Lecture 09: 02/13/2024

- Announcements
  - Homework extended
  - First exam – one week (2/20)
  - Sample test – in recitation … longer than real thing

- Last Time
  - Gauss's law
  - Field of symmetrical charge configurations

- Today
  - Some review
  - Gauss's law
  - More about flux and dot products
  - Fields and conductors

SCHEDULE

#	Dates	Reading	Topic	Lab.
1	Jan 16	B1Ch16	Intro, Waves ($v = f\lambda$, $v = \sqrt{T/\mu}$)	no lab
2	Jan 18		Superposition, Standing Waves	
3	Jan 23	B2Ch5	$F = q_1 q_2 / r^2 \hat{r}$, conductors/insulators	Wave Superposition
4	Jan 25		$\vec{E}$-field concept and multi-Q	
5	Jan 30	Ch 5	Field lines and dipoles	Oscilloscope
6	Feb 1	Ch 5	Flux concept and Gauss Law	
7	Feb 6	Ch 6	Field of line, point, plane	Coulomb's Law
8	Feb 8	Ch 6	Gaussian tricks!	
9	Feb 13	Ch 7	PE and Electric Potential	E-field and Superposition
10	Feb 15	Ch 7	$V = \int \vec{E} \cdot d\vec{s}$	
11	Feb 20		V for multi charges	Electric Field Mapping
12	Feb 22		Test 1	
13	Feb 27	Ch 8	Capacitance	Capacitors and Delectrics
14	Feb 29	Ch 8	Capacitance	
15	Mar 5	Ch 9	Current and Resistance	Ohm's Law
16	Mar 7	Ch 9	Current and Resistance	
17	Mar 12	Ch 10	DC Circuits	Kirchoff's Laws
18	Mar 14	Ch 10	Magnetic Forces & Fields	
	Mar 19/21		Spring Break	

What have we learned?

Coulomb's Law

How to use it!

Electric Fields $\vec{F} = Q\vec{E}$

Gauss's Law

Waves

What have we learned?

Waves

Wavelength, frequency, speed

Relation between tension/mass and speed on a string.

Standing waves.

Charges and Fields

Coulomb's law … calculating forces between charges, getting the vectors right

Electric field, superposition

Electric field lines

Gauss's law

Flux

Field of a line

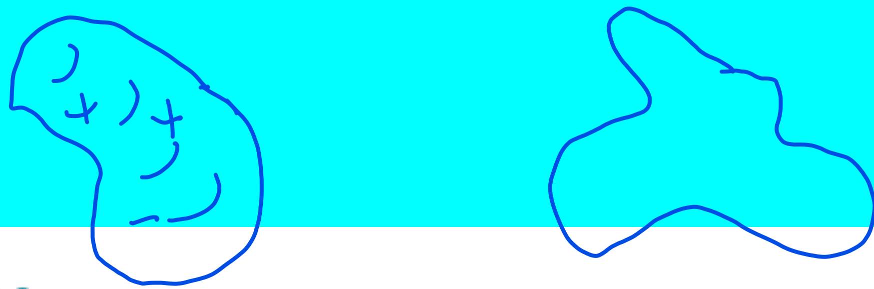
Field of a plane

Key Equations

Wave speed	$v = \frac{\lambda}{T} = \lambda f$
Linear mass density	$\mu = \frac{\text{mass of the string}}{\text{length of the string}}$
Speed of a wave or pulse on a string under tension	$\lvert v \rvert = \sqrt{\frac{F_T}{\mu}}$
Speed of a compression wave in a fluid	$v = \sqrt{\frac{B}{\rho}}$
Resultant wave from superposition of two sinusoidal waves that are identical except for a phase shift	$y_R(x,t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin\left(kx - \omega t + \frac{\phi}{2}\right)$
Wave number	$k \equiv \frac{2\pi}{\lambda}$
Wave speed	$v = \frac{\omega}{k}$

Key Equations

Coulomb's law	$\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$
Superposition of electric forces	$\vec{\mathbf{F}}(r) = \frac{1}{4\pi\varepsilon_0}Q\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$
Electric force due to an electric field	$\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$
Electric field at point P	$\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$
Field of an infinite wire	$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{\mathbf{k}}$
Field of an infinite plane	$\vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}}$
Dipole moment	$\vec{\mathbf{p}} = q\vec{\mathbf{d}}$



Key Equations

$\Phi = \int \vec{E} \cdot d\vec{A}$

Definition of electric flux, for uniform electric field	$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \to EA\cos\theta$
Electric flux through an open surface	$\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Electric flux through a closed surface	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Gauss's law	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{\text{enc}}}{\varepsilon_0}$
Gauss's Law for systems with symmetry	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E \oint_S dA = EA = \frac{q_{\text{enc}}}{\varepsilon_0}$
The magnitude of the electric field just outside the surface of a conductor	$E = \frac{\sigma}{\varepsilon_0}$

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Status

Online HW #4: Gauss's Law Begin Date: 2/9/2024 8:00:00 AM -- **Due Date:** 2/15/2024 11:59:00 PM **End Date:** 5/29/2024 11:59:00 PM

(13%) Problem 4: A circular loop of radius $R = 10.42$ cm is centered at the origin where there is a constant electric field

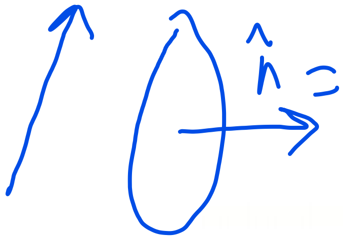
$$\vec{E} = (86.4\ \mathrm{N/C})\ \hat{i} + (183\ \mathrm{N/C})\ \hat{j}$$

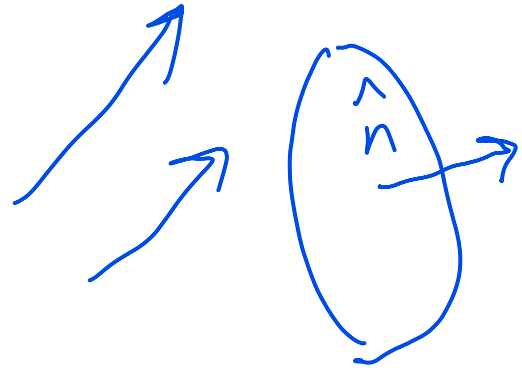
$\overline{86\,\hat{i} + 183\,\hat{j}}$

$\vec{A} = A\,\hat{n}$

$\vec{\Phi} = \vec{E} \cdot \vec{A}$

$\hat{n} = \hat{i}$





A message from your instructor

Problem 4: This one is easy if you can remember how to do dot products without the cosine theta. (Hint, multiply the i-component by the i-component, the j-by the j and the k by the k, then add them up)

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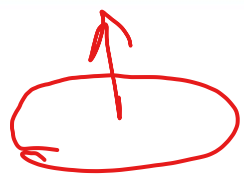
33% Part (a) What is the flux through the loop, in newton squared meters per coulomb, whe... e x direction?

$\Phi_1 =$ ______ $\mathrm{N \cdot m^2/C}$



sin()	cos()	tan()	π	(	)	7	8	9	HOME
cotan()	asin()	acos()	E	↑^	^↓	4	5	6	←
atan()	acotan()	sinh()		/	*	1	2	3	→
cosh()	tanh()	cotanh()		+	-	0		.	END
◉ Degrees	○ Radians			√()	BACKSPACE		DEL	CLEAR	

Submit | Hint | Feedback | I give up!

Grade Summary

Deductions	0%
Potential	100%

Submissions

Attempts remaining: 6

(1% per attempt)

detailed view

Hints: 0% deduction per hint. Hints remaining: 2

Feedback: 0% deduction per feedback.

