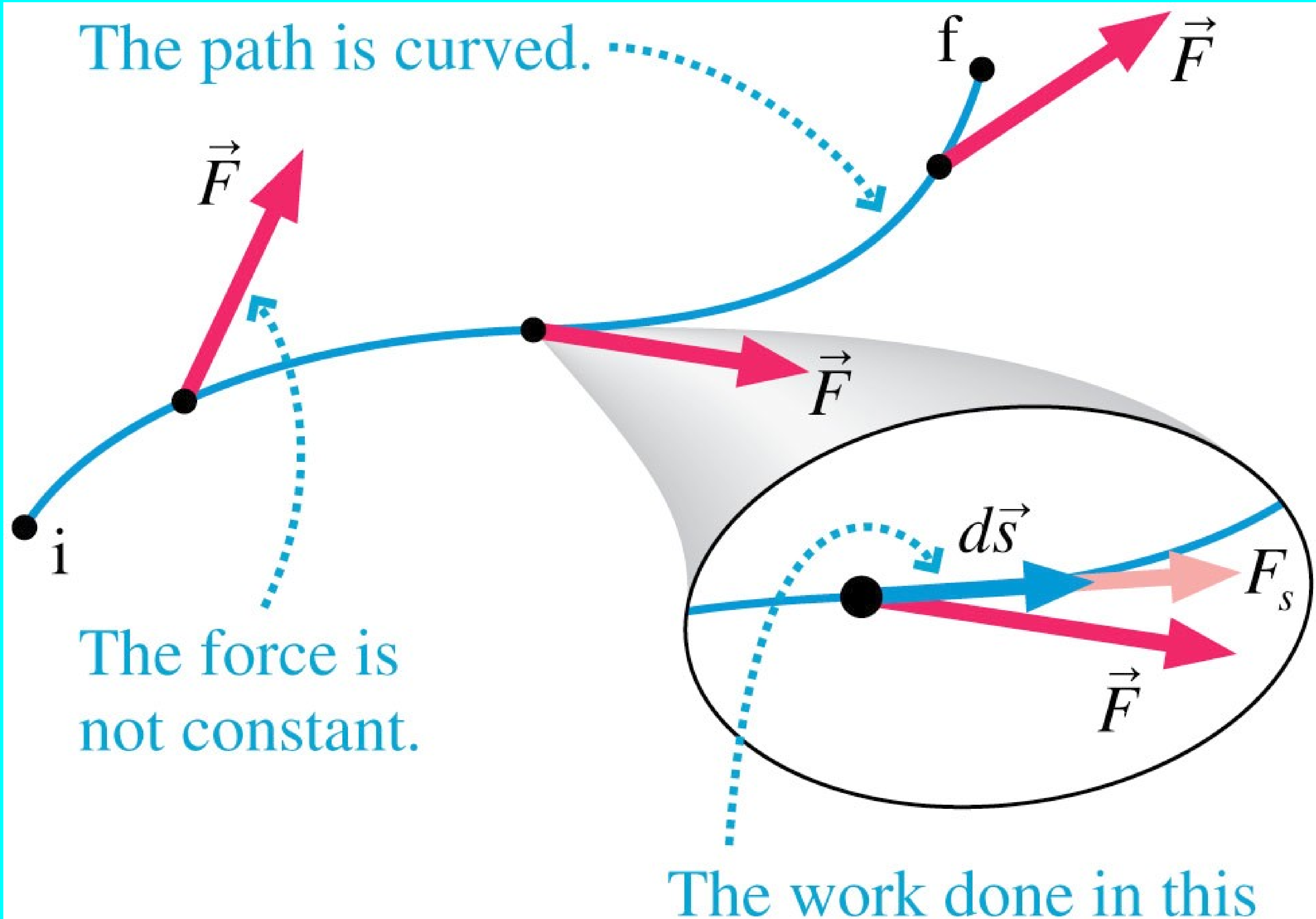
If you can do work and “Get it back”, the force is called “conservative”.

Gravity, Springs and Electric forces are conservative
Friction is NOT conservative

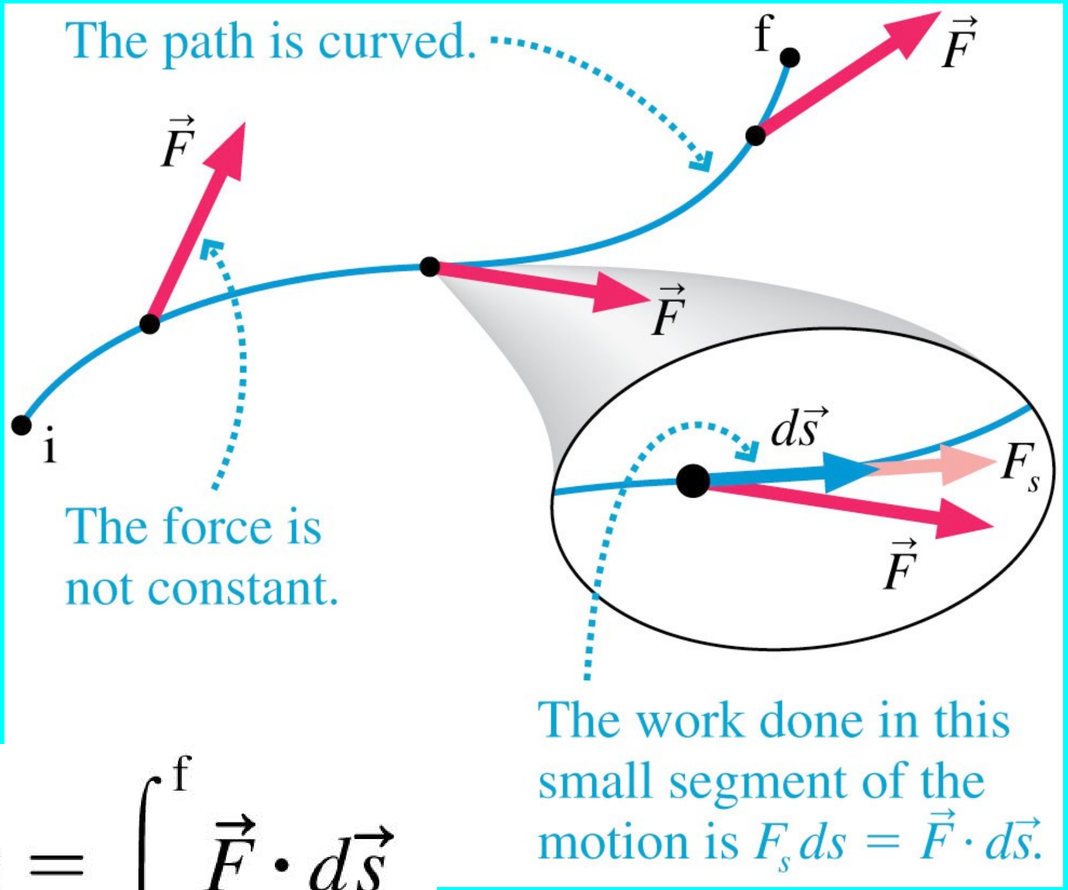
# Work




$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_i^f \vec{F}\cdot d\vec{s}$$

# Work

If the force is *not* constant or the displacement is *not* along a linear path, we can calculate the work by dividing the path into many small segments.




$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s \, ds = \int_i^f \vec{F} \cdot d\vec{s}$$

Both **dr** and **ds** mean "small displacement"
Books are inconsistent on which they use.

# Human bodies and work ...

## Confusion about Work and Potential Energy:

Distinguish between work done by a human and work done by a conservative force.

