Lecture 10: 02/15/2024

- Announcements
  First exam – Tuesday (2/20)
  Formula sheet / calculators only.
  Send Canvas message if you need one.
  Will publish answers to sample test tho' not solutions

- Last Time
  Gauss's law
  More about flux and dot products
  Fields and conductors
  Gauss's law
  Field of symmetrical charge configurations

- Today
  - Gauss law tricks
  - Introduction to electric potential

Stress

- Life is stressful, as is college

  It's OK to ask for help
  - From me, or any prof.
  - From fellow students
  - From relatives

  - From special crisis counselors
  - Text 741741 for instant crisis connection 24/7
  - Call New Mexico “warm line” 24/7

    855-466-7100

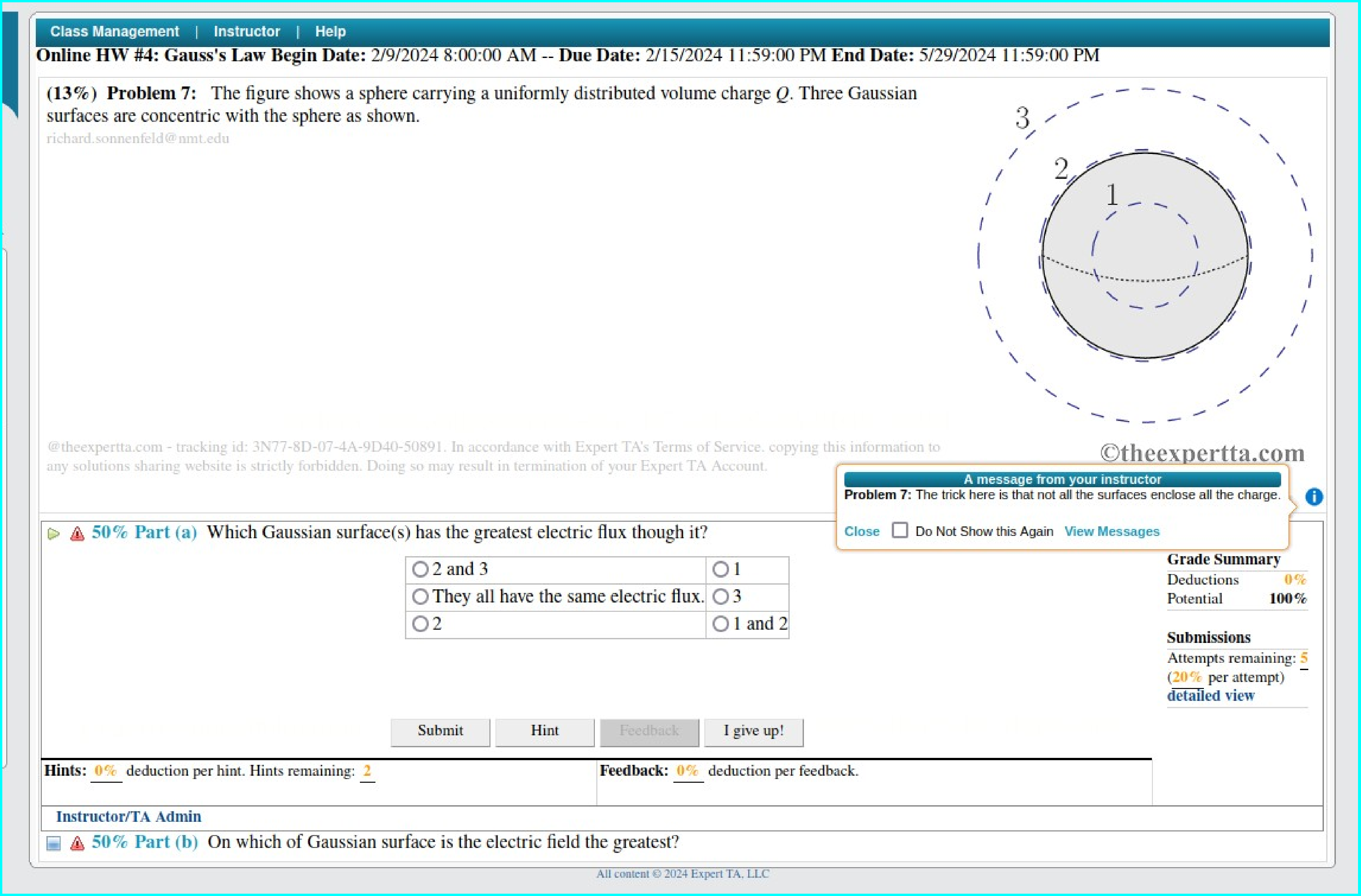
    “I just need to talk”
  - LGBTQ 866-485-7386
  - Socorro Mental Health 575-835-2444


CENTURY
CENTURY

Gauss's law

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$



Online HW #4: Gauss's Law Begin Date: 2/9/2024 8:00:00 AM -- **Due Date:** 2/15/2024 11:59:00 PM **End Date:** 5/29/2024 11:59:00 PM

(13%) Problem 7: The figure shows a sphere carrying a uniformly distributed volume charge Q. Three Gaussian surfaces are concentric with the sphere as shown.

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A message from your instructor

Problem 7: The trick here is that not all the surfaces enclose all the charge.

50% Part (a) Which Gaussian surface(s) has the greatest electric flux though it?

○ 2 and 3	○ 1
○ They all have the same electric flux.	○ 3
○ 2	○ 1 and 2

Grade Summary

Deductions	0%
Potential	100%

Submissions

Attempts remaining: 5

(20% per attempt)

detailed view

Hints: 0% deduction per hint. Hints remaining: 2

Feedback: 0% deduction per feedback.

50% Part (b) On which of Gaussian surface is the electric field the greatest?

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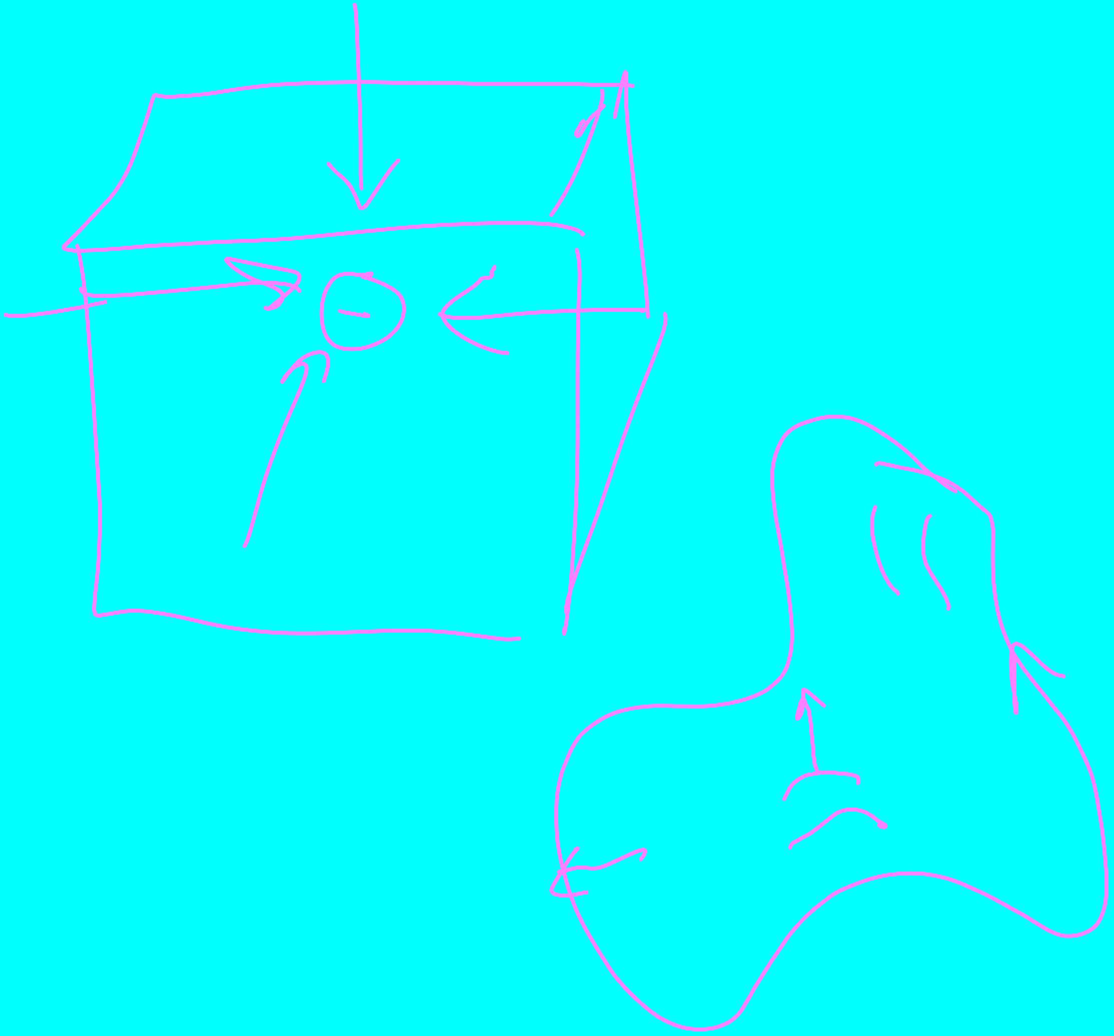
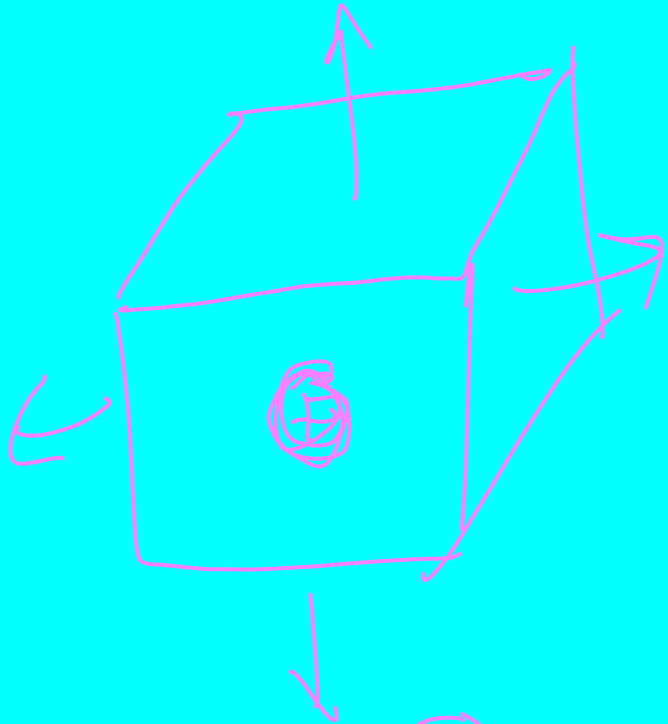
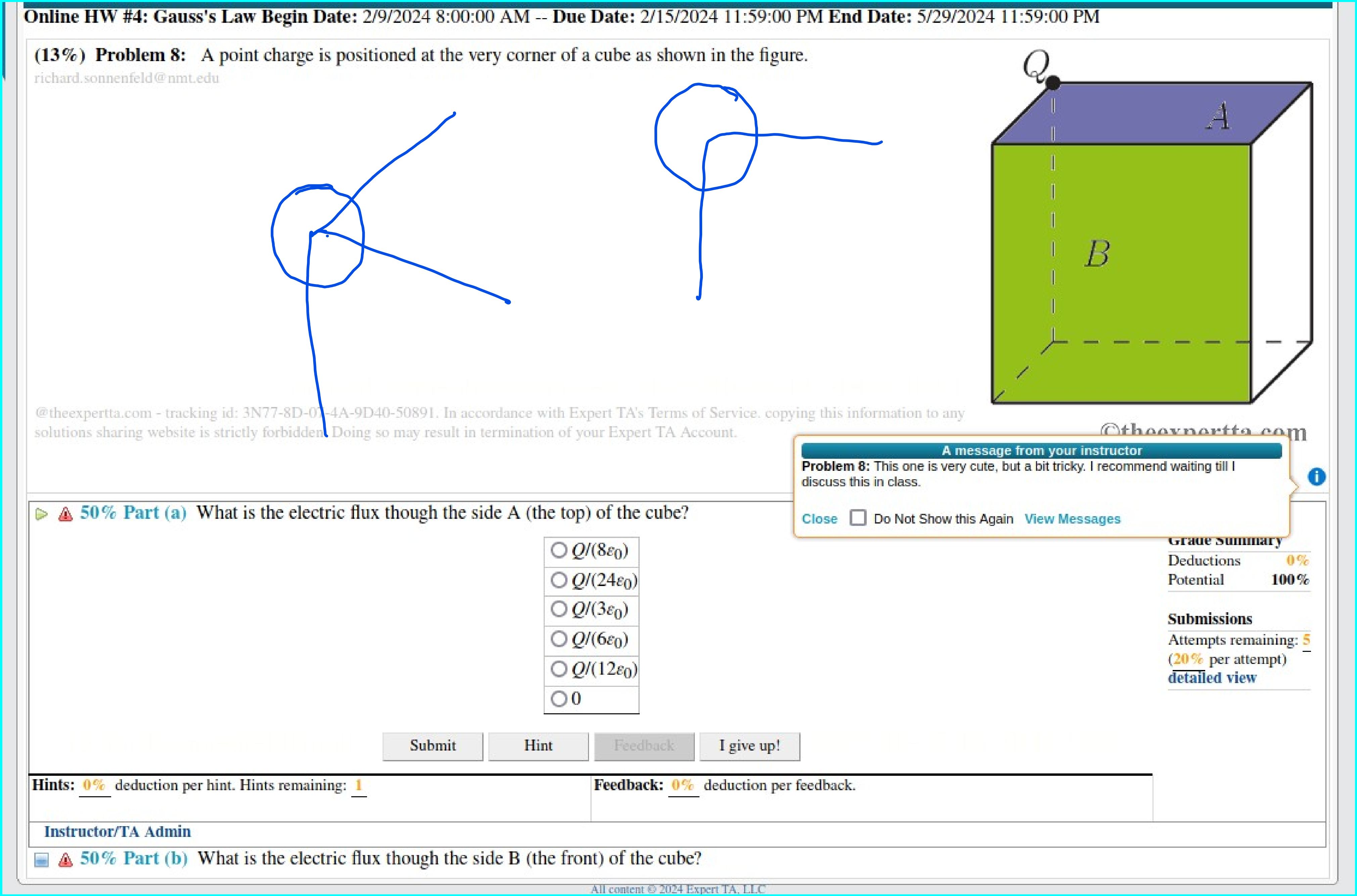