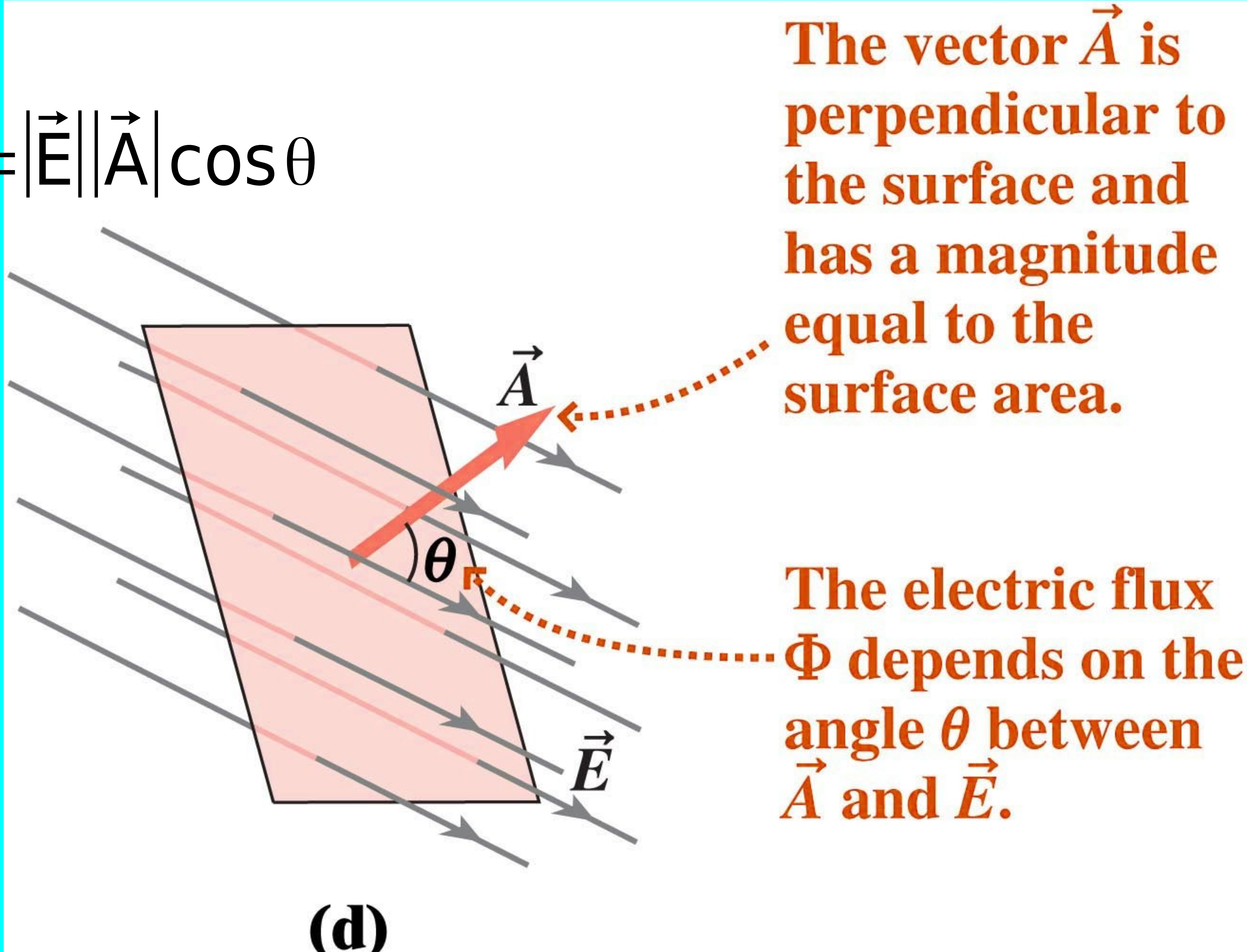
Instructor/TA Admin

33% Part (b) What is the flux through the loop, in newton squared meters per coulomb, when the loop is oriented such that its normal vector is in the negative y direction?

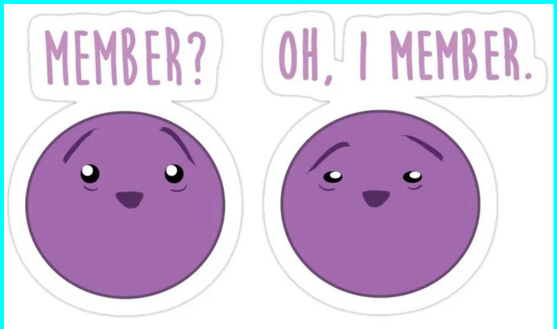
33% Part (c) What is the flux through the loop, in newton squared meters per coulomb, when the loop is oriented such that its normal vector is in the positive z direction?

$$\Phi = \vec{E} \cdot \vec{A}$$

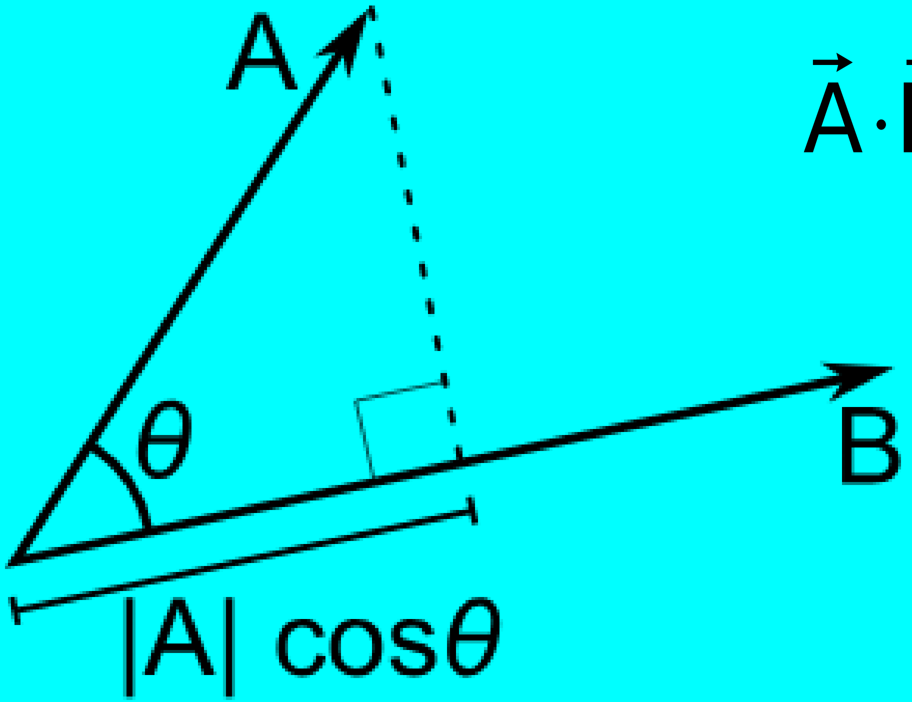
$$\Phi = \vec{E} \cdot \vec{A} = |\vec{E}||\vec{A}|\cos\theta$$



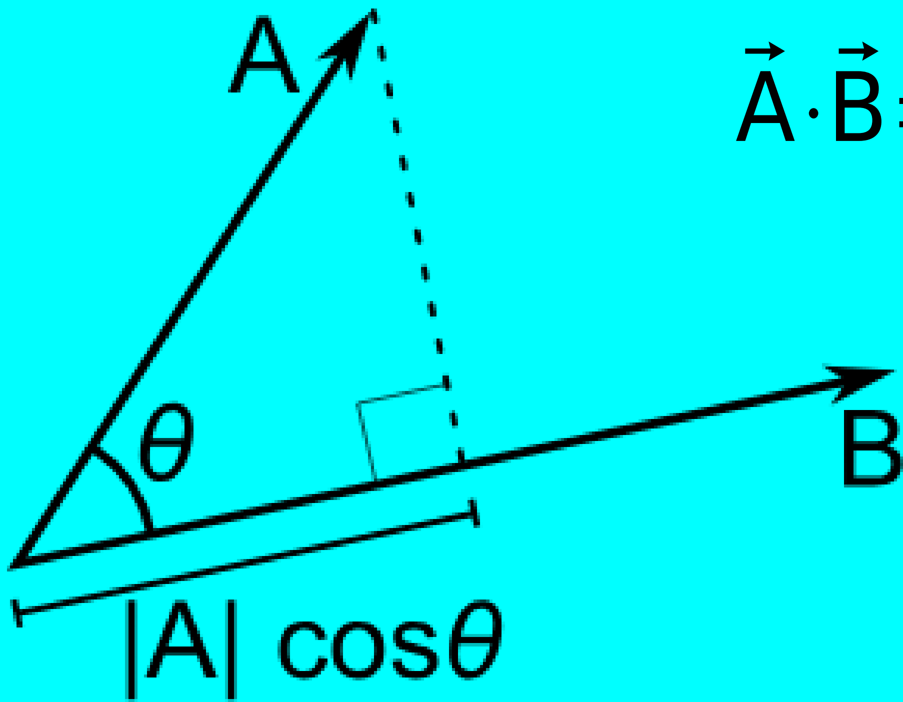
‘Member dot products?’



They convert two vectors to a scalar. They are zero when the vectors are perpendicular.



$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$



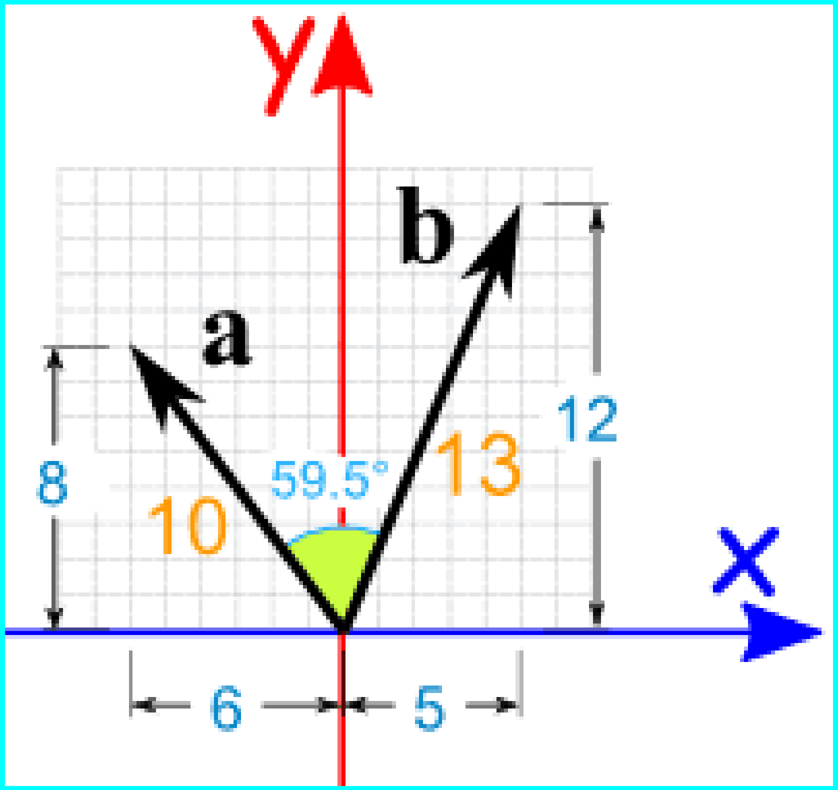
$$\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos\theta$$