A person lifting a weight does positive work on the weight, while gravity does negative work on it.

When we talk about potential energy, we are talking about the work done by gravity, not the person.

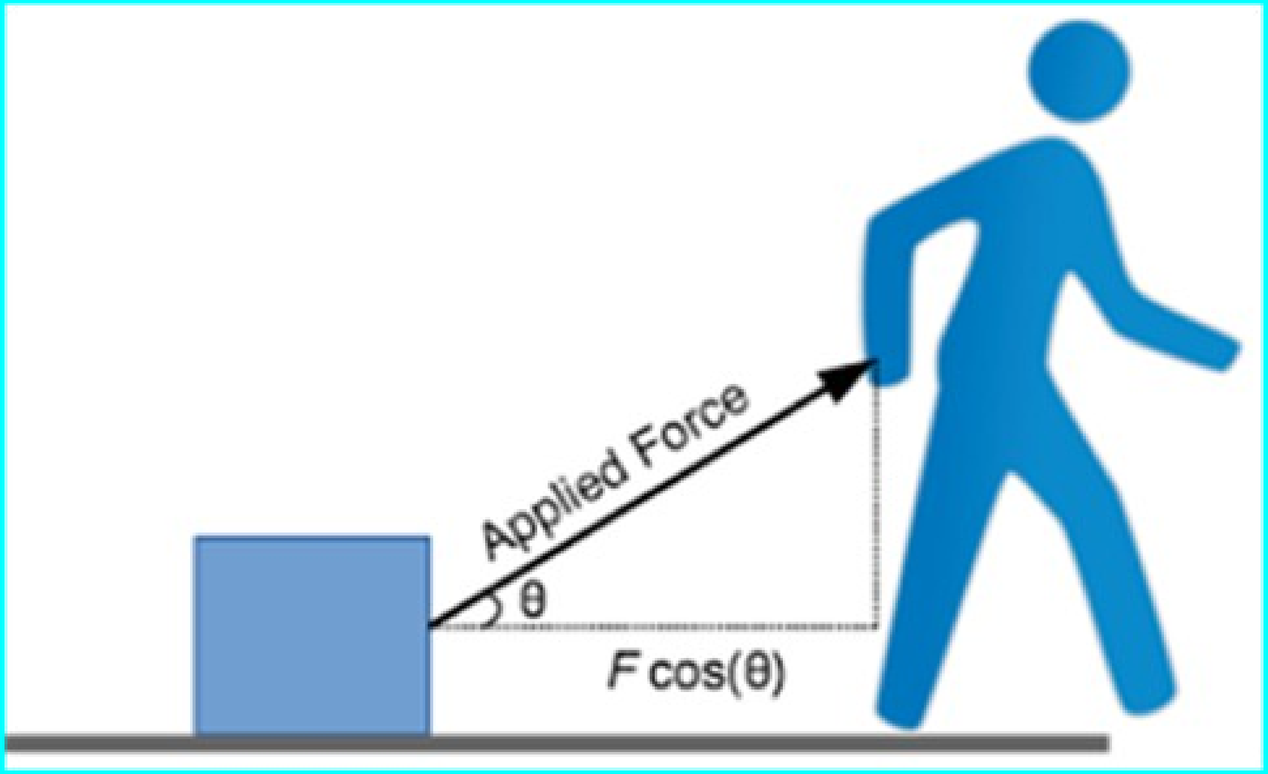
Holding a heavy object motionless is tiring, but is not considered work in physics.

Biological forces are not conservative.  We USUALLY are not considering them.

# Work and a conservative force

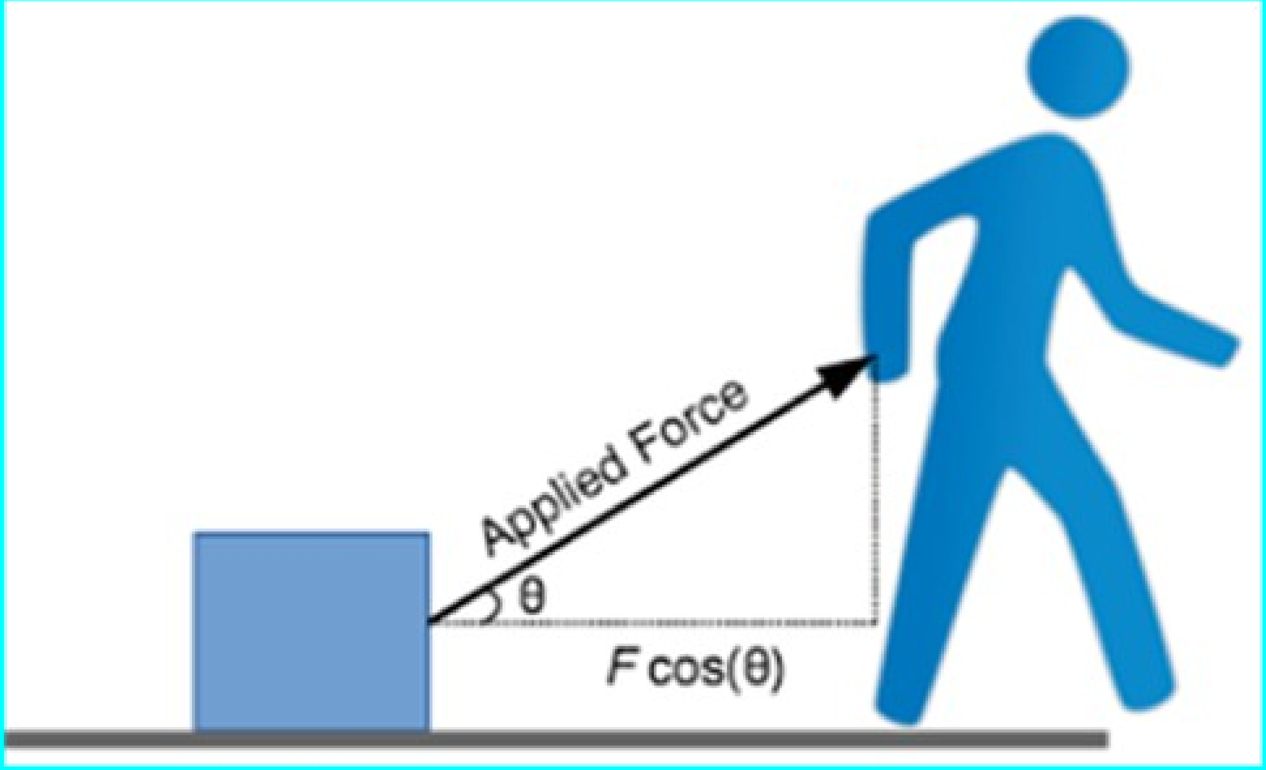
# Work and dot product

$$W = \vec{F}\cos(\theta)\Delta r = \vec{F}\cdot\Delta r$$



# Work and dot product and conservative force

$$W = \vec{F}\cos(\theta)\Delta r = \vec{F}\cdot\Delta r = -\Delta U$$



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# Work, Potential Energy, and Kinetic Energy