$$\Phi = \frac{q}{\epsilon_0}$$



Online HW #4: Gauss's Law Begin Date: 2/9/2024 8:00:00 AM -- **Due Date:** 2/15/2024 11:59:00 PM **End Date:** 5/29/2024 11:59:00 PM

(13%) Problem 8: A point charge is positioned at the very corner of a cube as shown in the figure.

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A message from your instructor

Problem 8: This one is very cute, but a bit tricky. I recommend waiting till I discuss this in class.

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50% Part (a) What is the electric flux though the side A (the top) of the cube?

○ $Q/(8\varepsilon_0)$
○ $Q/(24\varepsilon_0)$
○ $Q/(3\varepsilon_0)$
○ $Q/(6\varepsilon_0)$
○ $Q/(12\varepsilon_0)$
○ 0

Grade Summary

Deductions	0%
Potential	100%

Submissions

Attempts remaining: 5

(20% per attempt)

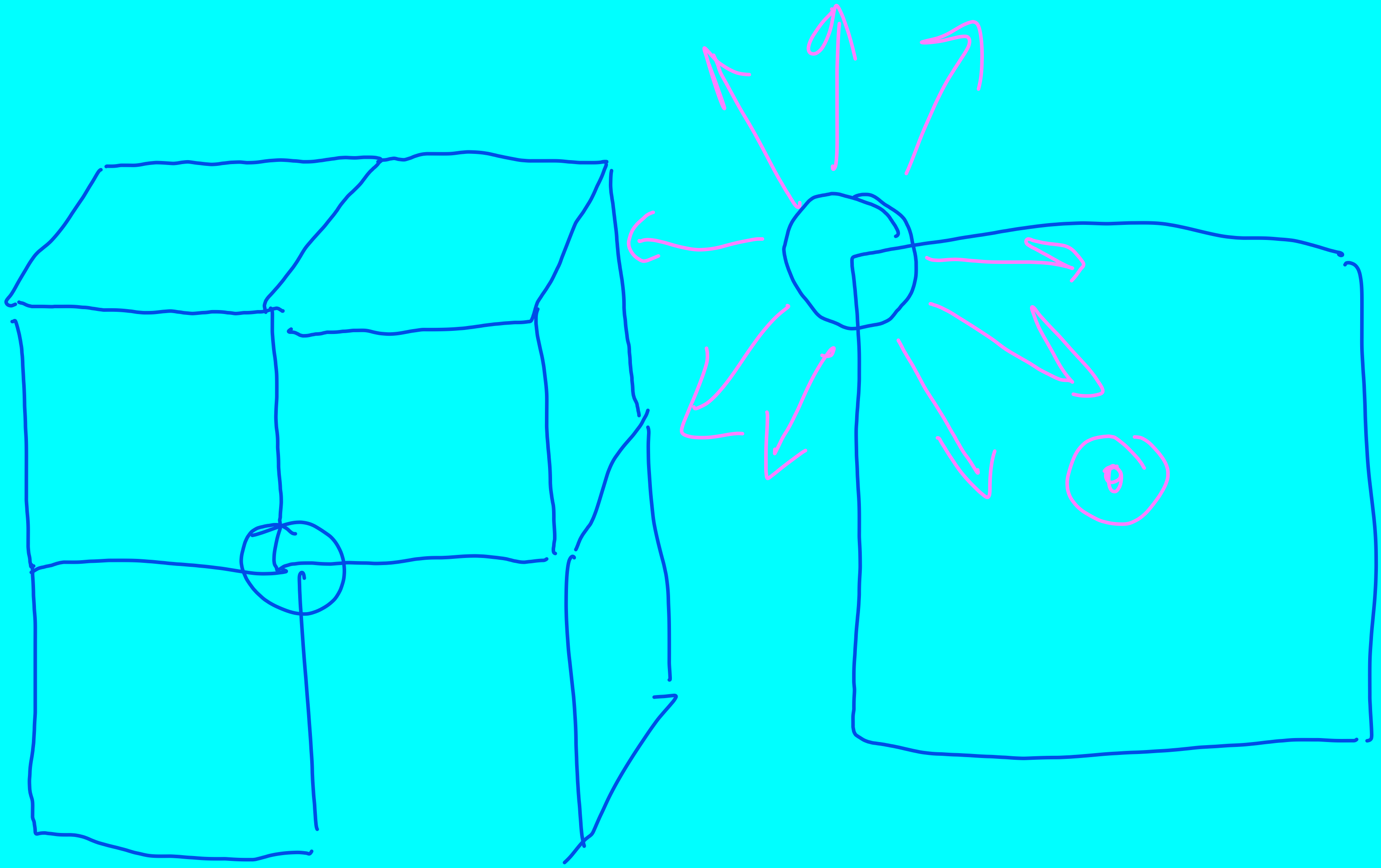
detailed view

Hints: 0% deduction per hint. Hints remaining: 1

Feedback: 0% deduction per feedback.

Instructor/TA Admin

50% Part (b) What is the electric flux though the side B (the front) of the cube?



Gauss's law for symmetrical cases

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

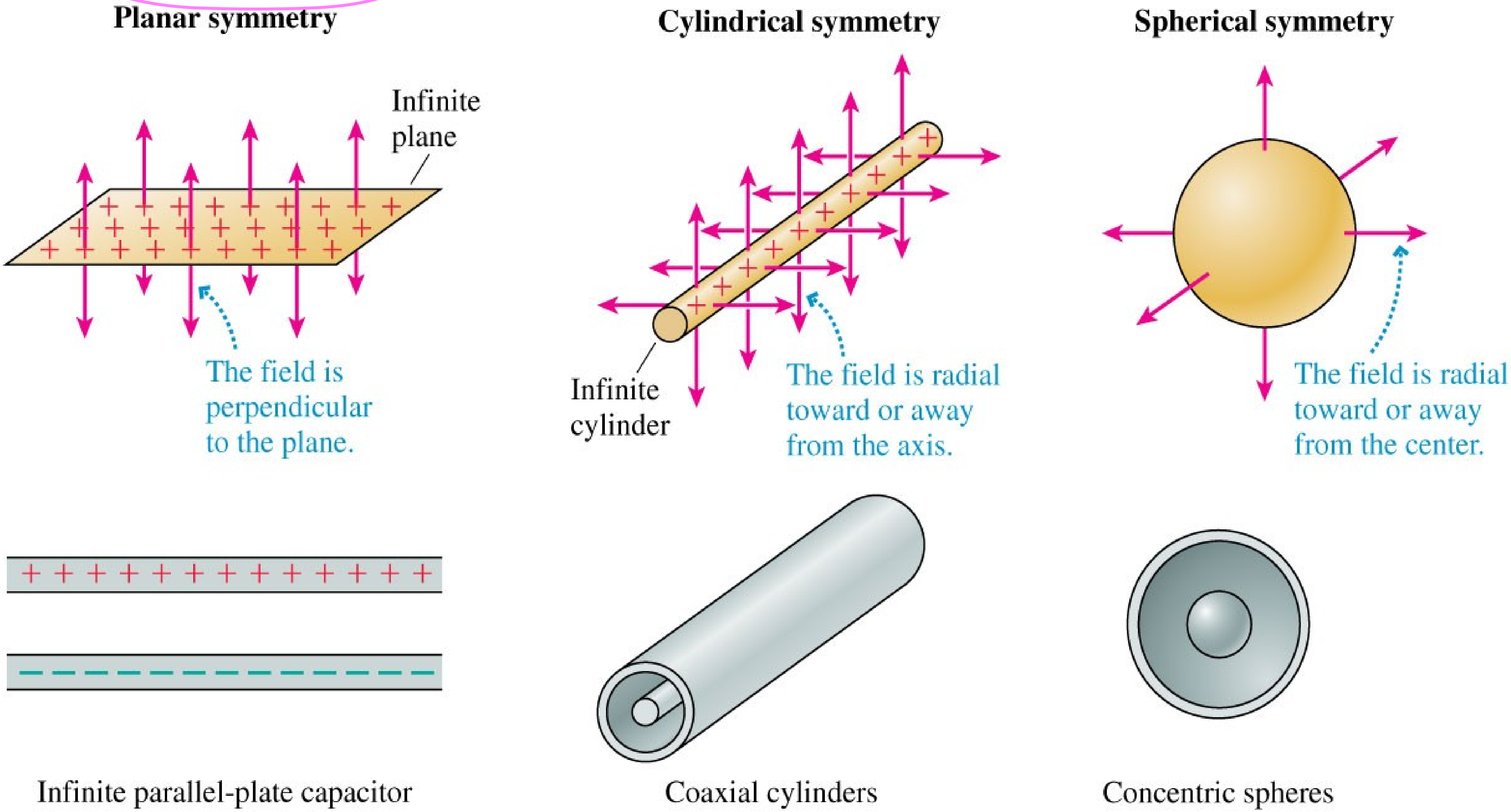
$$\underline{E \times (\text{Surface Area})} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$\oint \vec{E} \cdot d\vec{a}$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k}$$