$$\vec{a}\cdot\vec{b} = (10)(13)\cos(59.49)$$

$$=(10)(13)(.5077)$$

$$65.9995$$

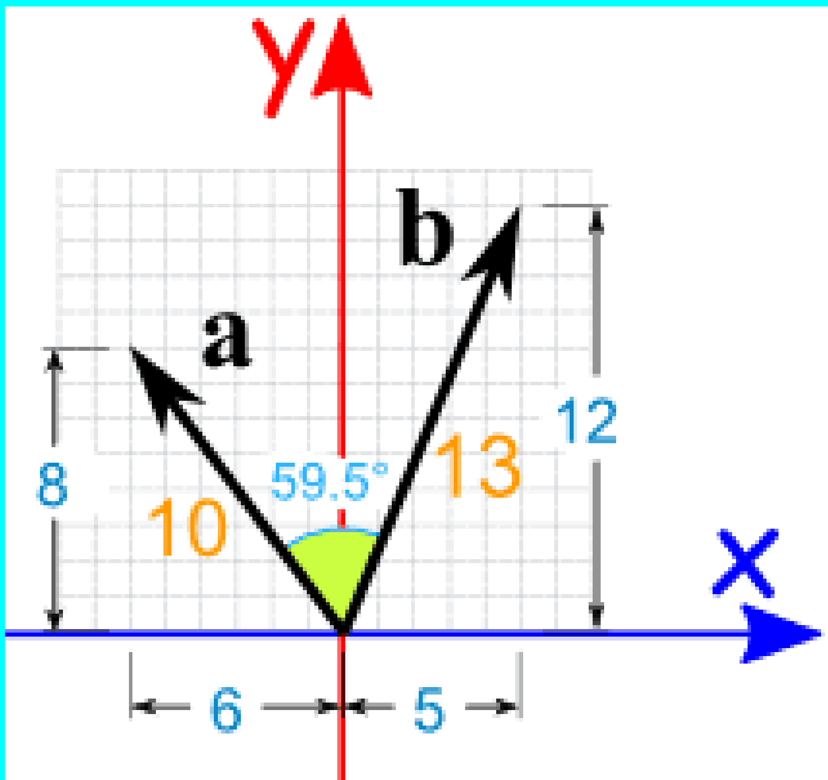
$$\vec{a}\cdot\vec{b}=a_x b_x+a_y b_y+a_z b_z$$



$\vec{a}\cdot\vec{b} = (-6)(5) + (8)(12) + (0)(0)$

$= -30 + 96 = 66$

$$\vec{a}\cdot\vec{b}=a_x b_x+a_y b_y+a_z b_z \qquad \hat{j}\cdot\hat{j}=1 \qquad \hat{i}\cdot\hat{j}=0$$



$|\vec{j}| = 1 \qquad |\hat{j}||\hat{j}| \cos(0)$

$= 1 \cdot 1 \cdot 1 = 1$

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

$\vec{a}\cdot\vec{b} = (a_x \hat{i})\cdot(b_x \hat{i}) + (a_x \hat{i} \cdot b_y \hat{j}) + (a_y \hat{j} \cdot b_x \hat{i})$

$(a_y \hat{j} \cdot b_y \hat{j})$

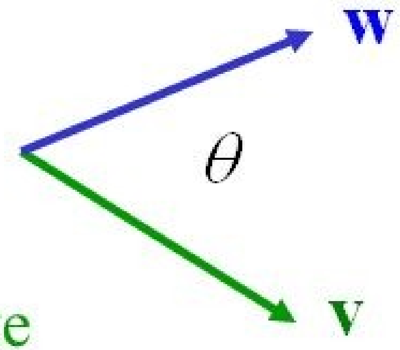
0 0

Dot Products

<u>There are two ways to multiply two vectors</u>

- The dot product produces a scalar quantity
  - It has no direction
  - It can be pretty easily computed from geometry
  - It can be easily computed from components

$$\mathbf{v} \cdot \mathbf{w} = vw\cos\theta = v_x w_x + v_y w_y + v_z w_z$$



- The dot product of two unit vectors is easy to memorize

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0$$
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$
$$\hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0$$

- The dot product is commutative

$$\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$$

$\vec{A} = 10\,\hat{i} + 20\,\hat{k} \qquad \vec{E} = 3\,\hat{j} + 10\,\hat{k}$

What is Φ?

(A) $200\,\hat{k}$ $\quad \frac{N}{C}\,m^2$

(B) 230

(C) $10\,\hat{i} + 3\,\hat{j} + 200\,\hat{k}$

(D) 200

(E) 213

Gauss's law

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

Online HW #4: Gauss's Law Begin Date: 2/9/2024 8:00:00 AM -- **Due Date:** 2/15/2024 11:59:00 PM **End Date:** 5/29/2024 11:59:00 PM

(13%) Problem 6: Consider a cubic surface surrounding a charge Q shown in the picture.

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A 0

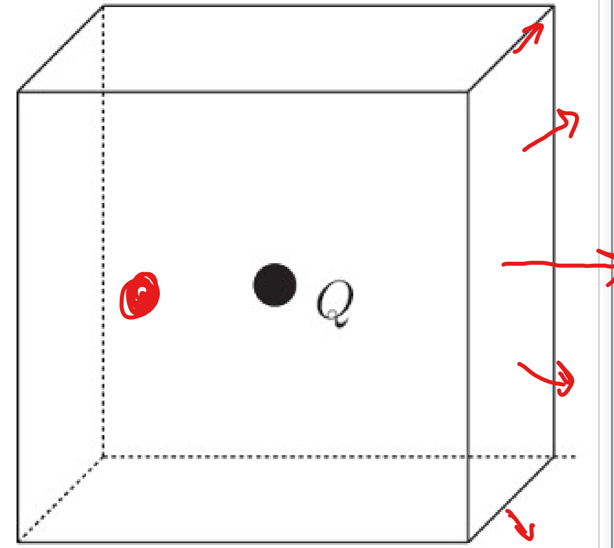
B $Q/3\varepsilon_0$

C $Q/6\varepsilon_0$

D Impossible

E Q/ε_0

$\oint \vec{E} \cdot d\vec{A}$



If the charge is directly in the center of the cube, what is the flux through each face of the cube?

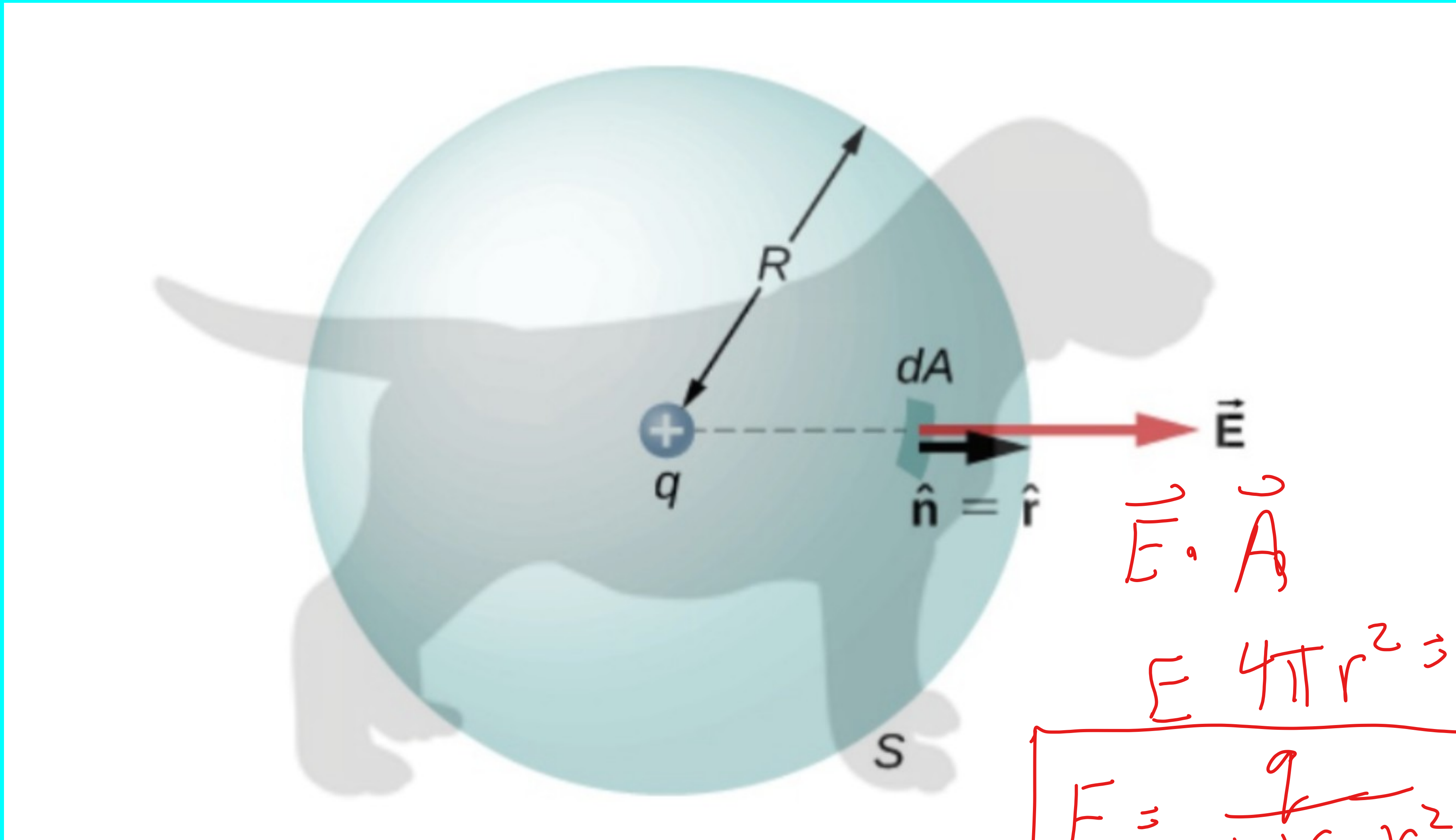
○ 0
○ $Q/(3\ \varepsilon_0)$
○ $Q/(6\ \varepsilon_0)$
○ It is impossible to give the answer without exact integration over the surface of a cube.
○ Q/ε_0

Grade Summary
Deductions 0%
Potential 100%

Submissions
Attempts remaining: 4
(25% per attempt)
detailed view

Submit | Hint | Feedback | I give up!