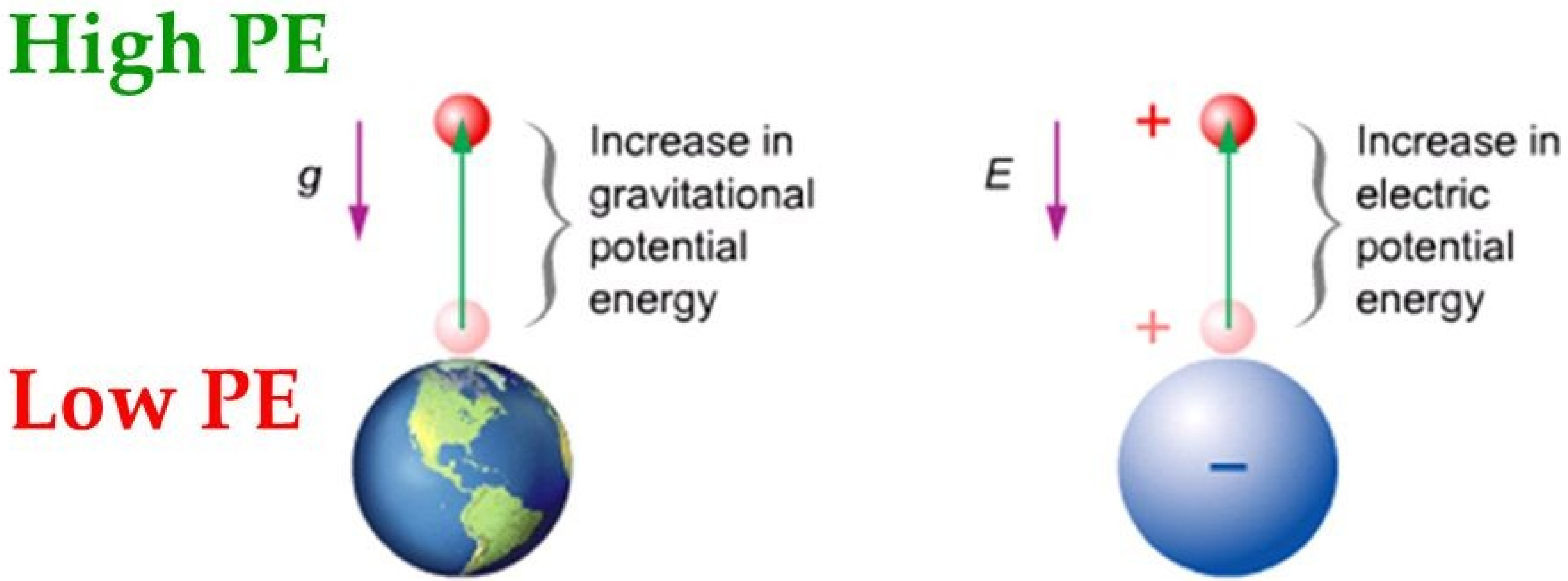
$$W = \vec{F} \cdot \Delta r = -\Delta U$$

$$W = \Delta KE = \frac{1}{2} m v_{FINAL}^2 - \frac{1}{2} m v_{INITIAL}^2$$

$$\Delta U + \Delta KE \text{ is constant!}$$

**Gravity and Electrostatic Force are both CONSERVATIVE**

# Potential Energy




$$\vec{F} = m\,\vec{g}$$

$$U = m\,g\,h$$

$$\text{geopotential} = g\,h$$

$$\vec{F} = Q\,\vec{E}$$

$$U = Q\,E\,h$$

$$V = E\,h$$

## **Electric Potential and Electric field:**

Positive charges move in direction of electric field

Negative charges move against an electric field

Positive charges will move to lower voltage

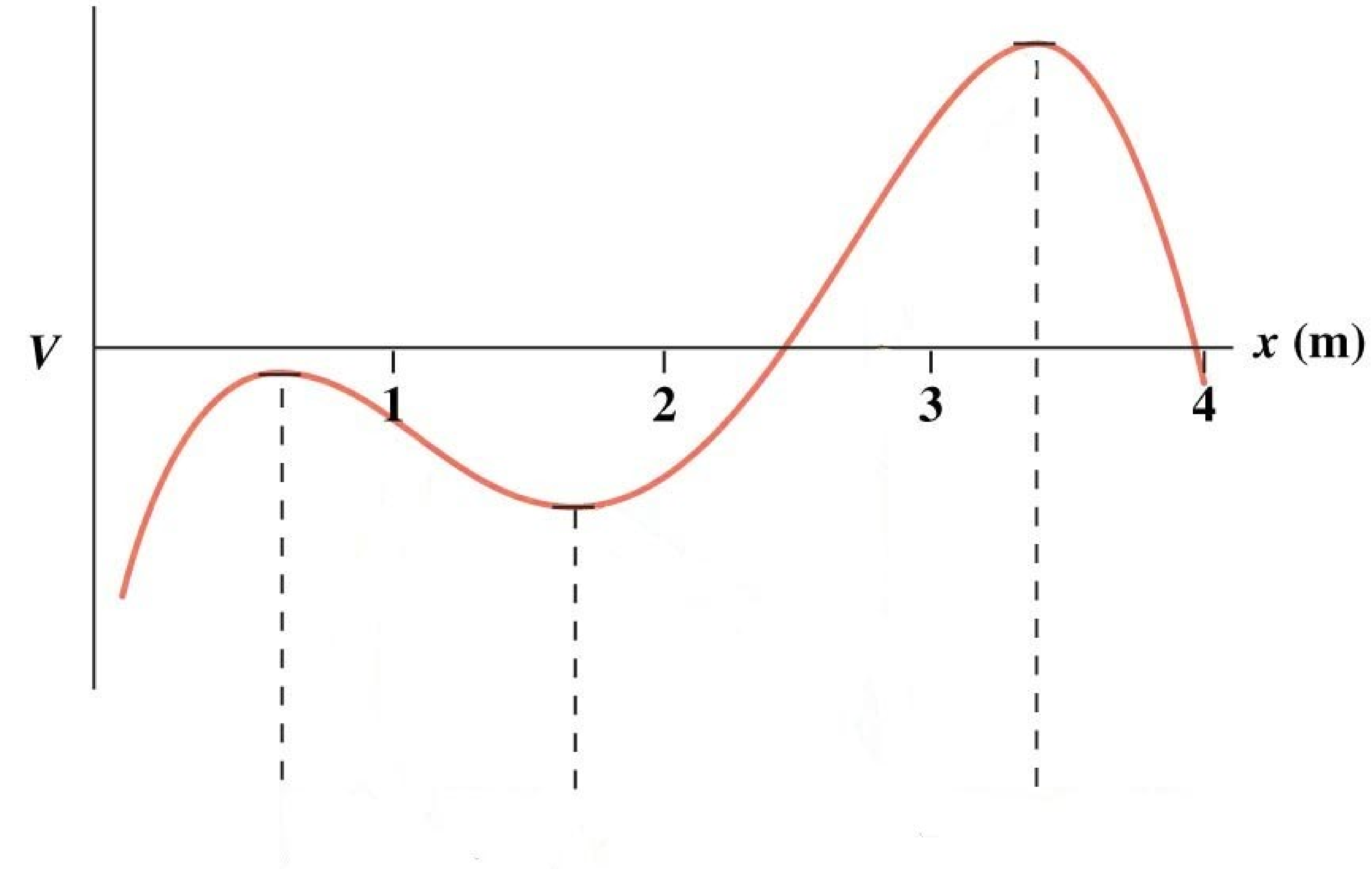
Negative charges move to higher voltages.