$$\vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r}$$

$$\vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}$$



Planar symmetry

The field is perpendicular to the plane.

Infinite parallel-plate capacitor

Cylindrical symmetry

The field is radial toward or away from the axis.

Coaxial cylinders

Spherical symmetry

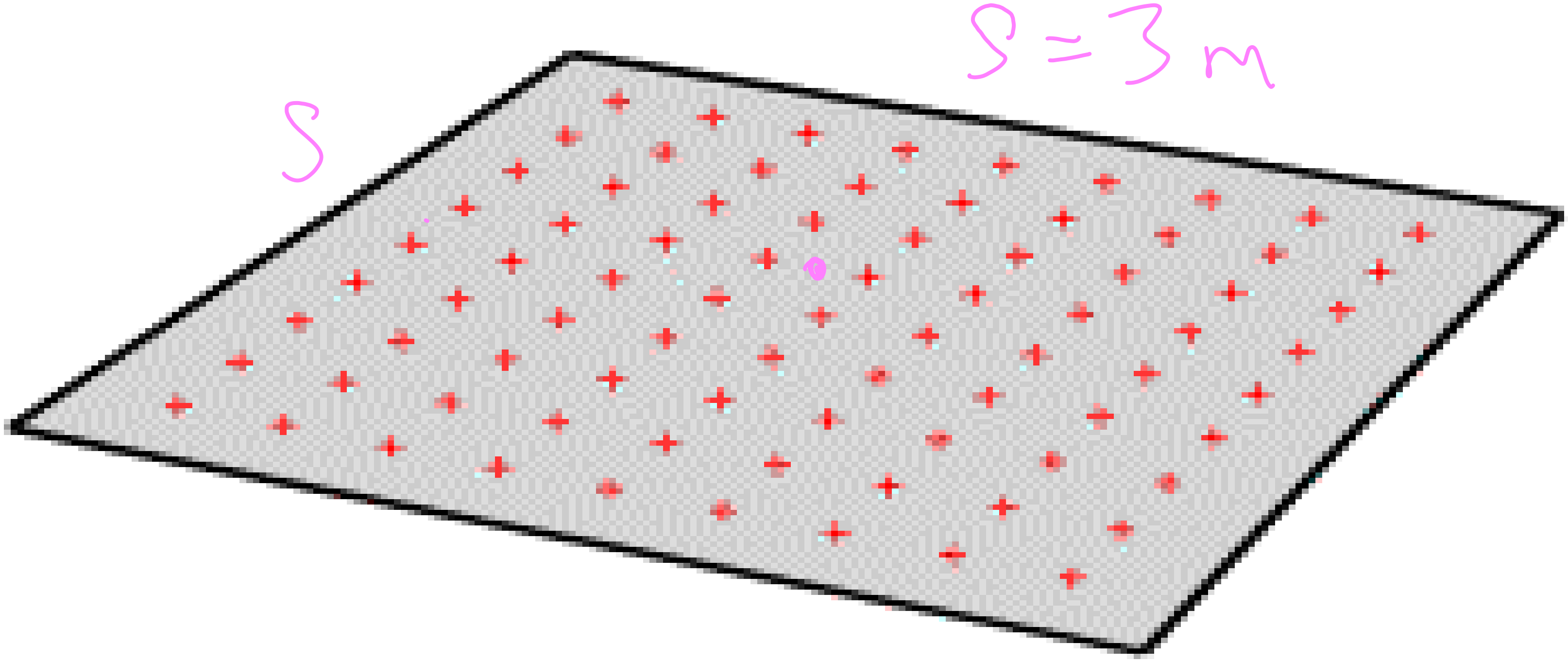
The field is radial toward or away from the center.

Concentric spheres

ucation, Inc.

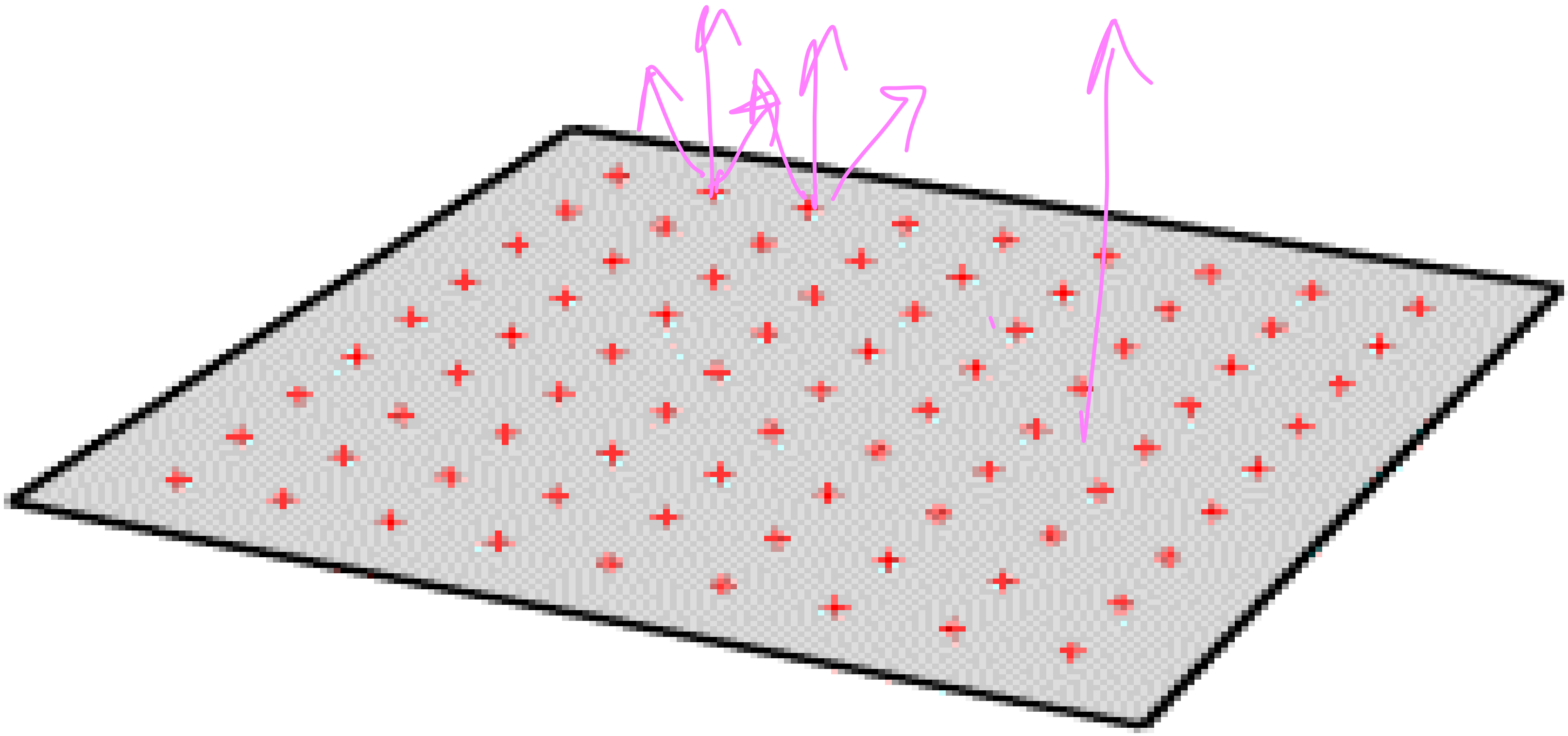
Relation between symmetry and Electric Field

Imagine an infinite plane of charge.



Relation between symmetry and Electric Field

Because you can't tell what direction you are facing, the field must be ONLY Perpendicular to the plane.