Hints: 0% deduction per hint. Hints remaining: 2 **Feedback:** 0% deduction per feedback.

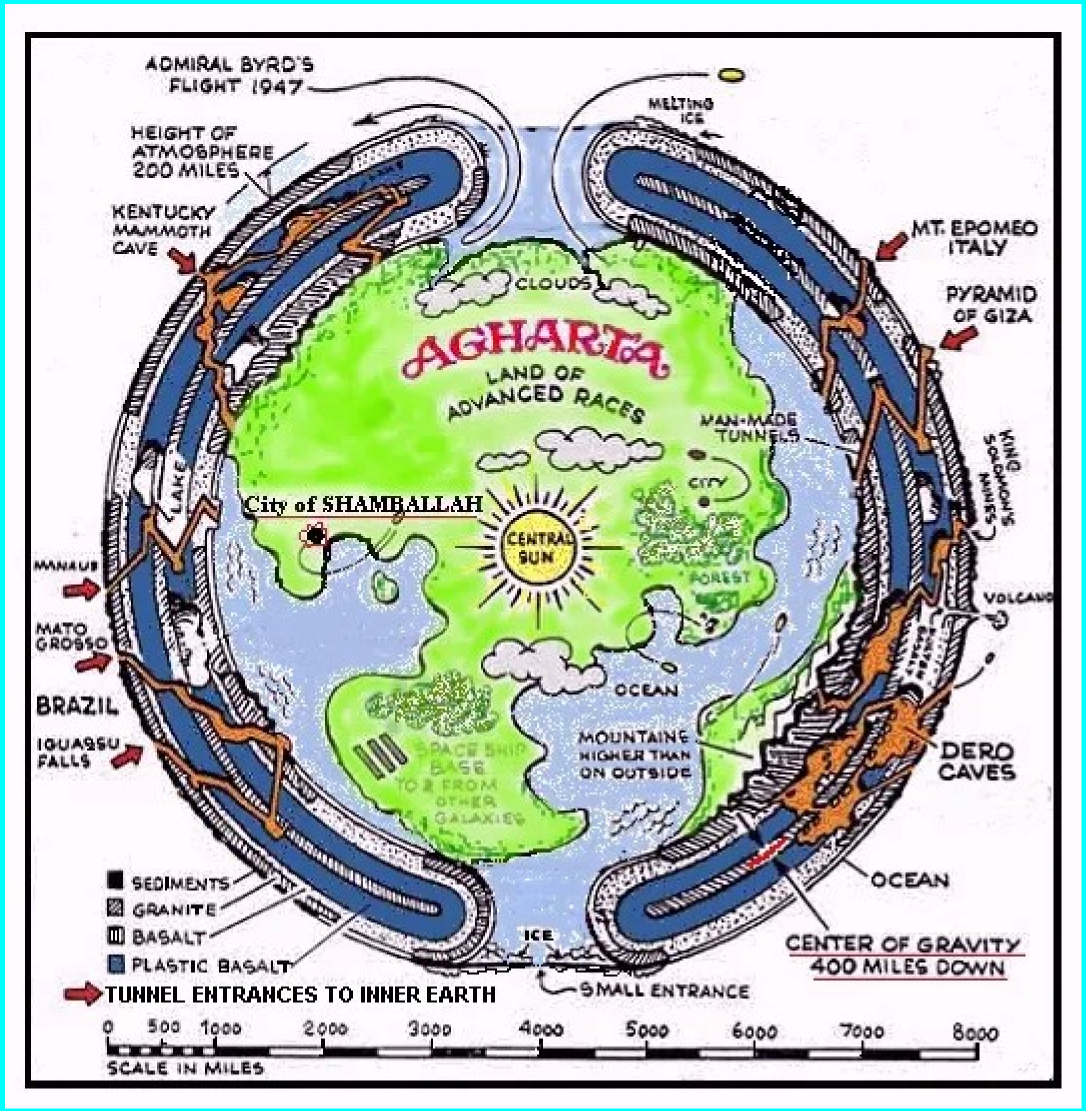


$$\vec{E} \cdot \vec{A}$$

$$E \, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

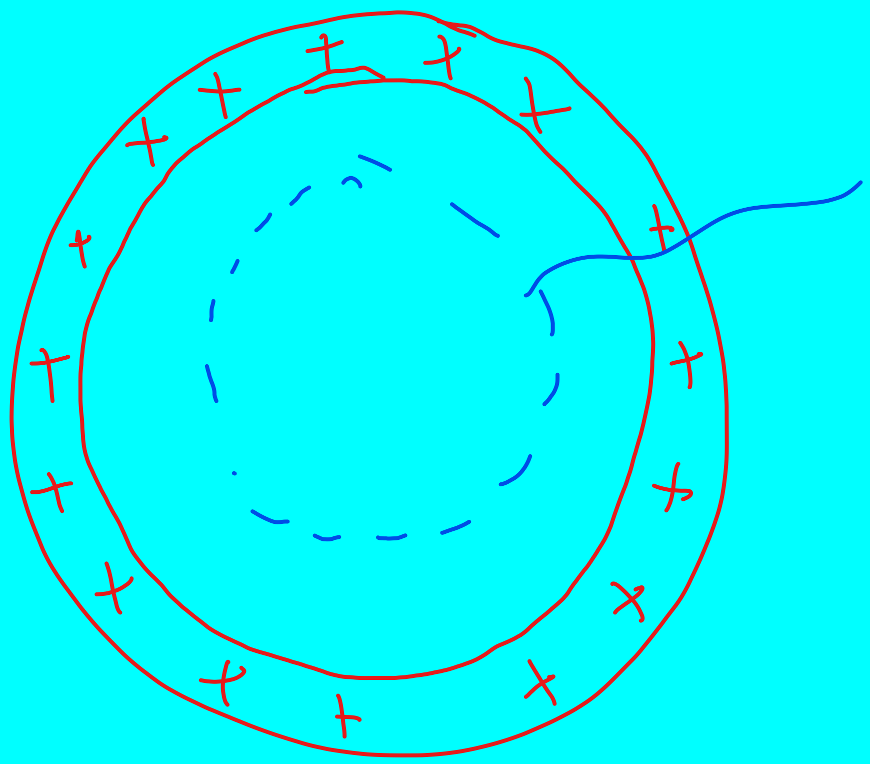
What about a hollow sphere?



$$\int \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

What about a hollow sphere of charge?