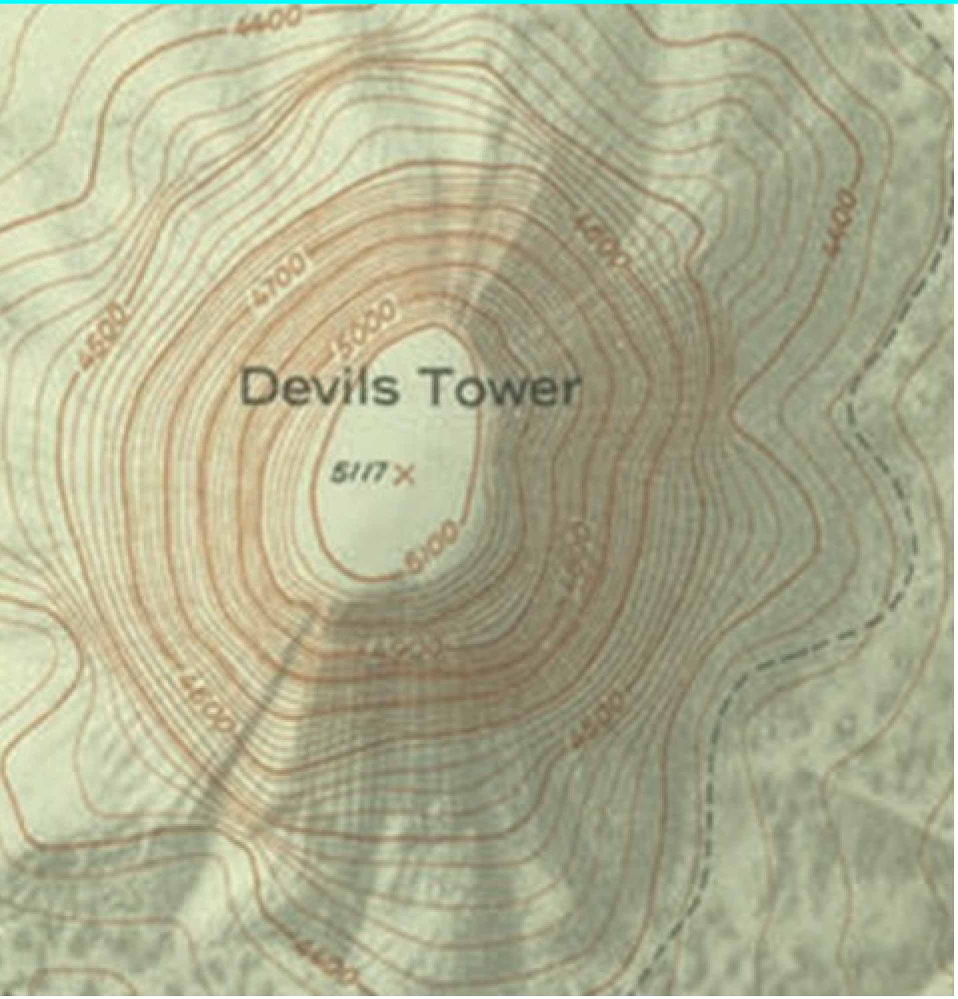
$$\mathbf{U = Q\,V} \qquad \mathbf{V = \frac{U}{Q}}$$

$$\vec{\mathbf{F}} = \mathbf{Q}\,\vec{\mathbf{E}} \qquad \vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{\mathbf{Q}}$$

Positive charges will move to lower voltage

Negative charges move to higher voltages.






(a)

(b)

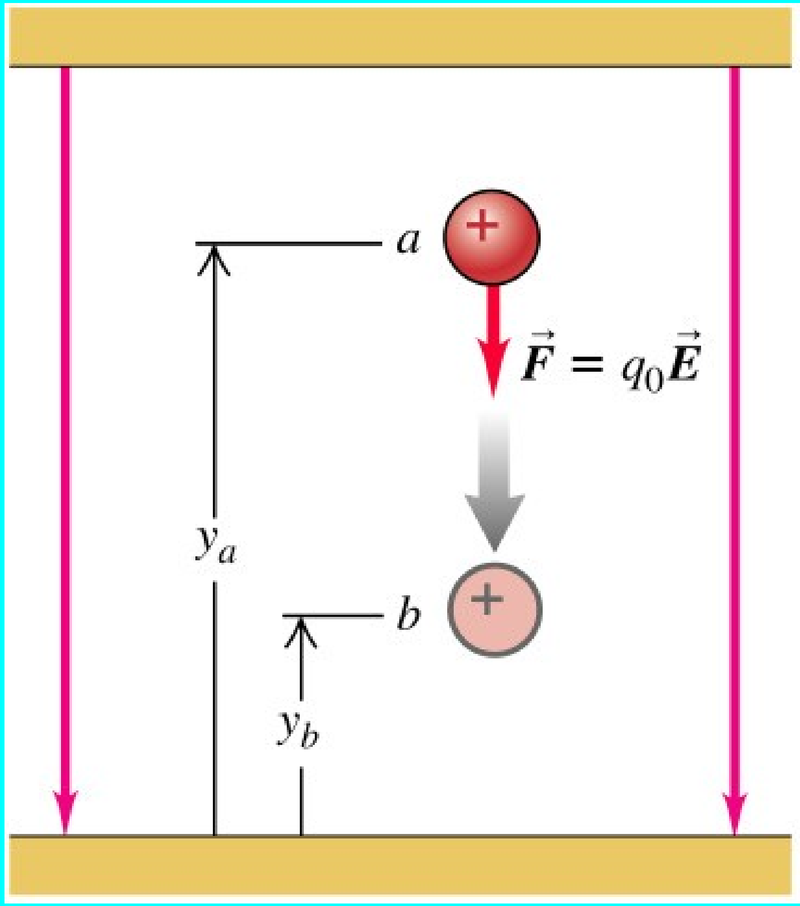
When a positive charge moves in the direction of the electric field,

A. the field does positive work on it and the potential energy increases

B. the field does positive work on it and the potential energy decreases

C. the field does negative work on it and the potential energy increases

D. the field does negative work on it and the potential energy decreases



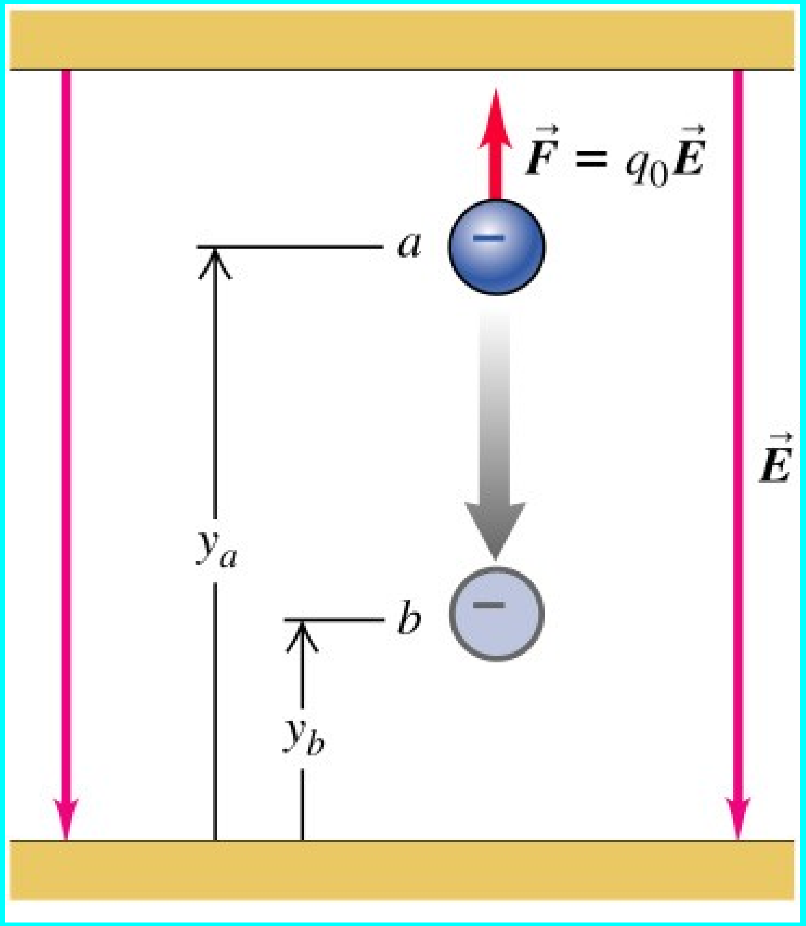
When a negative charge moves in the direction of the electric field,