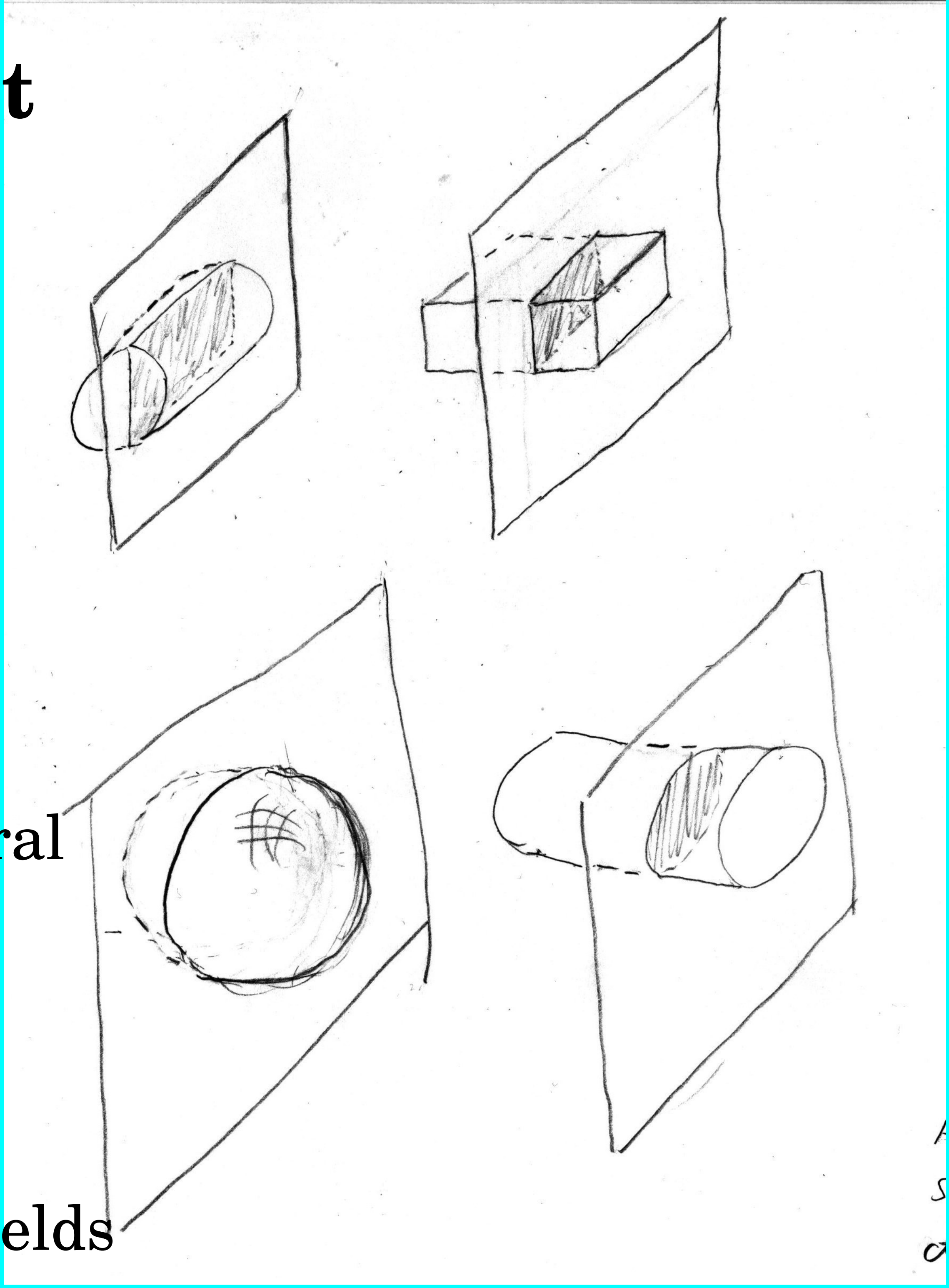


Why would I want a Gaussian surface?

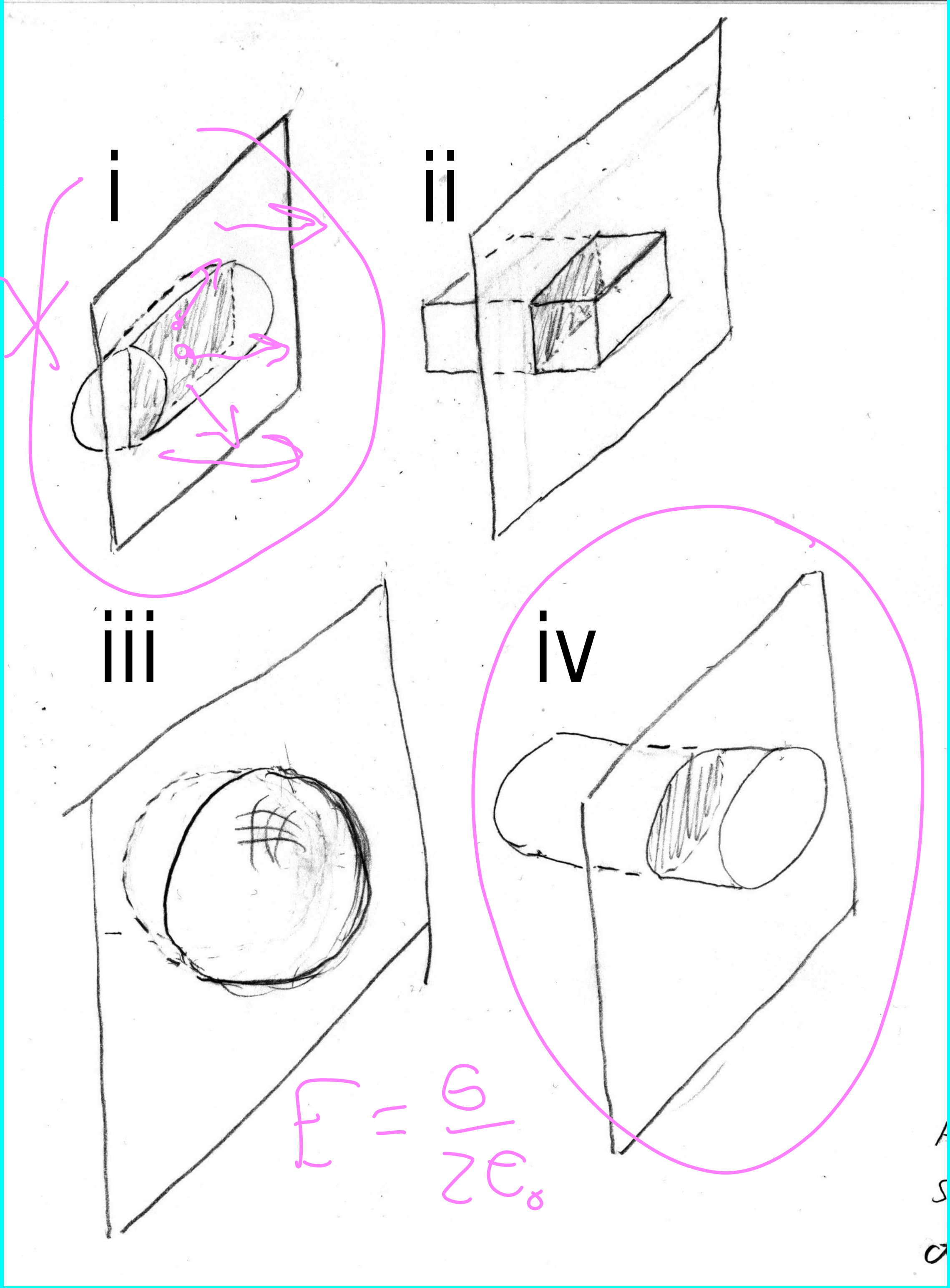
A Gaussian surface
is any surface encloses
some of a
charge distribution.

You draw it to have the
correct symmetry to
to turn the surface integral
into mere multiplication.

It can lead directly to
A formula or provide
Inside into the behavior
Of materials in electric fields



Which of these surfaces are best to calculate the field of a charged plate?



$E = \frac{\sigma}{2\epsilon_0}$

(A) i

(B) i or ii

(C) ii or iii

(D) i or iv

(E) ii or iv

Symmetrical Case: Large (infinite) Plane

$\vec{E} \cdot \text{Surface Area} = \frac{q_{enc}}{\epsilon_0}$

$\sigma = \frac{Q}{a} = \frac{C}{m^2}$

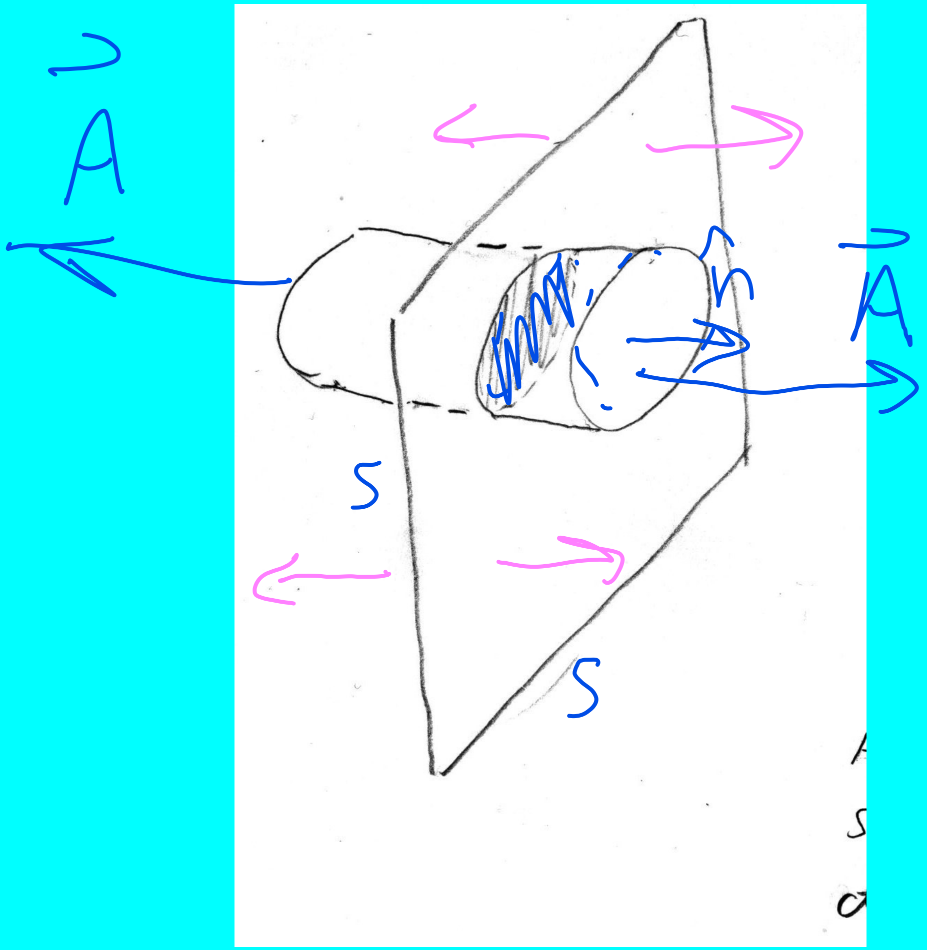
$A = 2\pi R^2$ $\quad E = q_{enc}/\epsilon_0$

$a = s^2$

$\Phi = \frac{q}{\epsilon_0}$

$\lambda = \frac{Q}{L}$

$\rho = \frac{Q}{V}$



$q_{enc} = \sigma A$
$= \sigma \pi R^2$

$E(2\pi R^2) = \frac{\sigma \pi R^2}{\epsilon_0}$

$\boxed{E = \frac{\sigma}{2\epsilon_0}}$

Infinite Plane

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\epsilon_0 = 8.85 \times 10^{-12}$$

(A) 8.85×10^{-5} N/C

(B) 4.43×10^{-5} N/C

[✓ **(C)** 5.00×10^{6} N/C]