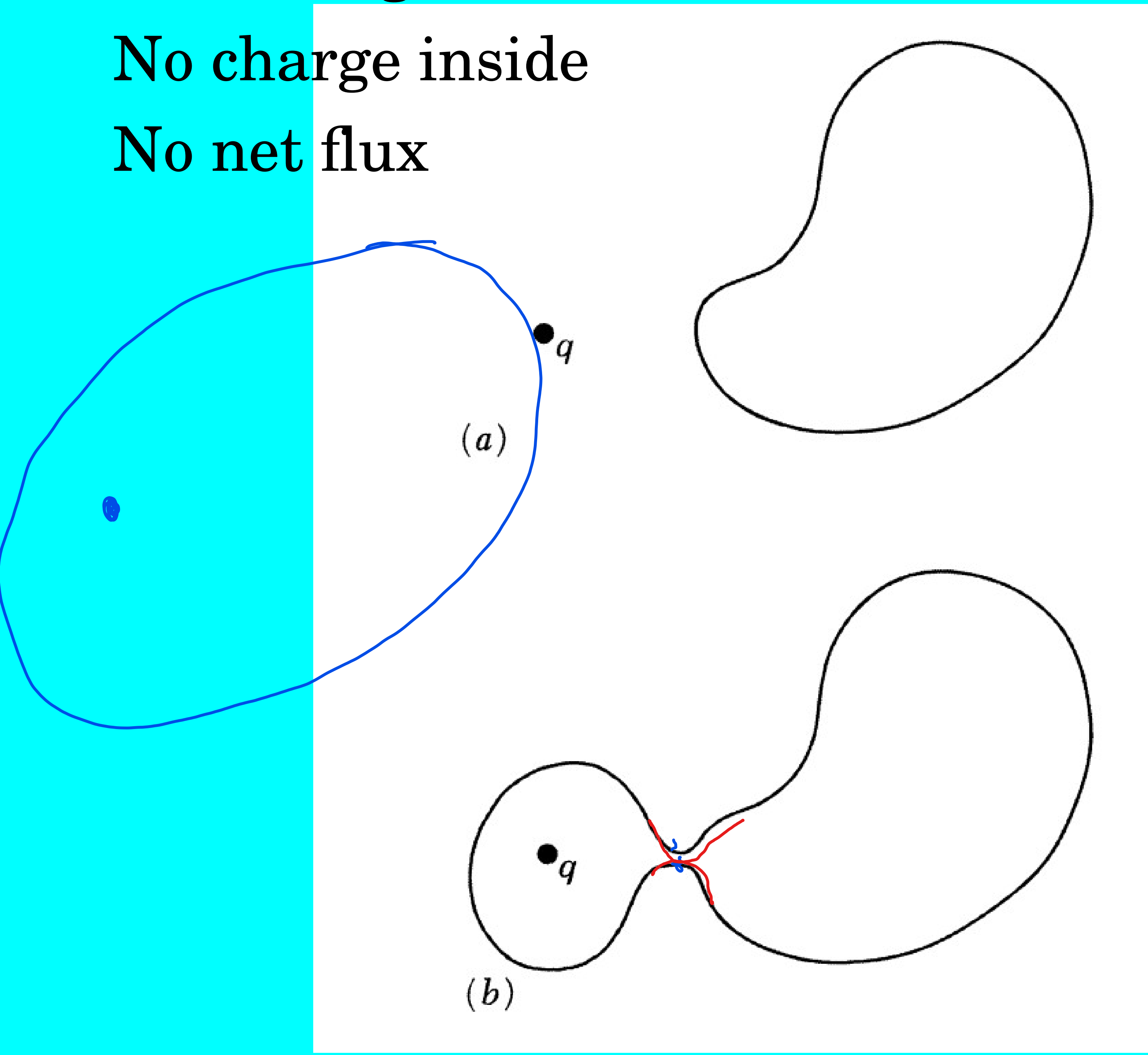


$$\Phi = \frac{q_{enc}}{\epsilon_0}$$

Gaussian logic trick

No charge inside

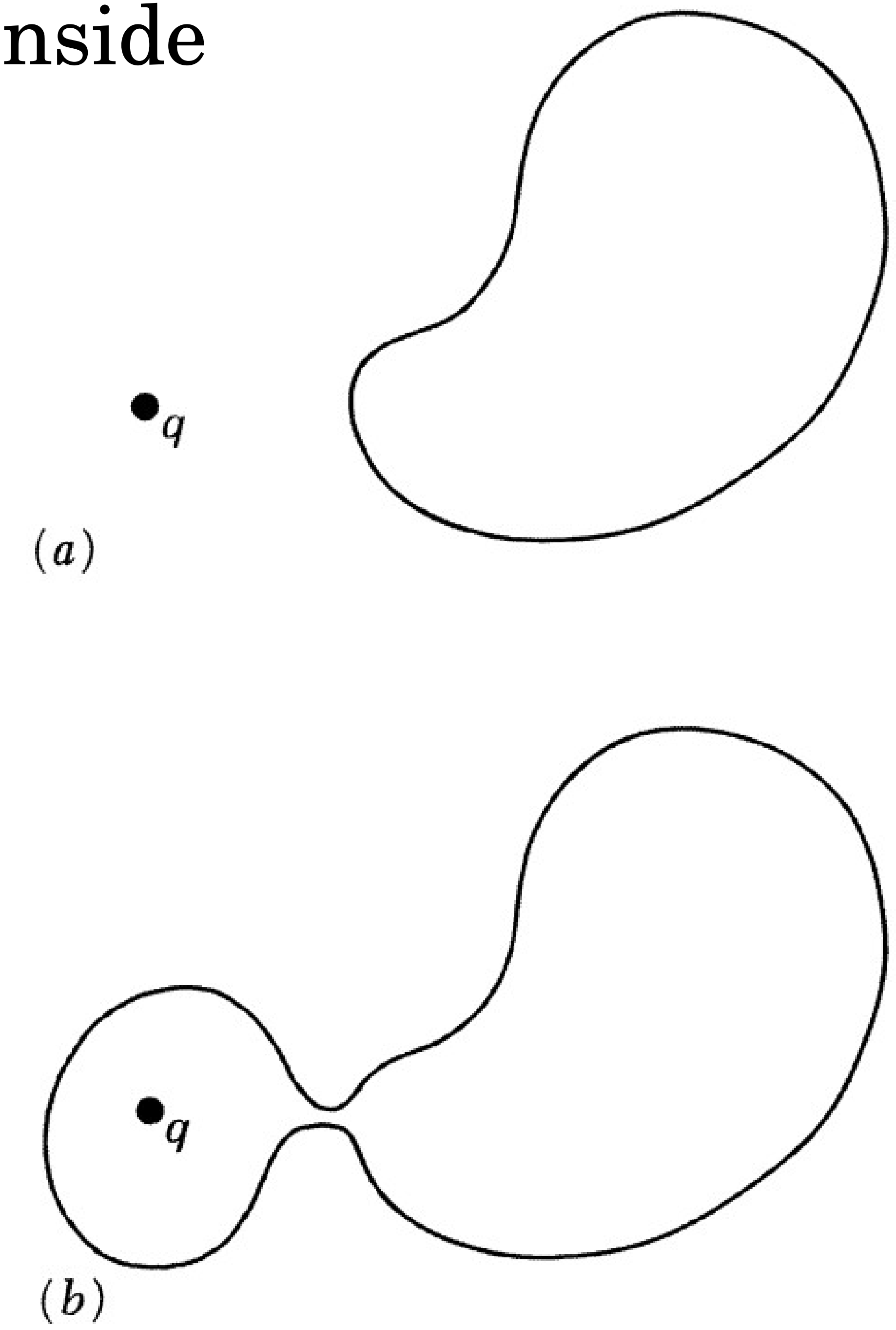
No net flux



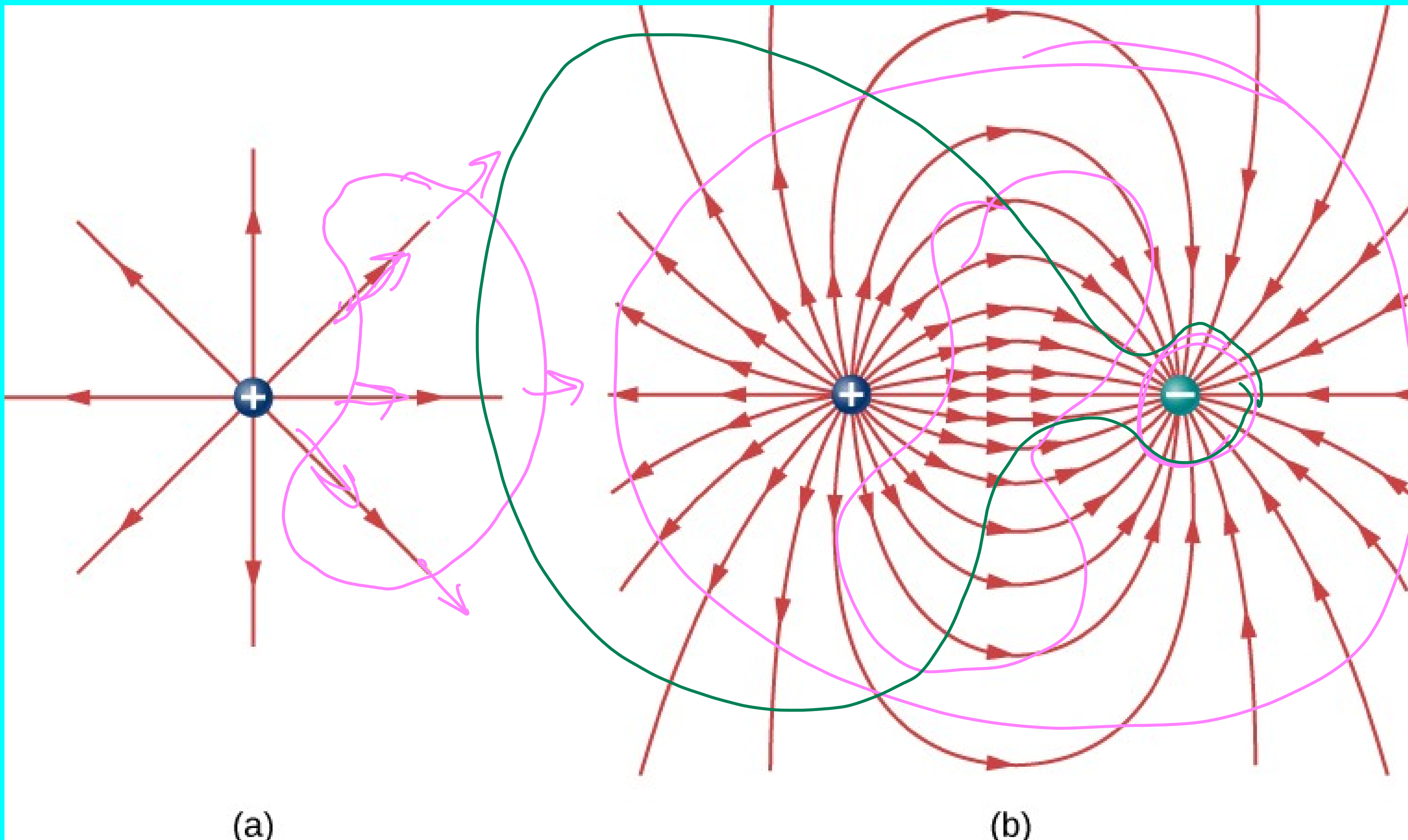
Gaussian logic trick

No charge inside

No net flux



Field line views



What about a hollow sphere of charge?

Class Management | Instructor | Help

Online HW #4: Gauss's Law Begin Date: 2/9/2024 8:00:00 AM -- **Due Date:** 2/15/2024 11:59:00 PM **End Date:** 5/29/2024 11:59:00 PM

(13%) Problem 7: The figure shows a sphere carrying a uniformly distributed volume charge Q. Three Gaussian surfaces are concentric with the sphere as shown.