A. the field does positive work on it and the potential energy increases

B. the field does positive work on it and the potential energy decreases

C. the field does negative work on it and the potential energy increases

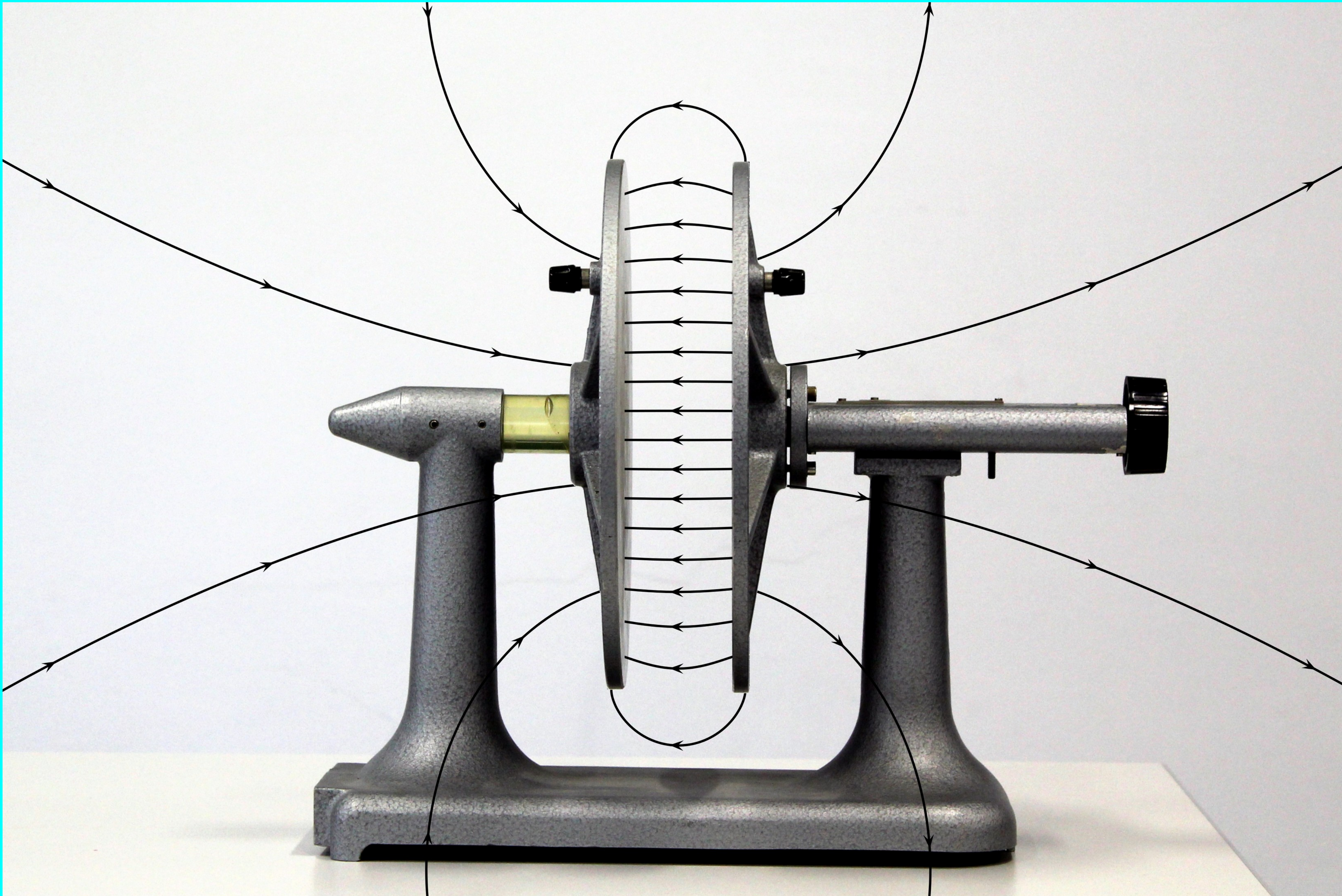
D. the field does negative work on it and the potential energy decreases

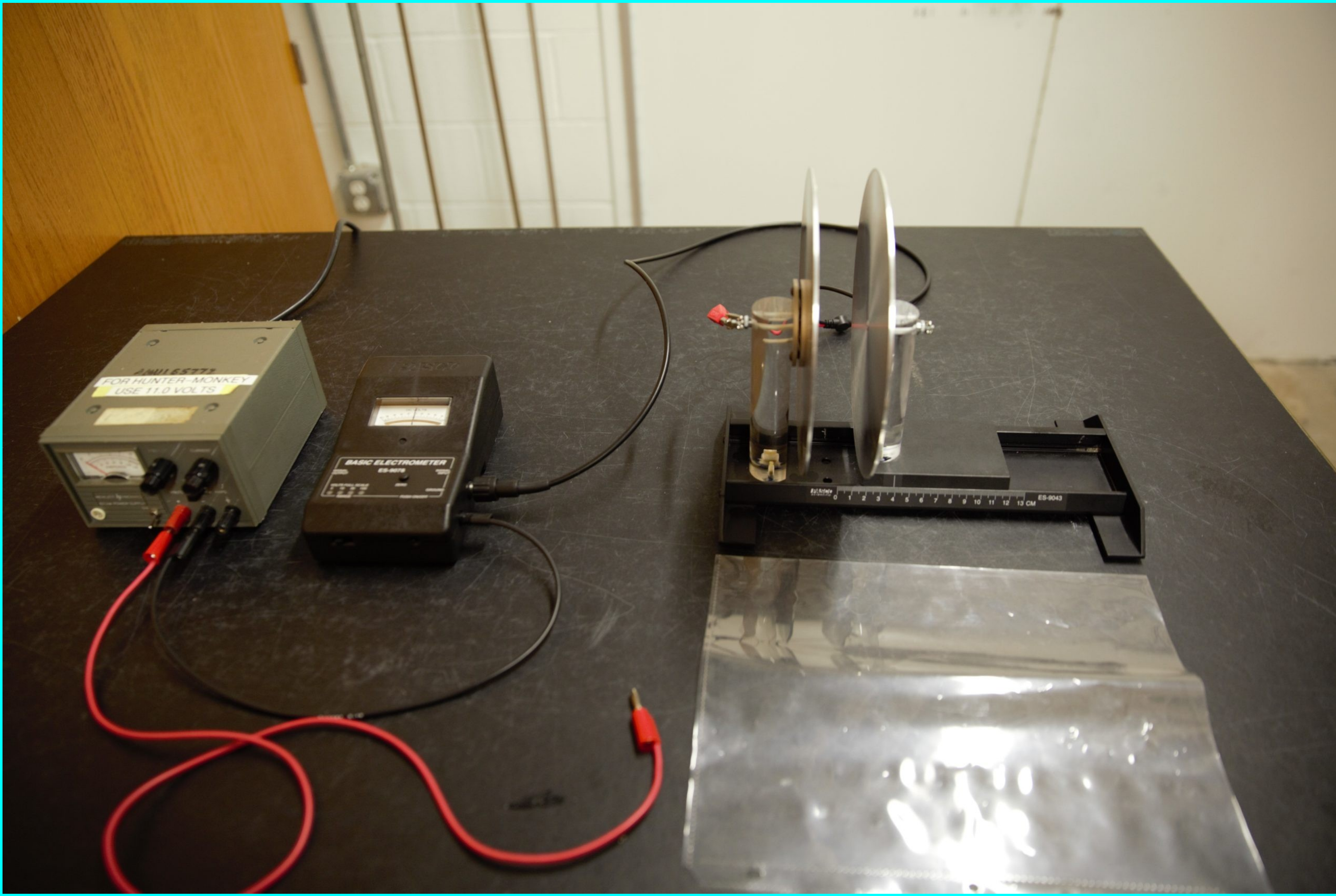


## How do you get a constant electric field?

Voltage is easiest to calculate with constant field.