(D) 1.00×10^{7} N/C

(E) 1.00×10^{8} N/C

Infinite Plane

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

$$\epsilon_0 = 8.85\times10^{-12}$$

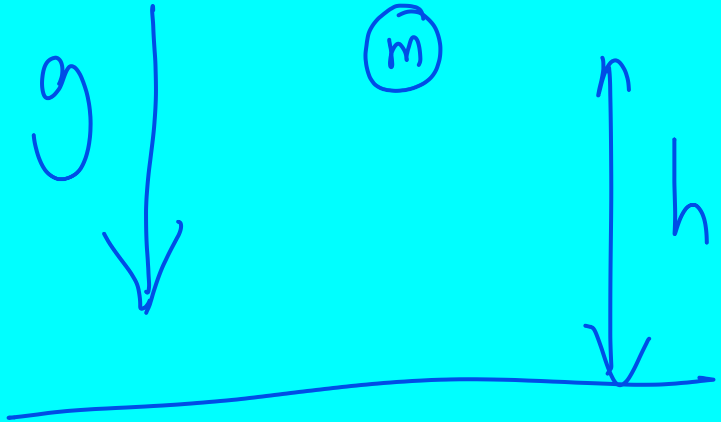
$$E = \frac{\sigma}{2\epsilon_0}$$

$$= \frac{8.85\times10^{-5}}{(2)(8.85\times10^{-12})}$$

$$\frac{10^7}{2} = 5\times10^6$$

$$\sigma = \frac{Q}{A} = \frac{Q}{s^2}$$

$$\sigma = \frac{Q}{10^2} = 8.85\times10^{-5}\,\frac{C}{m^2}$$



$$h = \frac{1}{2} g t^2$$

$$v = g t$$

$$\boxed{v_f^2 - v_i^2 = 2 g \Delta x}$$

$$U = mgh \qquad \Delta U = \Delta K$$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_f^2 - \frac{1}{2} \cancel{m} v_i^2$$

$E = 10^4\ N/C$

$v = ?$

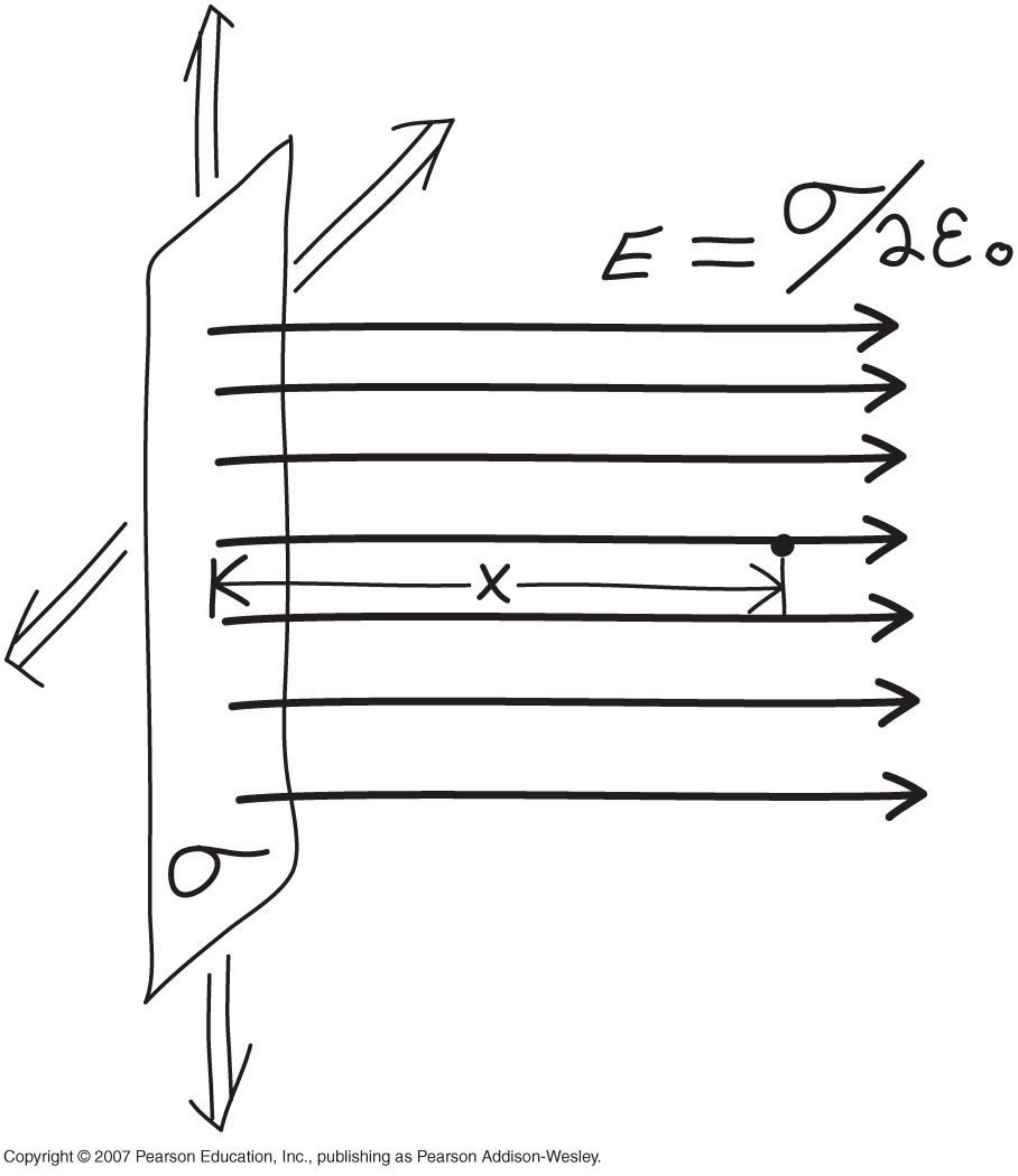
$\vec{F} = Q\vec{E} = (1.6 \times 10^{-19})(10^4)$

$m\vec{a} = Q\vec{E}$

$$a = \frac{(1.6 \times 10^{-19})(10^4)}{9.0 \times 10^{-31}\ kg} = \frac{1.6 \times 10^{-15}}{9 \times 10^{-31}} = 2 \times 10^{16}$$

$a = 2.54 \times 10^{15}\ m/s^2$

$v_f^2 - v_i^2 = 2a\Delta X$



An infinite sheet of charge creates a field of 10000 V/m. N/C

An electron moves 10 cm in this field.

What is the final speed of the electron?

$$m_e = 9.0 \times 10^{-31}\,\text{kg}$$

Electric Potential:
What's a volt anyway?

Electric Potential:

Is also called voltage. They are the same thing.

Electric potential is measured in Volts. (SI unit)

The term potential ties back to potential energy because potential IS the potential energy per unit charge.

$$\mathbf{U = Q\,V} \qquad \mathbf{V = \frac{U}{Q}}$$

$$U = mgh = F \text{ distance}$$

$$U = qEh$$

$$V = \int \vec{E}(x)\,dx$$

Electric Potential and Electric field:

We did a similar trick with electric field and force

Electric field ties back to force because field is the force per unit charge.

$$\mathbf{U} = \mathbf{Q}\,\mathbf{V} \qquad \mathbf{V} = \frac{\mathbf{U}}{\mathbf{Q}}$$

$$\vec{\mathbf{F}} = \mathbf{Q}\,\vec{\mathbf{E}} \qquad \vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{\mathbf{Q}}$$

V

You have a 12.0-V motorcycle battery that can move 5000 C of charge, and a 12.0-V car battery that can move 60,000 C of charge. How much energy does each deliver?

Q

$U = QV$

(A) 60 and 720 kiloJoules

(B) 5 and 60 kiloJoules

(C) 417 and 5000 Joules

(D) 417 and 5000 Watts

(E) Both the same because both 12 Volts

In a large lightning flash, the potential difference between a cloud and the ground is 100 Million Volts and the quantity of charge transferred is 30 C.

How much potential energy is Released?

$V = 10^8$ Volts

$Q = 3 \times 10^1$ C

$U = QV = 3 \times 10^9$ J

$$\text{Power} = \frac{U}{t} = \frac{3 \times 10^9}{10^{-3}} = 3 \times 10^{12}$$

$\underline{\text{3 TeraWatts}}$

To what speed could you accelerate a 2000 kg car with this much energy?

$U = 3 \times 10^9$ Joules ←

$\Delta U = \Delta KE = \frac{1}{2} m v_f^2 = 10^9$

3×10^9

(A) 1.73 m/s

(B) 12.2 m/s ¿

(C) 122 m/s

[✓ **(D)** 1730 m/s]

(E) Mach 2

$3 \times 10^9 = \frac{1}{2} 2000 v_f^2$

$\frac{3 \times 10^9}{1000} = 3 \times 10^6 = v^2$

1.73×10^3

Electric potential is usually expressed in Volts, but it can be expressed in terms of other SI Units.

What other units are appropriate for electric potential?