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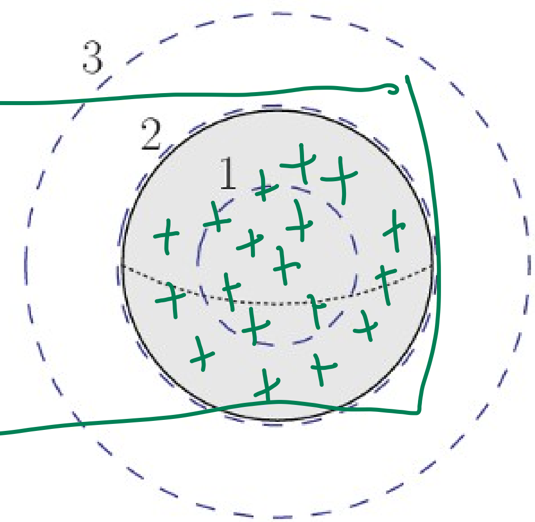
A) 2 + 3   C) 1
B) same   D) 3
B) 2   E) 1 and 2

$\Phi = \frac{q_{enc}}{\epsilon_0}$



A message from your instructor

Problem 7: The trick here is that not all the surfaces enclose all the charge.

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50% Part (a) Which Gaussian surface(s) has the greatest electric flux though it?

○ 2 and 3	○ 1
○ They all have the same electric flux.	○ 3
○ 2	○ 1 and 2

Grade Summary

Deductions	0%
Potential	100%

Submissions

Attempts remaining: 5
(20% per attempt)
detailed view

Submit | Hint | Feedback | I give up!

Hints: 0% deduction per hint. Hints remaining: 2

Feedback: 0% deduction per feedback.

Instructor/TA Admin

50% Part (b) On which of Gaussian surface is the electric field the greatest?

(13%) Problem 8: A point charge is positioned at the very corner of a cube as shown in the figure.

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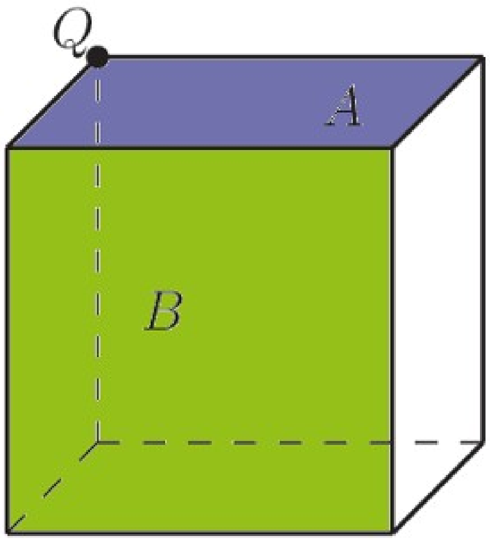

A message from your instructor

Problem 8: This one is very cute, but a bit tricky. I recommend waiting till I discuss this in class.

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50% Part (a) What is the electric flux though the side A (the top) of the cube?

- ○ $Q/(8\varepsilon_0)$
- ○ $Q/(24\varepsilon_0)$
- ○ $Q/(3\varepsilon_0)$
- ○ $Q/(6\varepsilon_0)$
- ○ $Q/(12\varepsilon_0)$
- ○ 0

Grade Summary

Deductions	0%
Potential	100%

Submissions

Attempts remaining: 5 (20% per attempt)

detailed view

Submit | Hint | Feedback | I give up!

Hints: 0% deduction per hint. Hints remaining: 1

Feedback: 0% deduction per feedback.

Instructor/TA Admin

50% Part (b) What is the electric flux though the side B (the front) of the cube?

Class Management | Instructor | Help

Online HW #4: Gauss's Law Begin Date: 2/9/2024 8:00:00 AM -- **Due Date:** 2/15/2024 11:59:00 PM **End Date:** 5/29/2024 11:59:00 PM

(13%) Problem 5: An infinite charged wire with charge per unit length λ lies along the central axis of a cylindrical surface of radius *r* and length *L*.

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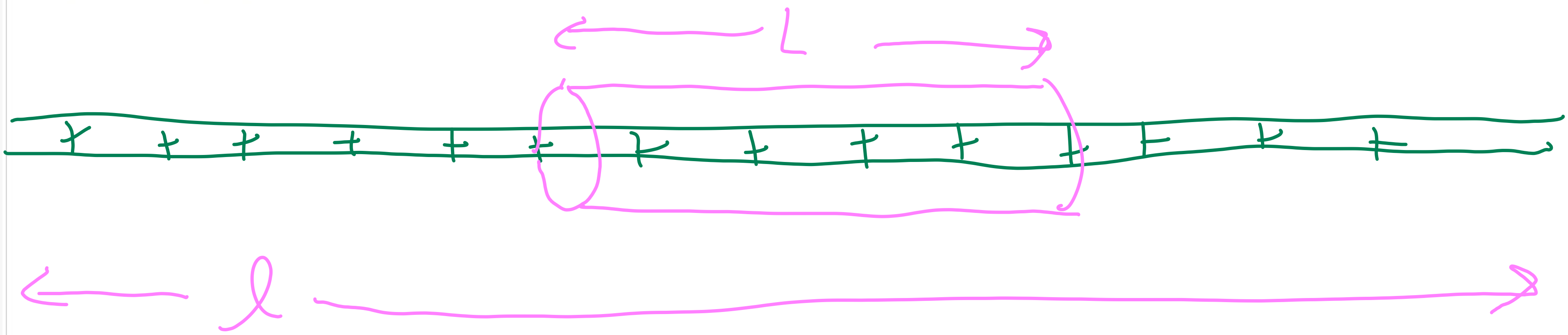


What is the flux through the surface due to the electric field of the charged wire?

$\Phi =$

Grade Summary	
Deductions	0%
Potential	100%

$$\lambda = \frac{Q}{\ell} \qquad \Phi = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

Charged wire problem

A wire is 70 meters long and has a total charge of 35 picocoulombs.
What is the total flux through a cylinder 3-m long centered on the wire?

Simple Case I: Long (infinite) Wire

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

Gauss's law for simple cases

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Simple Case I: Long (infinite) Wire

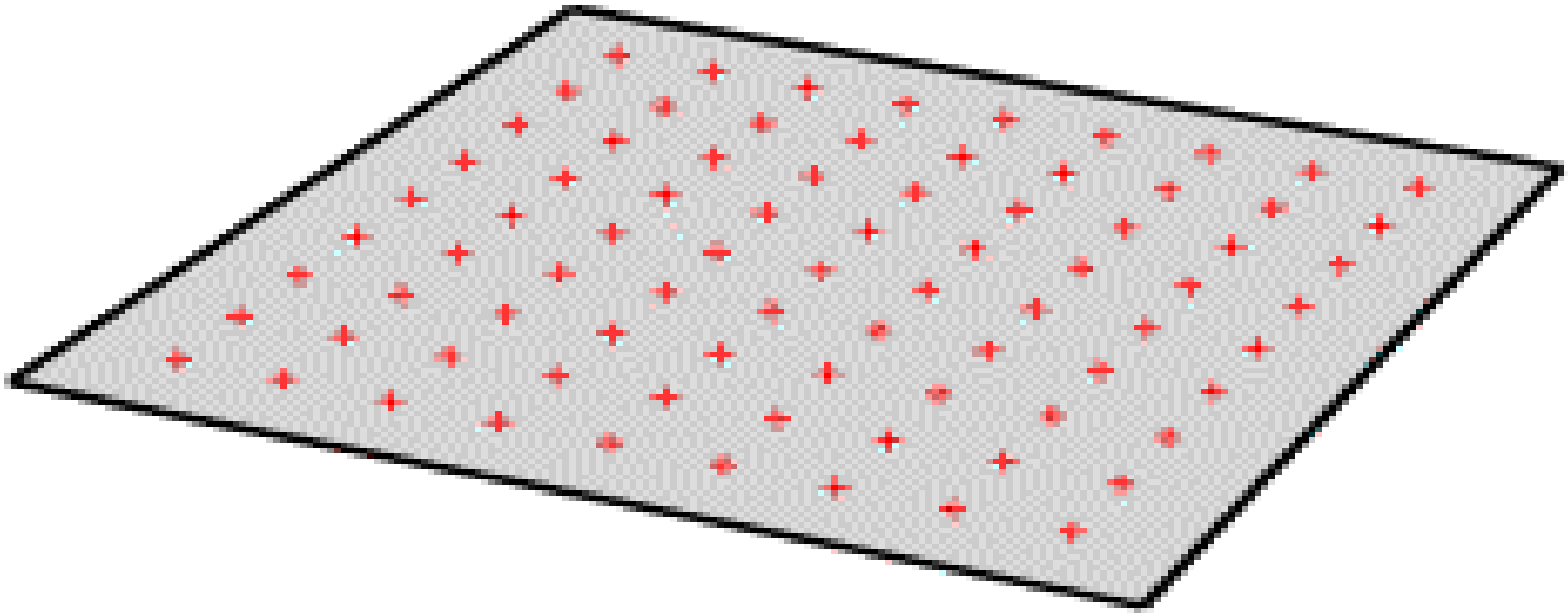
$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

Simple Case II: Large (infinite) Plane

$$E \times \left(\text{Surface Area}\right) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