It's also easy to make a constant field!





FOR HUNTER-MONKEY
USE 11.0 VOLTS
BASIC ELECTROMETER
0 1 2 3 4 5 6 7 8 9 10 11 12 13 CM
ES-9043

**Problem 13:** Two very large parallel plates are perpendicular to the $x$ axis and have a small separation, $d$. (The dimensions are distorted for purposes of visualization.) The first plate, located at $x = 0$, has a negative uniform charge density, $-\sigma$, and is designated as the zero of electric potential. The second plate, located at $x = d$, has a positive uniform charge density, $+\sigma$.

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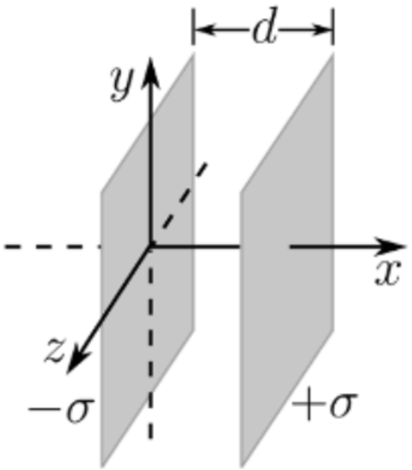





**Part (a)** In terms of the variables provided in the problem statement, enter a vector expression for the electric field, $\vec{E}$, that is valid in the gap between the two plates.

**Expression** :

$\vec{E}$ = ________________________________________

Select from the variables below to write your expression. Note that all variables may not be required.

**$\varepsilon_0$, λ, $\mu_0$, $\Phi_E$, π, ρ, σ, î, ĵ, k̂, e, n, x, y, z**

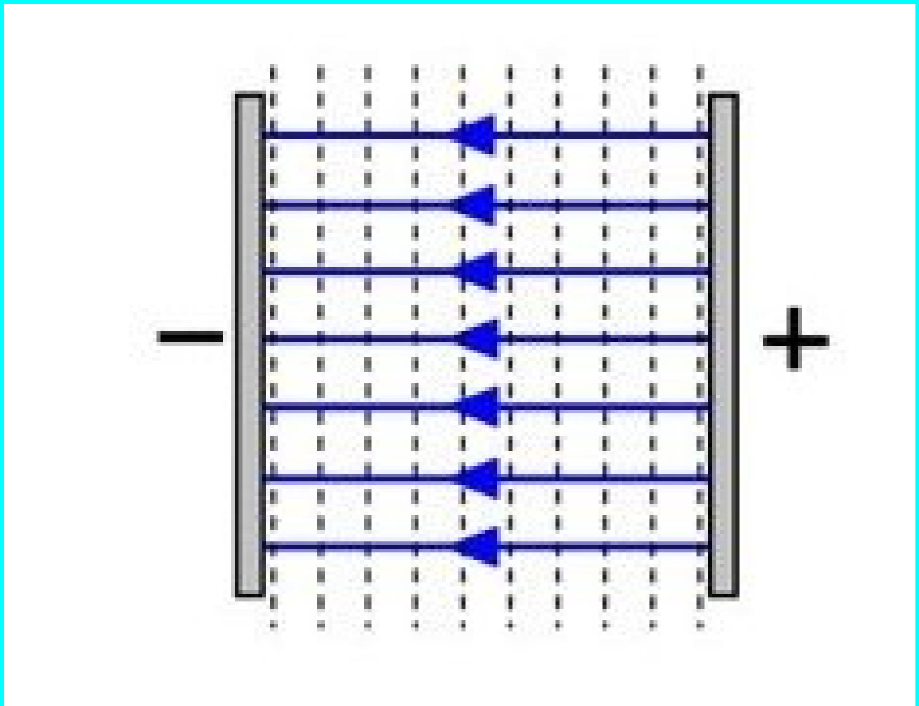
**Part (b)** In terms of the variables provided in the problem statement, enter an expression for the electric potential that is valid in the gap between the

Electric potential of TWO large plates (Area "A" separation "d") …
Electric field is doubled because two charged plates of opposite sign.

$$\vec{E} = \frac{-\sigma}{\epsilon_0}\hat{i}$$

$$V = -\vec{E}\cdot\Delta\vec{r} = E\,d$$

$$V = \frac{\sigma}{\epsilon_0}x = \frac{\sigma}{\epsilon_0}d$$



What does work have to do with Electric Potential?

$$W = \vec{F} \cdot \Delta \vec{r} = Q \vec{E} \cdot \Delta \vec{r}$$

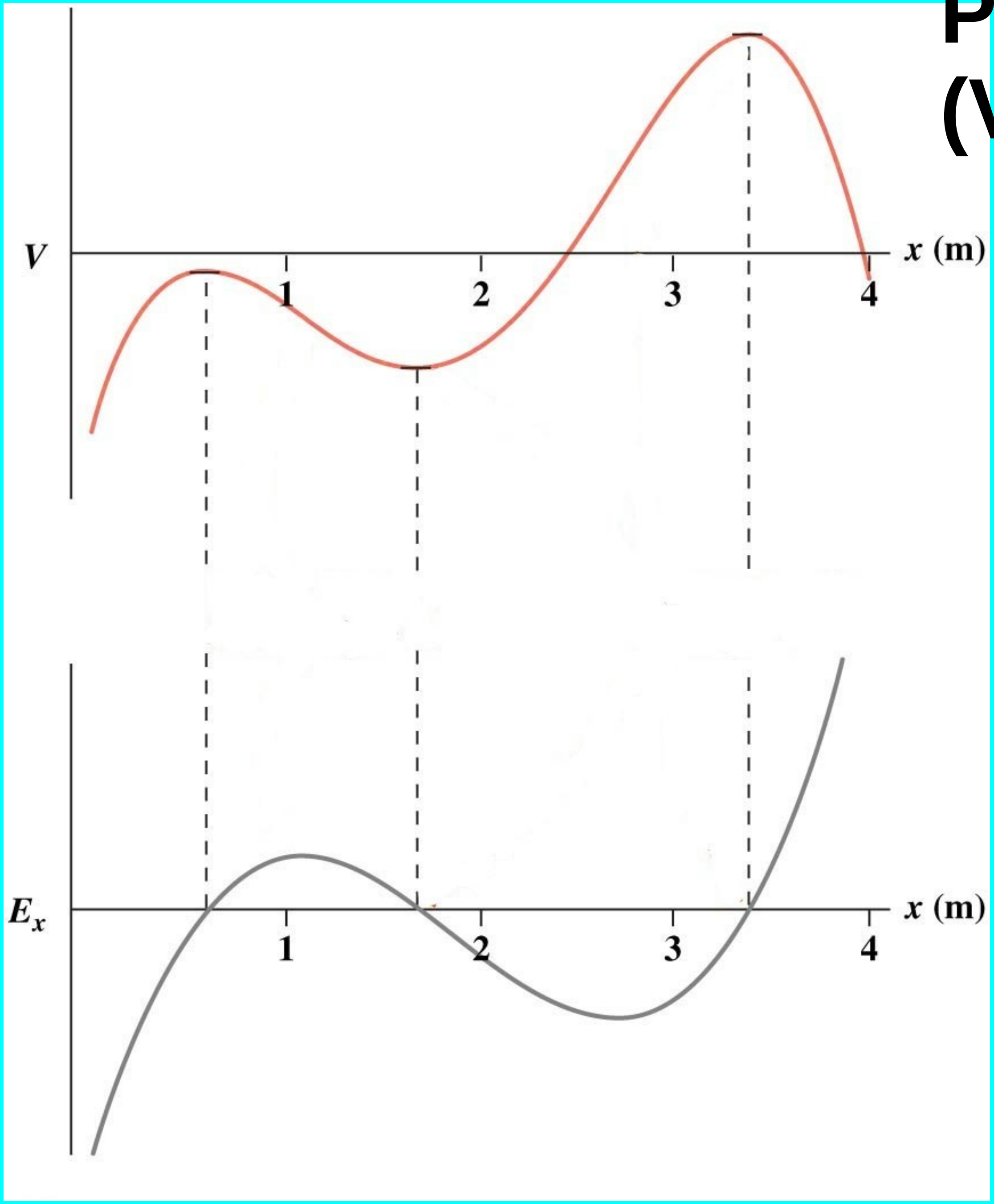
What if force or Electric field are not constant?