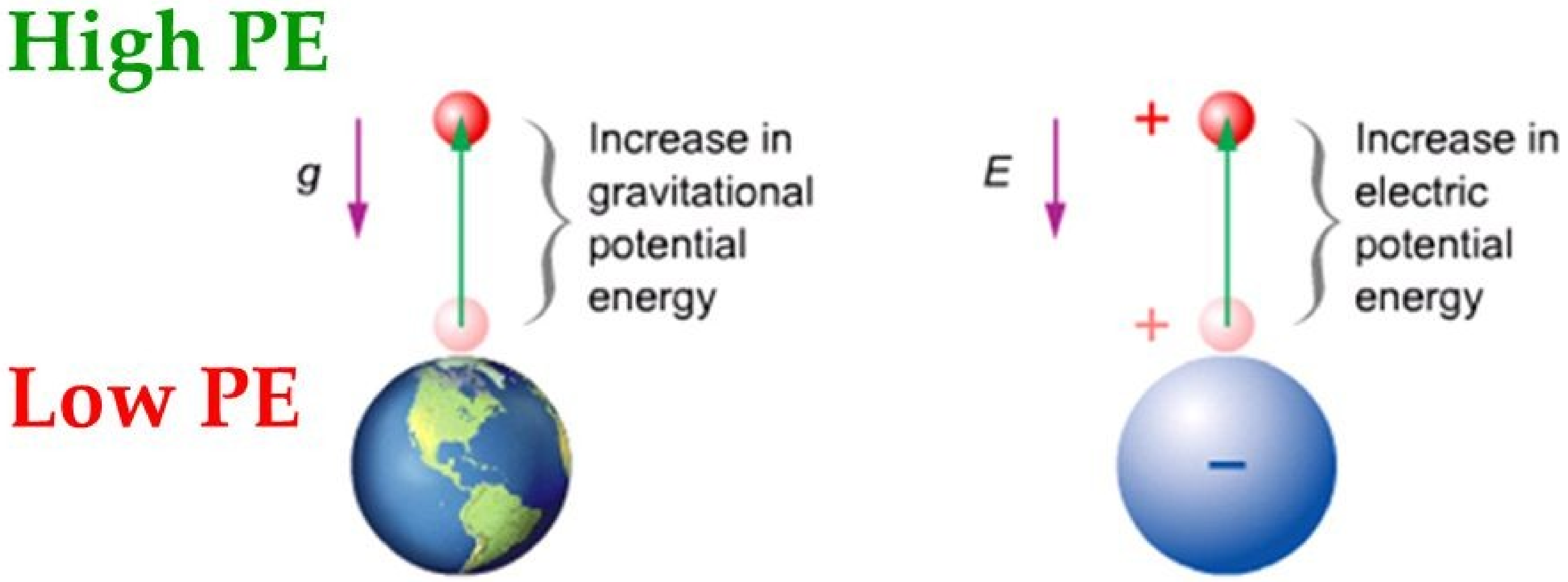
(A) $\frac{\text{N}}{\text{C}}$

(B) $\frac{\text{V}}{\text{m}}$

(C) $\frac{\text{J}}{\text{C}}$

(D) $\text{N} \cdot \text{m}$

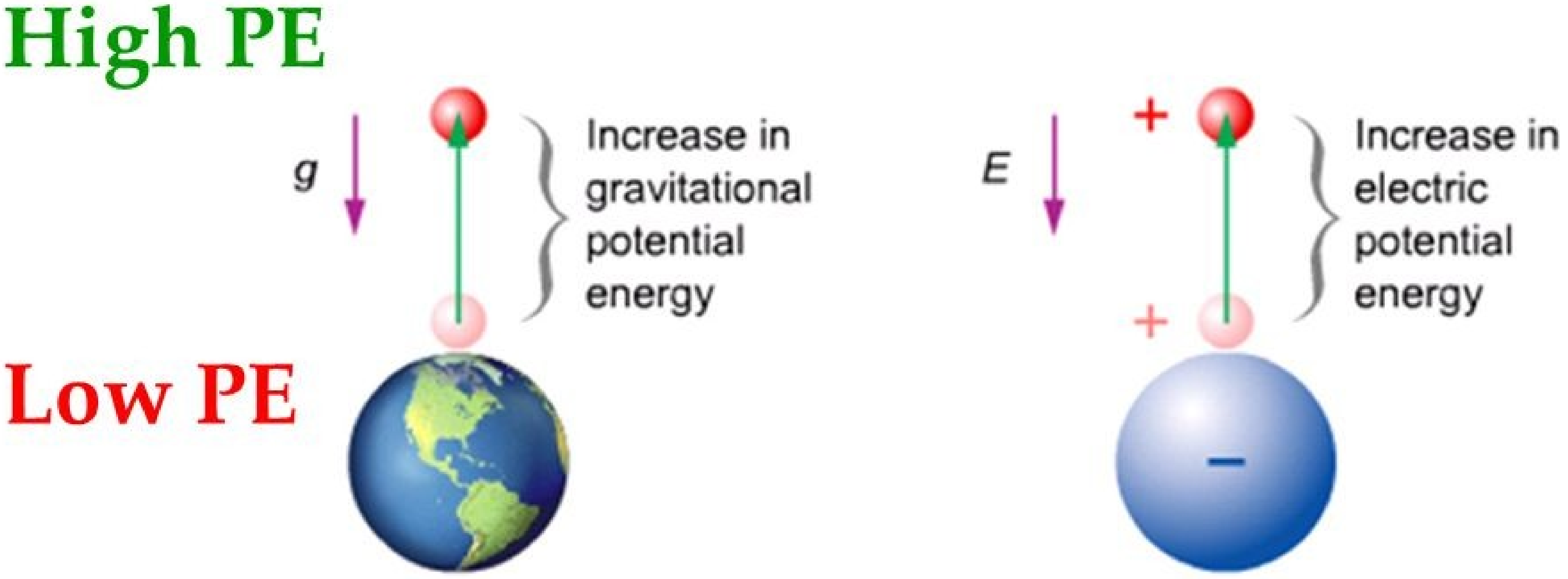
Potential Energy



Doing work against a gravity field raises the gravitational potential energy of a mass

Doing work against an electric field raises the electric potential energy of a charge

Potential Energy



$$\vec{F} = m\vec{g}$$

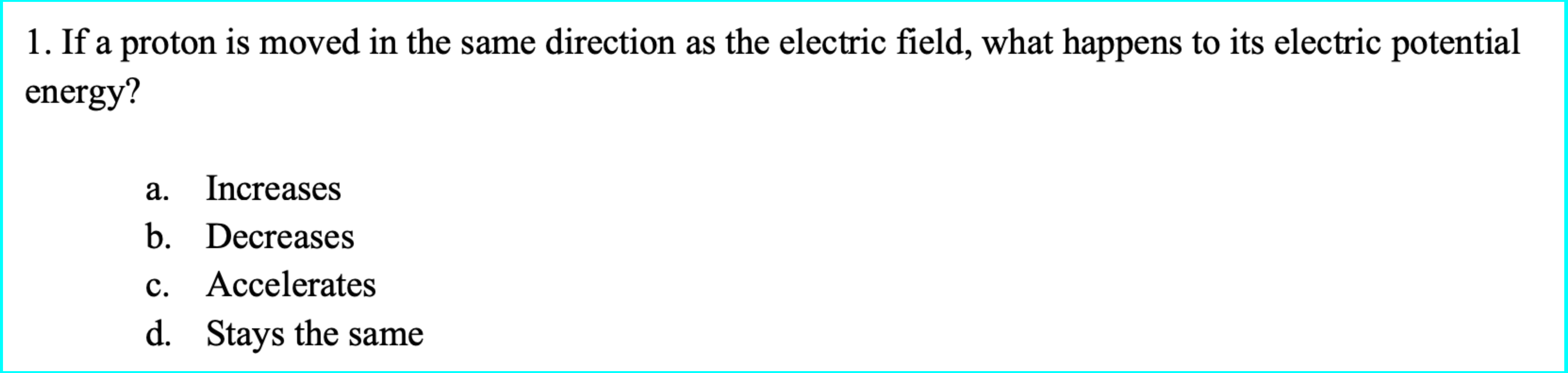
$$U = mgh$$

$$\text{geopotential} = gh$$

$$\vec{F} = Q\vec{E}$$

$$U = QEh$$

$$V = Eh$$



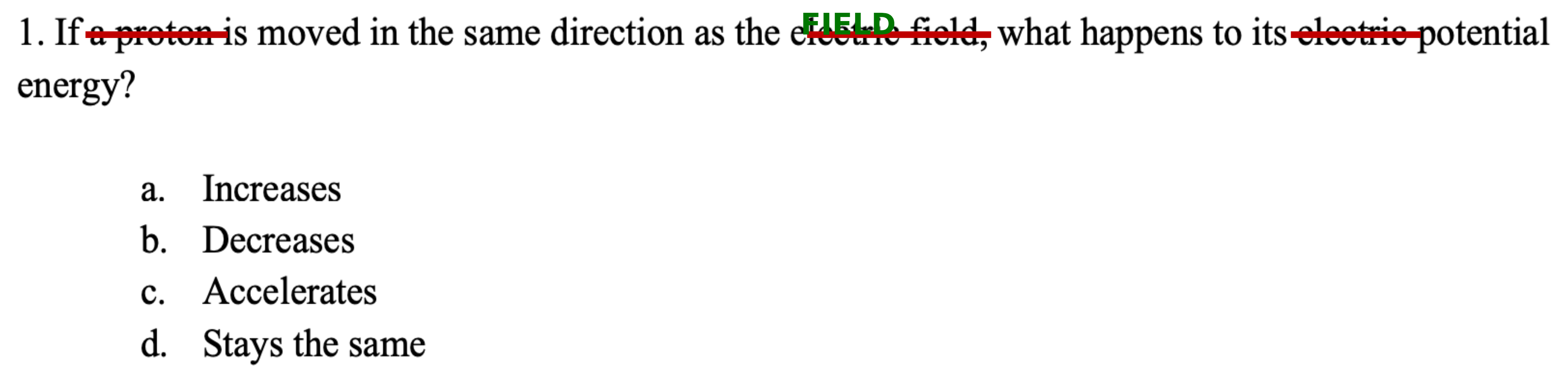
1. If a proton is moved in the same direction as the electric field, what happens to its electric potential energy?

   a. Increases
   b. Decreases
   c. Accelerates
   d. Stays the same

POSITIVE MASS

GRAVITY FIELD

GRAVITATIONAL



1. If ~~a proton~~ is moved in the same direction as the ~~electric field,~~ what happens to its ~~electric~~ potential energy?

a. Increases
b. Decreases
c. Accelerates
d. Stays the same

Work, Potential Energy, and Kinetic Energy

$$W = \vec{F} \cdot \Delta \vec{r} = -\Delta U$$

$$W = \Delta KE = \frac{1}{2} m v_{FINAL}^2 - \frac{1}{2} m v_{INITIAL}^2$$

$\Delta U + \Delta KE$ is constant!

Gravity and Electrostatic Force are both CONSERVATIVE

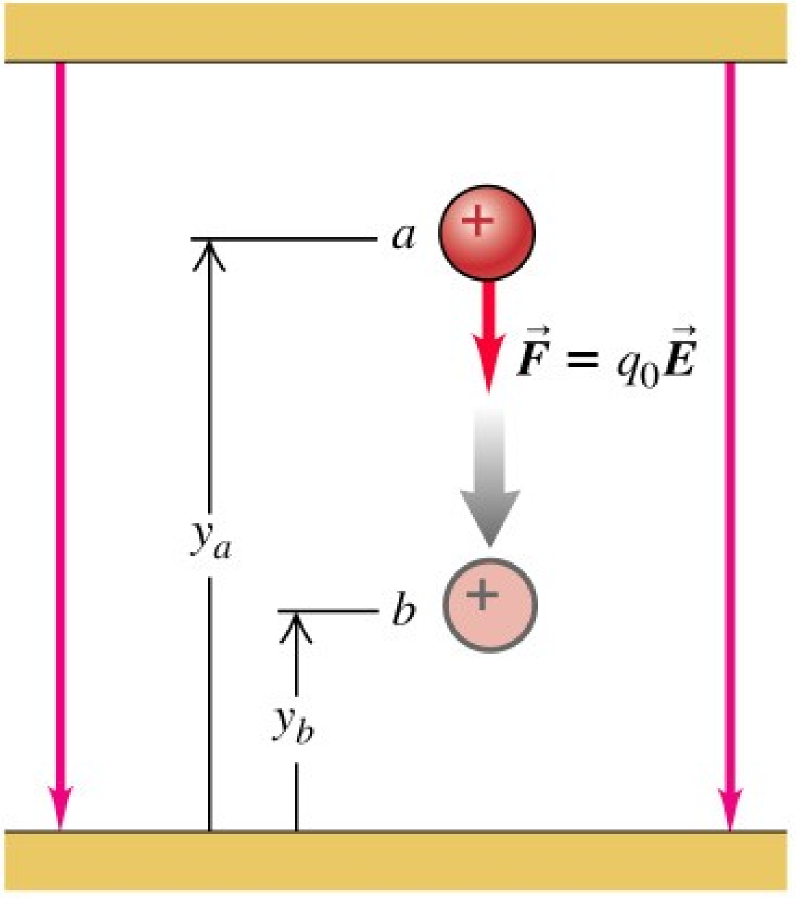
When a positive charge moves in the direction of the electric field,