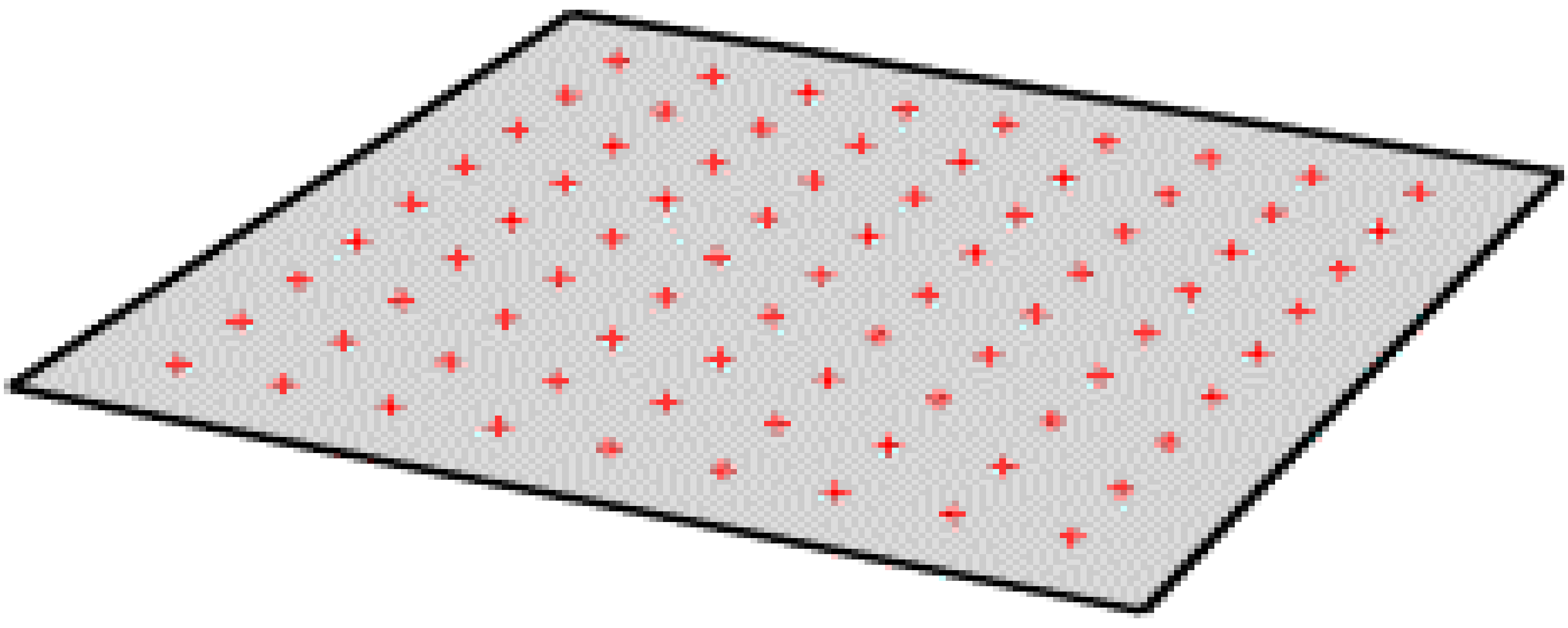
Relation between symmetry and Electric Field

Imagine an infinite plane of charge.



Relation between symmetry and Electric Field

Because you can't tell what direction you are facing, the field must be ONLY Perpendicular to the plane.

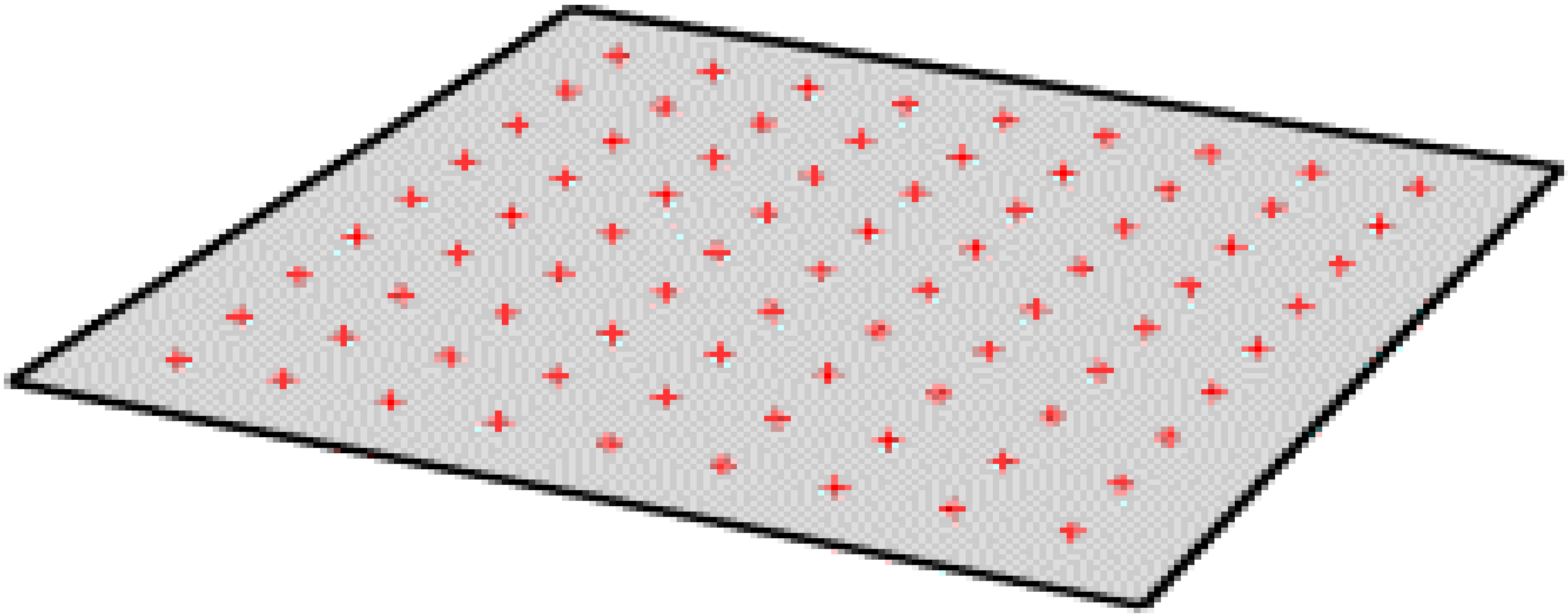


How large is this area?

[A] Floor tiles (4'x6')
[B] Painting (12”x18”)
[C] Warehouse (60'x90')
[D] Airfield (1000'x1500')
[E] Not enough Info, can't tell

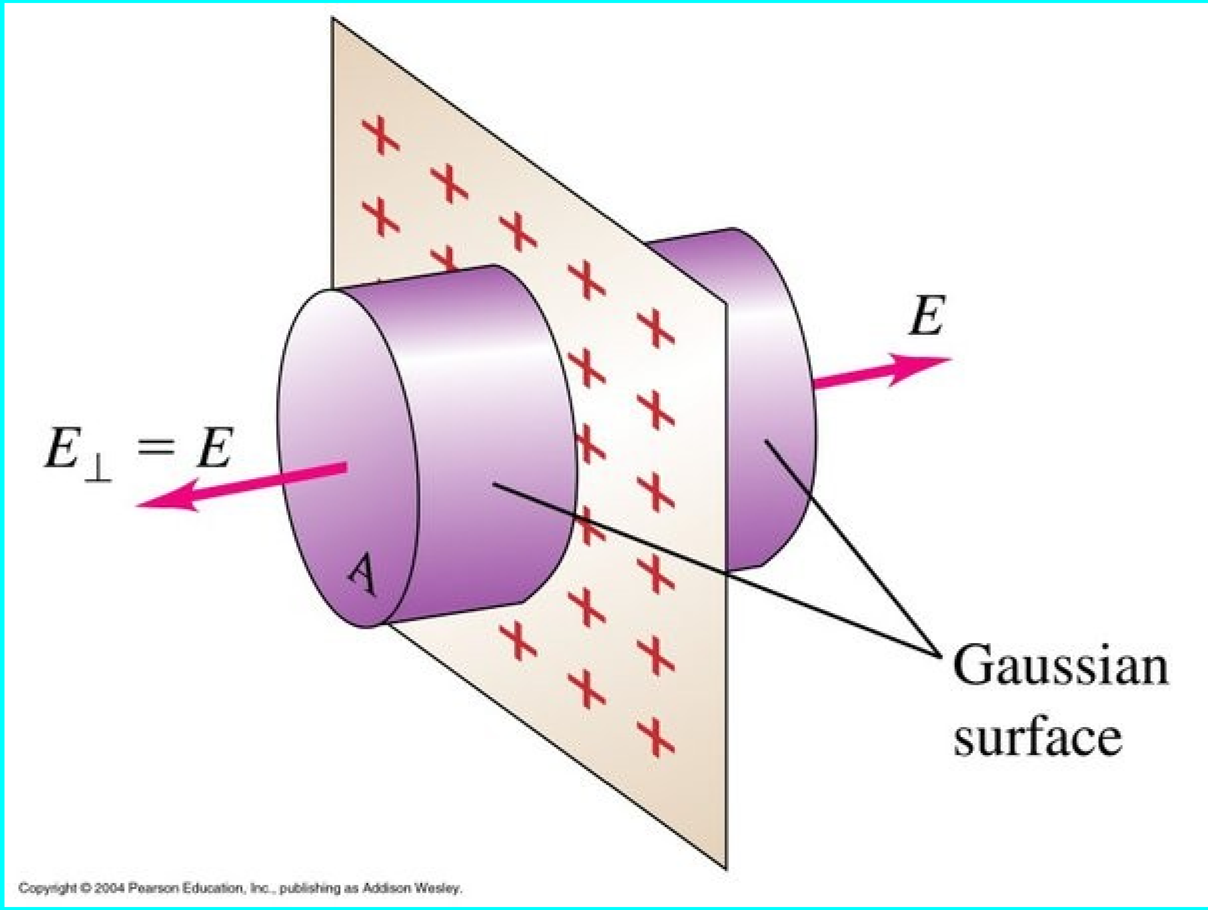
Electric field of a plane of charge

Because you ALSO can't tell how far away you are from the plane, the field cannot change magnitude. It must be constant.