$$W = \int \vec{F} \cdot d\vec{r} = Q \int \vec{E} \cdot d\vec{r}$$

$$\Delta U = -W = QV = -Q \int \vec{E} \cdot d\vec{r}$$

$$\text{therefor } V \stackrel{\text{def}}{=} -\int \vec{E} \cdot d\vec{r}$$

# Potential (Voltage) in 1-D




$$V(x) = -\int E_x \, dx$$

$$E_x = -\frac{dV}{dx}$$

**You are told that the electric potential is zero At some point "P". Which statement is correct?**

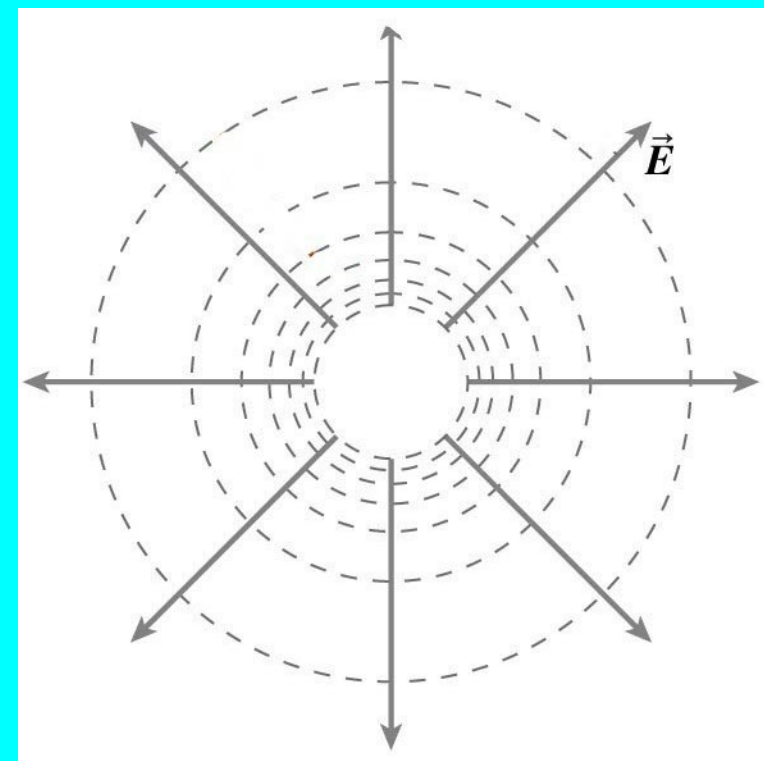
A. A charge placed at *P* would feel no electric force

B. The electric field at *P* is also zero.

C. The electric field at *P* is negative

D. Both A and B

E. not enough information given to decide



$\vec{E}$

**Voltage of a point charge:**

(Or a spherically symmetric charge)

$$\Delta V_{AB} = -\int_A^B \frac{kq}{r^2} \hat{r} \cdot d\vec{r} \qquad + \qquad A \rightarrow B$$

$$\Delta V_{AB} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

**If A goes to infinity:**

$$\Delta V_{\infty B} = kq\frac{1}{r_B} \qquad V = \frac{kq}{r}$$

It takes work to move a charge in from infinity toward another charge.

What is the potential (Voltage) (relative to infinity) halfway between two 1 nC charges spaced 2 cm apart?

$$V_1 = kq_1/r_1$$

$$V_1 = \frac{\left(9\text{x}10^9 \frac{\text{N m}^2}{\text{C}^2}\right)\left(10^{-9}\ \text{C}\right)}{10^{-2}\ \text{m}}$$

$$V_1 = \left(9\text{x}1\frac{\text{N m}}{\text{C}}\right)(1)\text{x}10^2 = 900\,\text{Volts}$$

# Potential of point charges

# Potential of point charges

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

for this system, we have

$$V_r = -\int_\infty^r \frac{kq}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} dr,$$