A. the field does positive work on it and the potential energy increases

B. the field does positive work on it and the potential energy decreases

C. the field does negative work on it and the potential energy increases

D. the field does negative work on it and the potential energy decreases



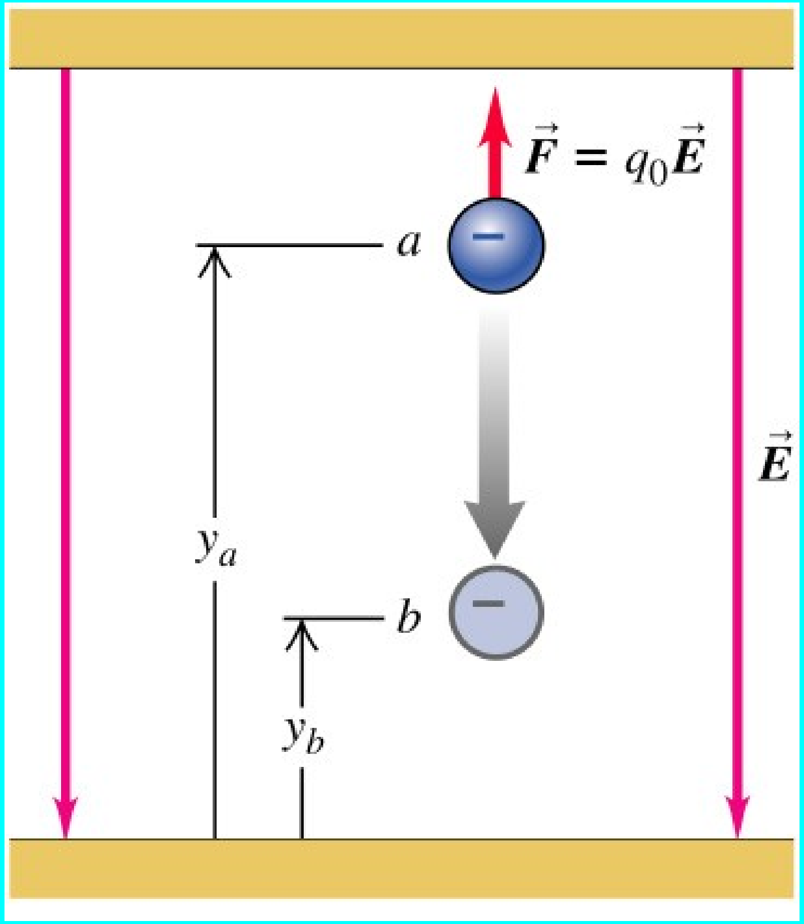
When a negative charge moves in the direction of the electric field,

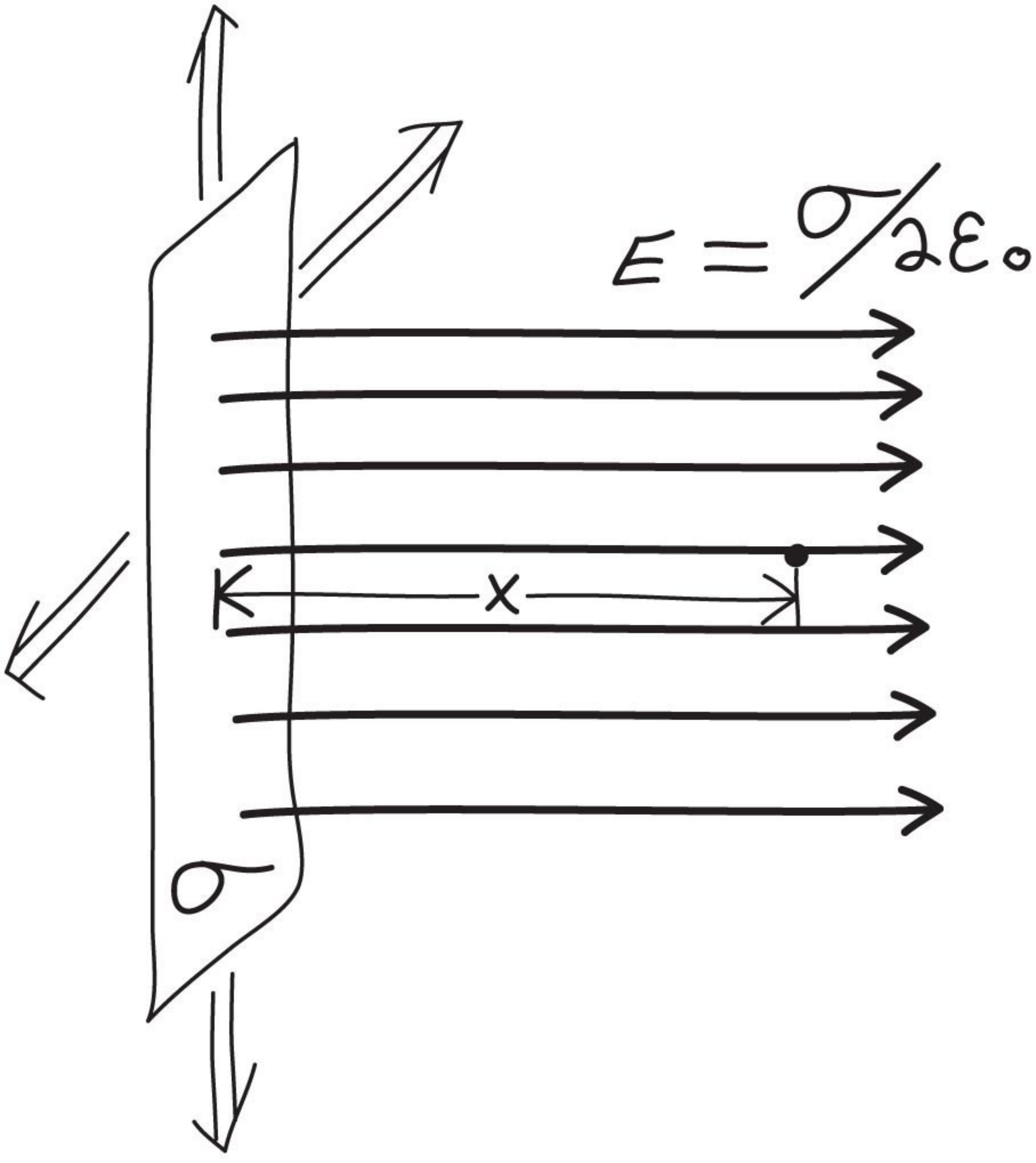
A. the field does positive work on it and the potential energy increases

B. the field does positive work on it and the potential energy decreases

C. the field does negative work on it and the potential energy increases

D. the field does negative work on it and the potential energy decreases





An infinite sheet of charge creates a field of 10000 V/m.

An electron moves 10 cm in this field.

What is the final speed of the electron?

$$m_e = 9.0 \times 10^{-31}\,\mathrm{kg}$$

What does work have to do with Electric Potential?

$$W = \vec{F} \cdot \Delta \vec{r} = Q \vec{E} \cdot \Delta \vec{r}$$

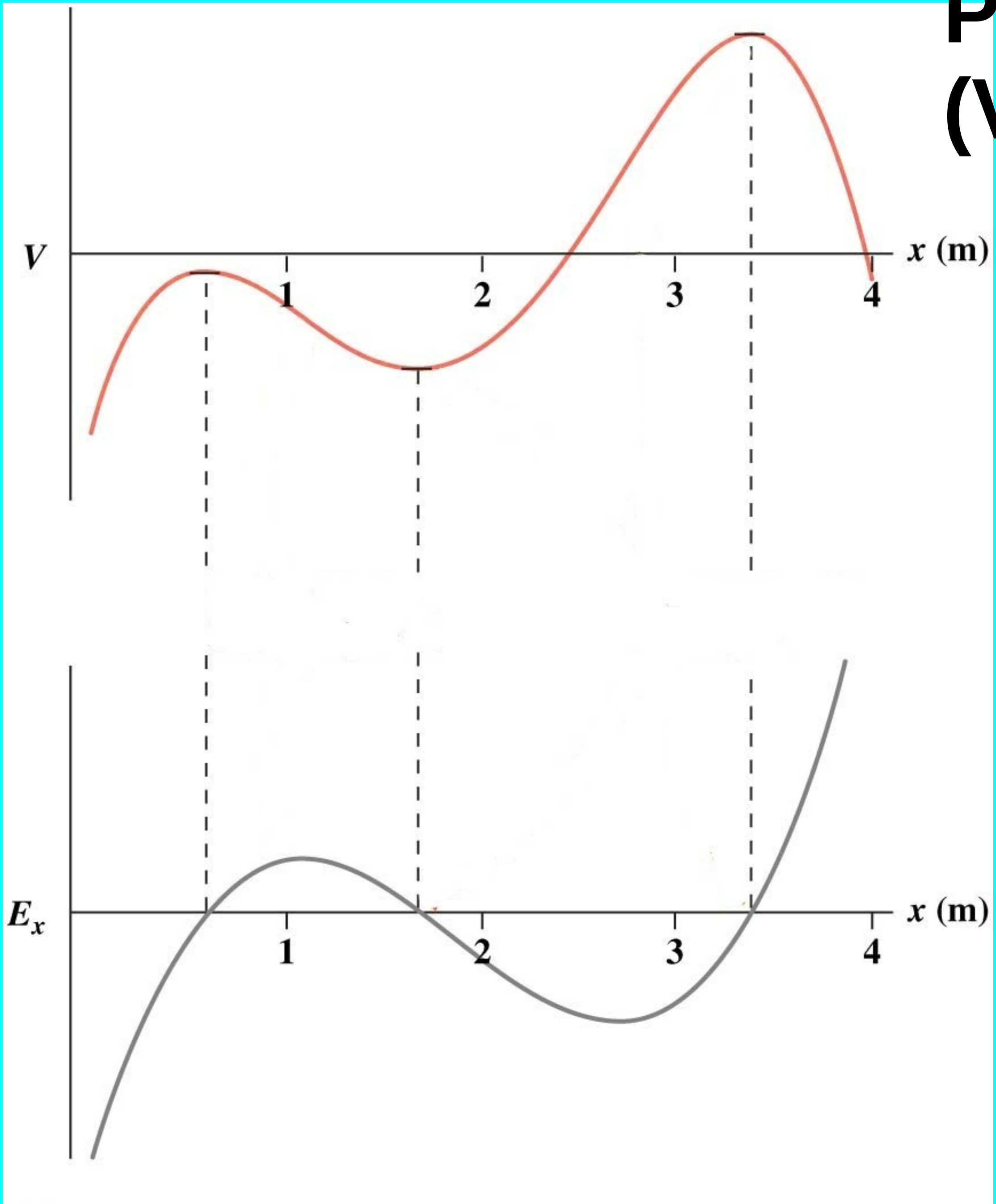
What if force or Electric field are not constant?

$$W = \int \vec{F} \cdot d\vec{r} = Q \int \vec{E} \cdot d\vec{r}$$

$$\Delta U = -W = QV = -Q \int \vec{E} \cdot d\vec{r}$$

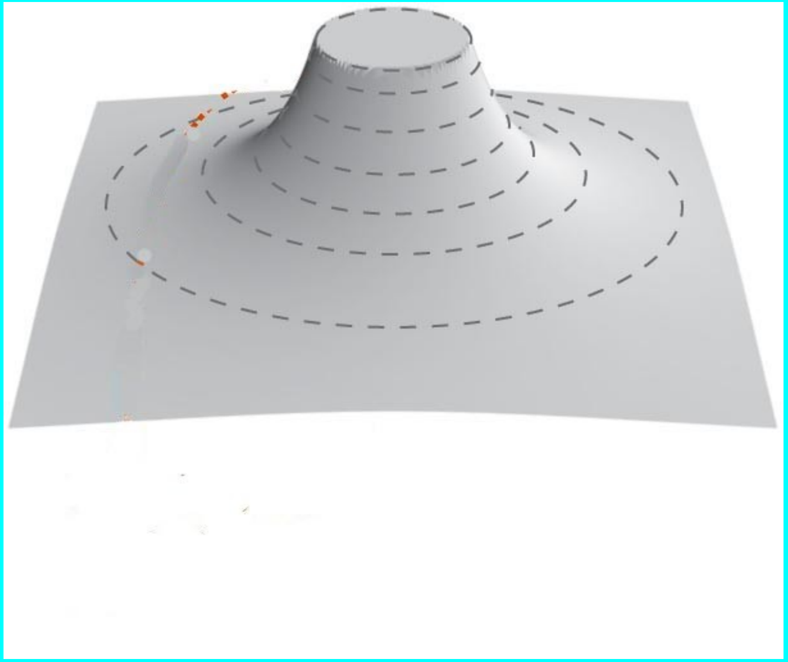
$$\text{therefor } V \overset{\text{def}}{=} -\int \vec{E} \cdot d\vec{r}$$

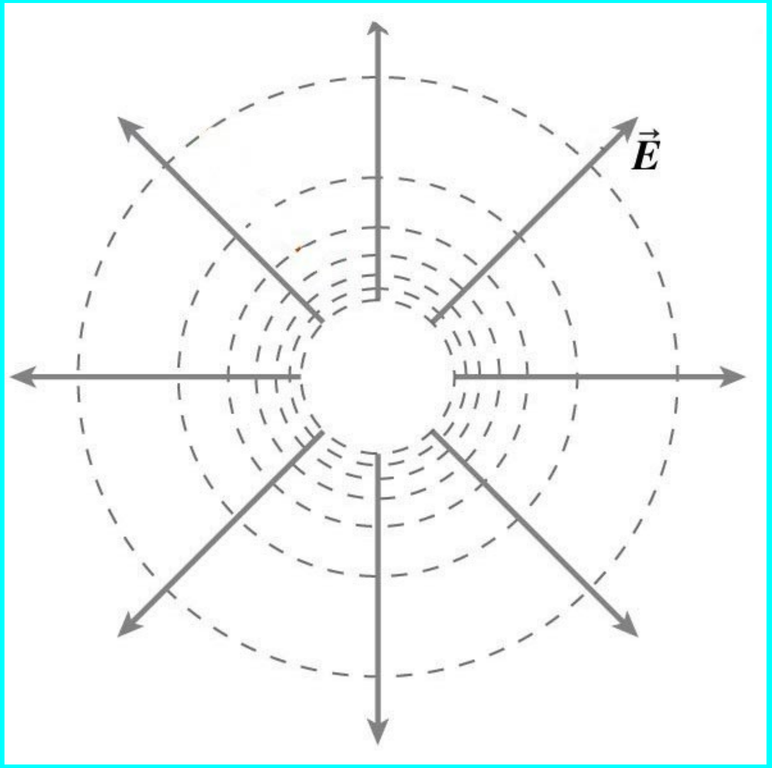
Potential (Voltage) in 1-D



$$V(x) = -\int E_x\,dx$$

$$E_x = -\frac{dV}{dx}$$




$\vec{E}$

Work, Potential Energy, and Kinetic Energy

$$W = \vec{F} \cdot \Delta r = -\Delta U$$

$$W = \Delta KE = \frac{1}{2} m v_{FINAL}^{2} - \frac{1}{2} m v_{INITIAL}^{2}$$

$\Delta U + \Delta KE$ is constant!

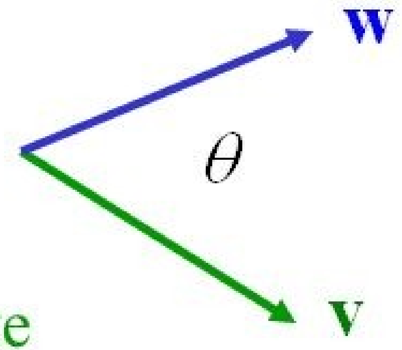
Gravity and Electrostatic Force are both CONSERVATIVE

Dot Products

<u>There are two ways to multiply two vectors</u>

- The dot product produces a scalar quantity
  - It has no direction
  - It can be pretty easily computed from geometry
  - It can be easily computed from components

$$\boxed{\mathbf{v}\cdot\mathbf{w} = vw\cos\theta = v_x w_x + v_y w_y + v_z w_z}$$



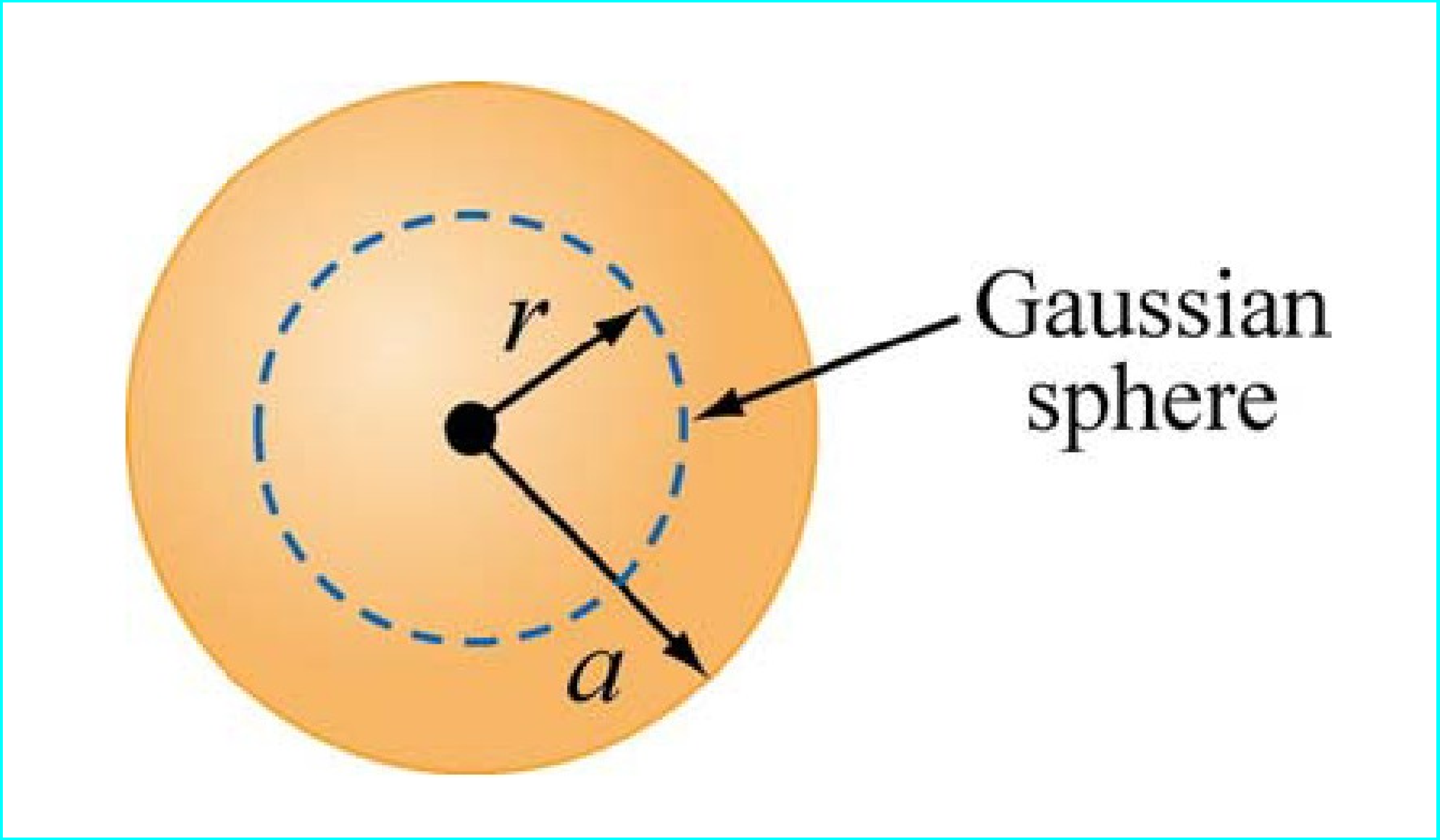
- The dot product of two unit vectors is easy to memorize

$$\boxed{\begin{aligned}&\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1\\ &\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = 0\\ &\hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0\\ &\hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{j}} = 0\end{aligned}}$$

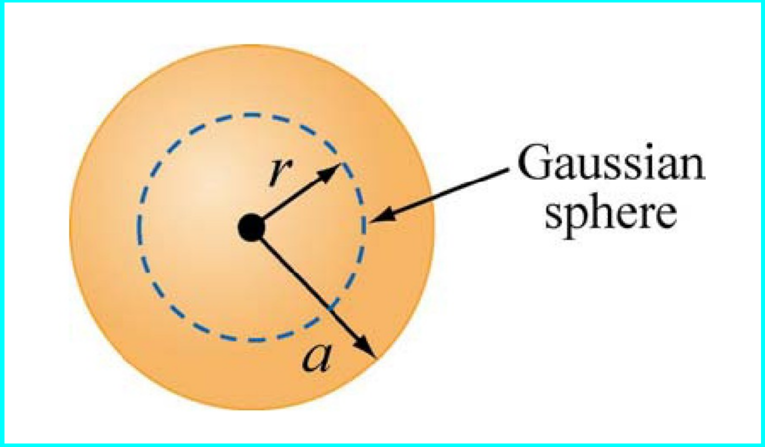
- The dot product is commutative

$$\boxed{\mathbf{v}\cdot\mathbf{w} = \mathbf{w}\cdot\mathbf{v}}$$

What about a solid sphere of charge?



What about a solid sphere of charge?



What about a solid sphere of charge?

What about a solid sphere of charge?