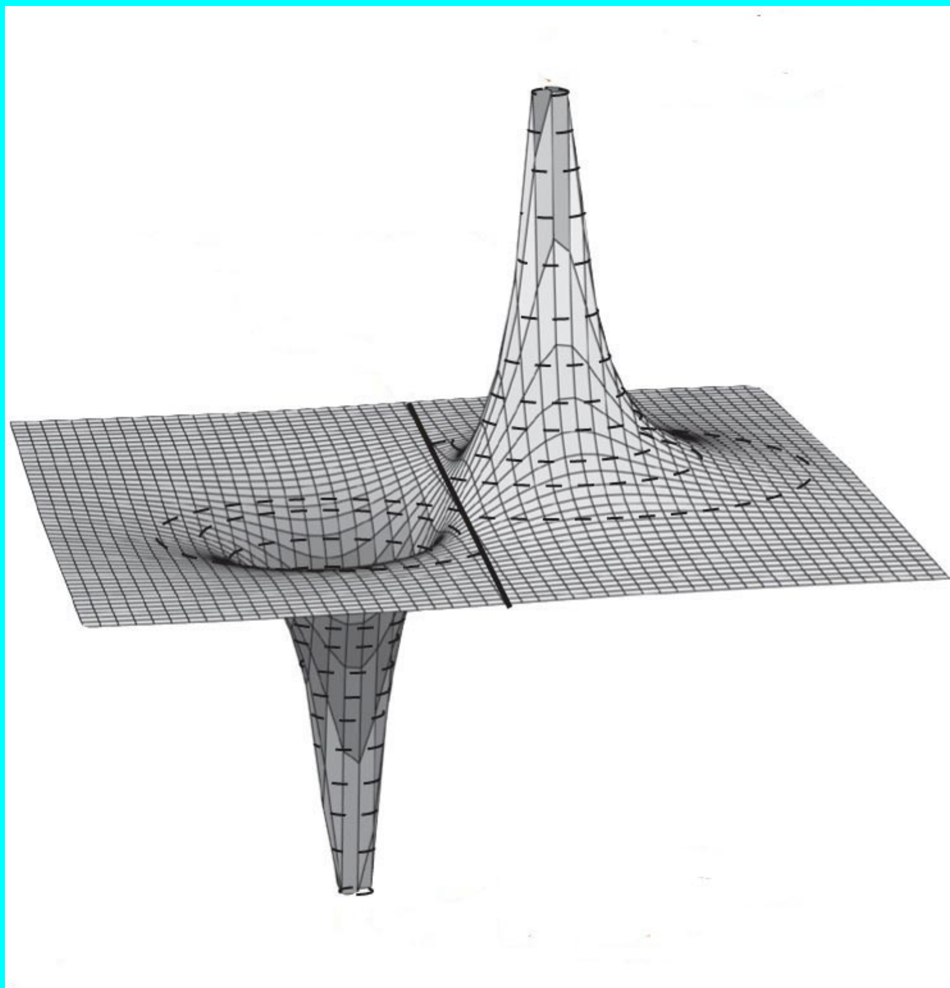
which simplifies to

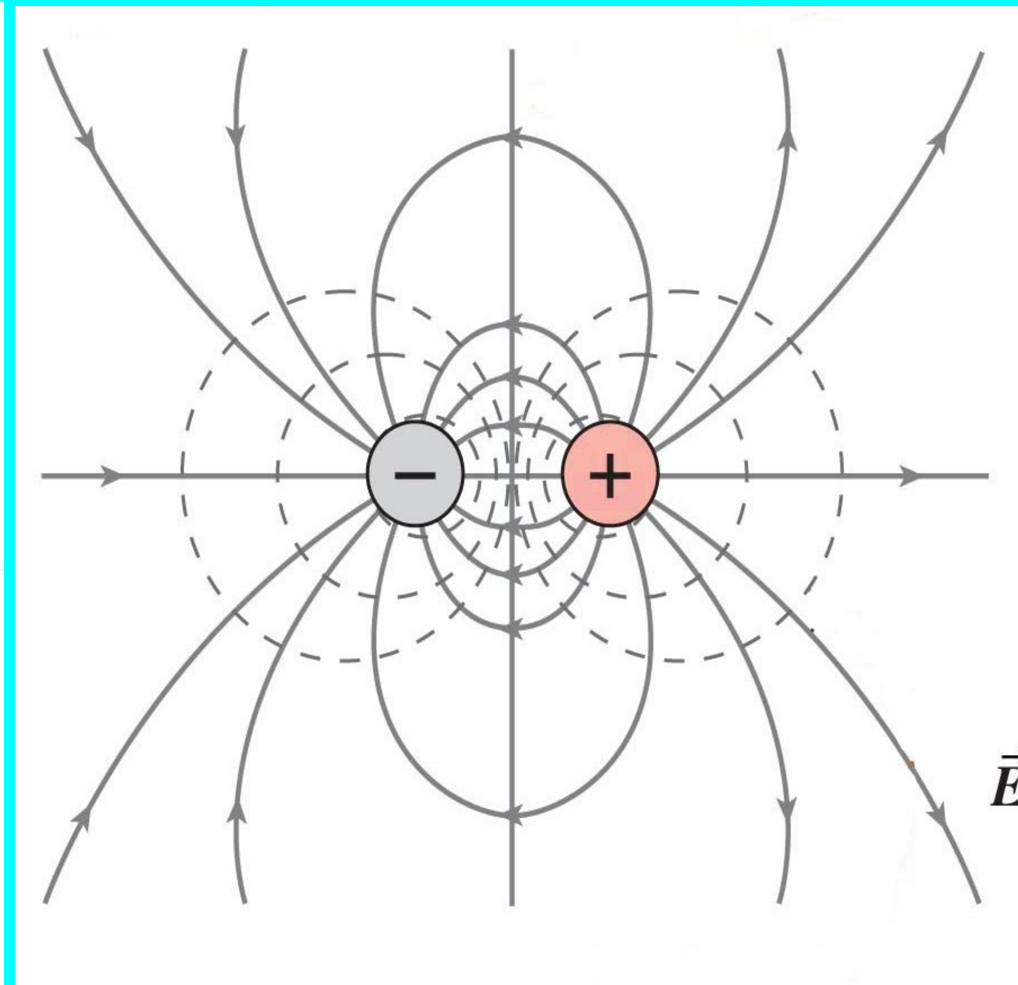


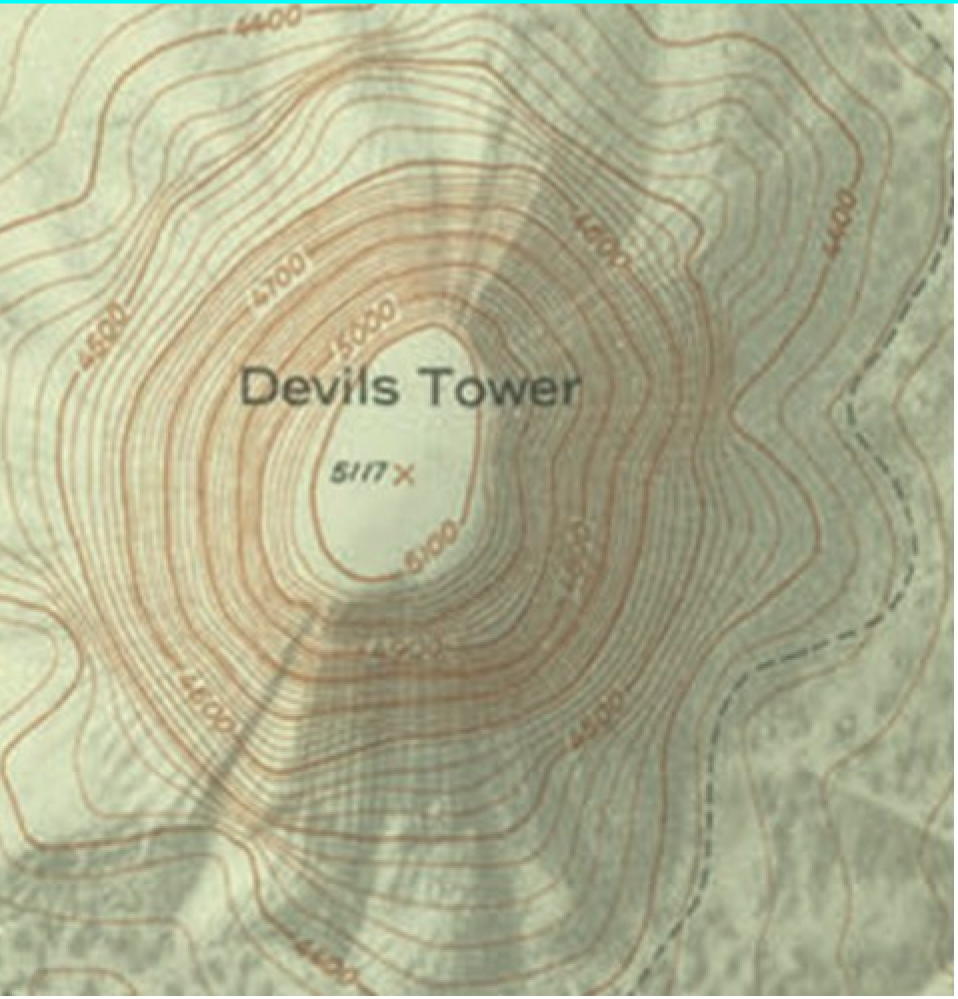


$$V_r = -\int_\infty^r \frac{kq}{r^2}\, dr = \frac{kq}{r} - \frac{kq}{\infty} = \frac{kq}{r}.$$

# Potential surface of a dipole








(a)

(b)