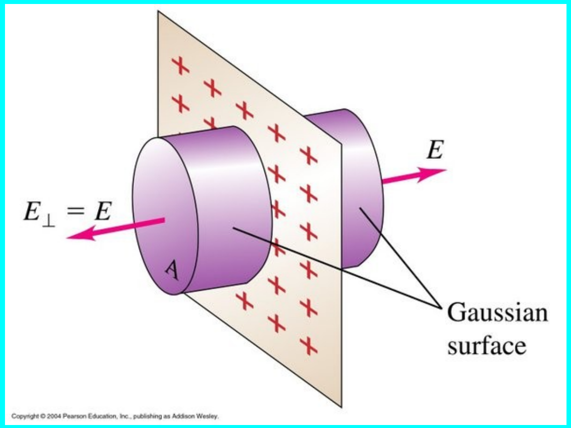
Electric field of a plane of charge

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$



Electric field of a plane of charge

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$



Infinite Plane I

A square plate is 10 meters on a side and you are 10 cm away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 20 cm away?

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

Infinite Plane II

A square plate is 10 meters on a side and you are 100 m away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 200 m away?

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

Infinite Plane III

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

(A) 8.85×10^{-5} N/C

(B) 4.43×10^{-5} N/C

(C) 5.00×10^{6} N/C

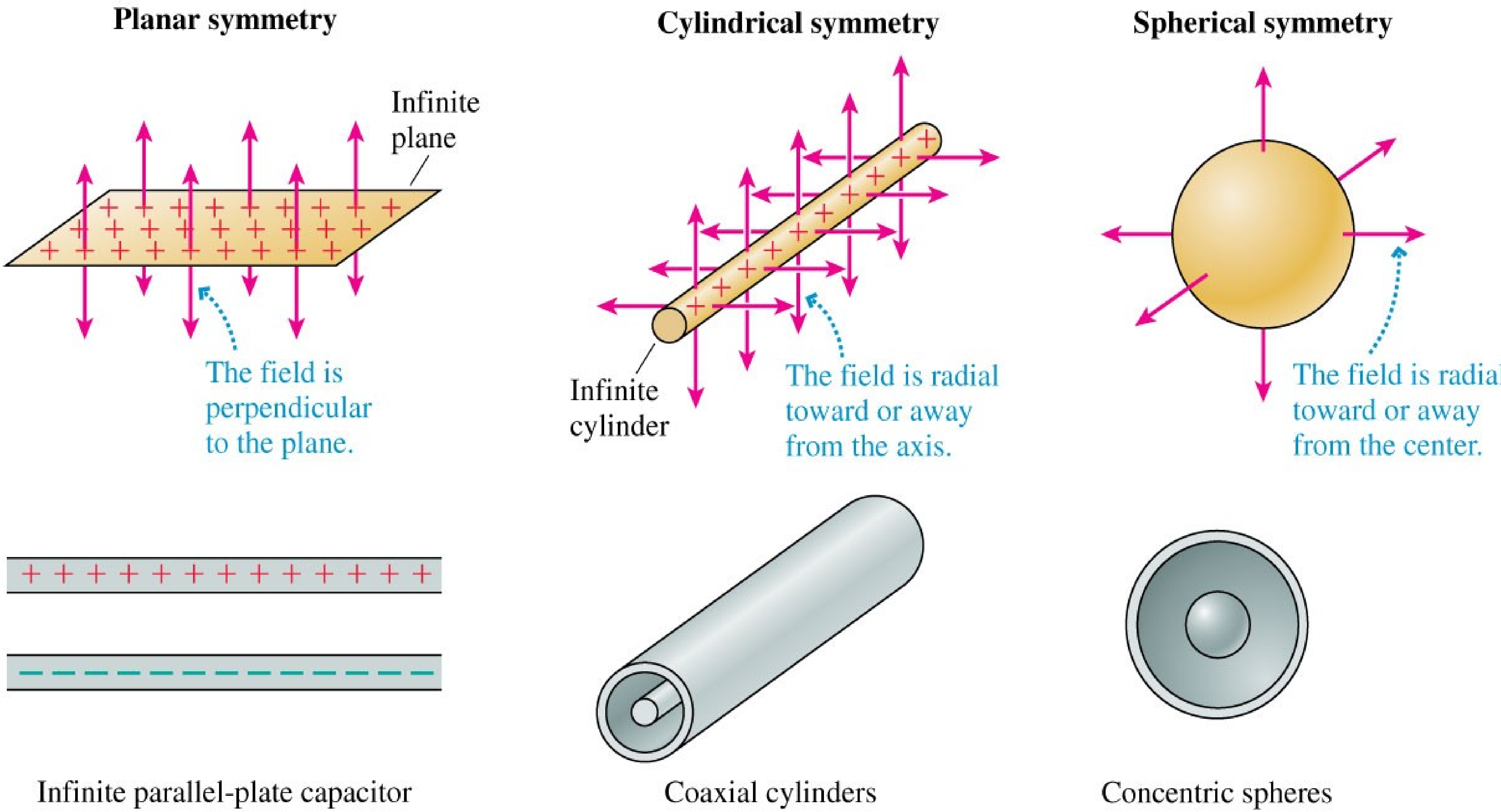
(D) 1.00×10^{7} N/C

(E) 1.00×10^{8} N/C

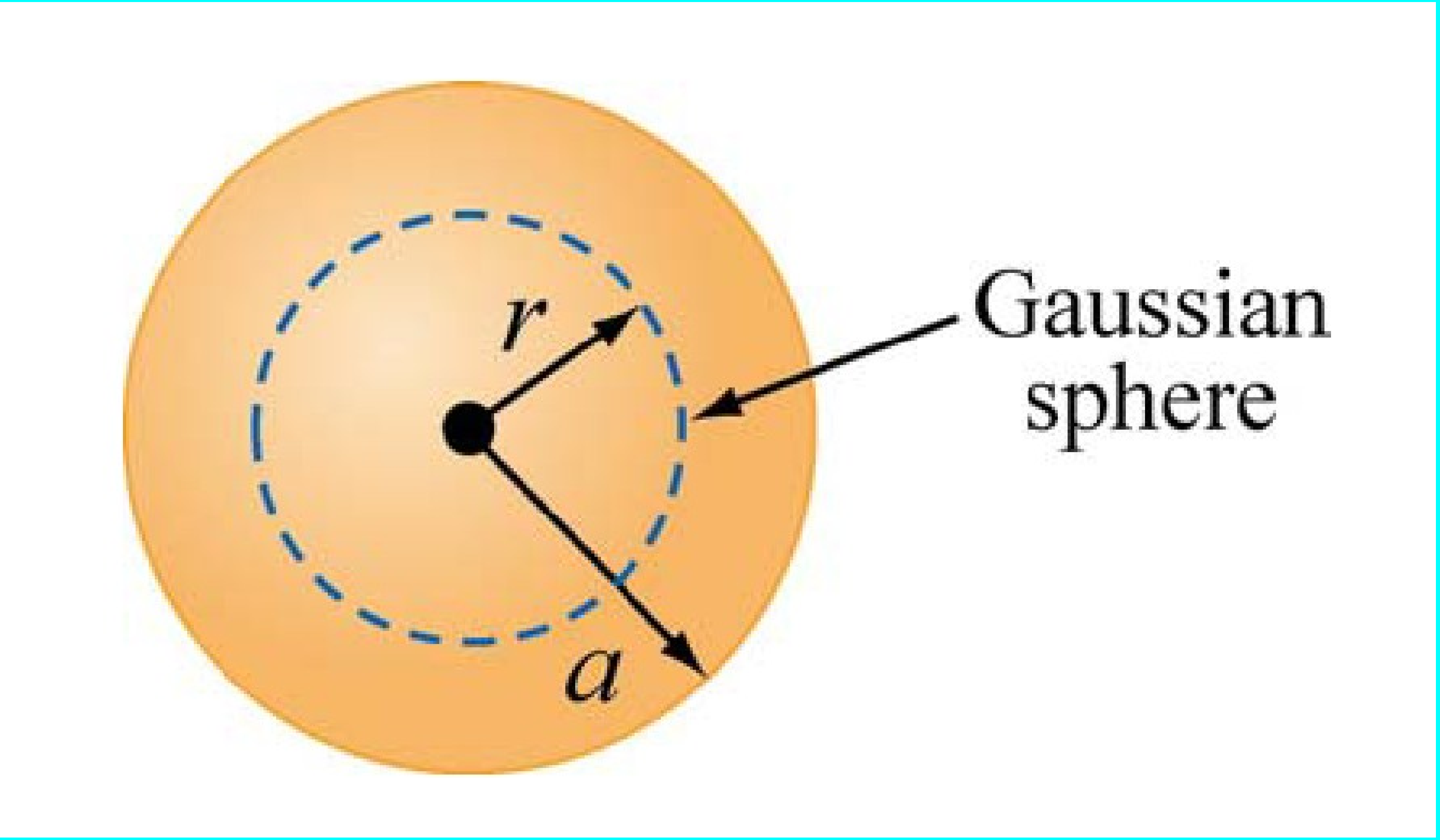
Infinite Plane III

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

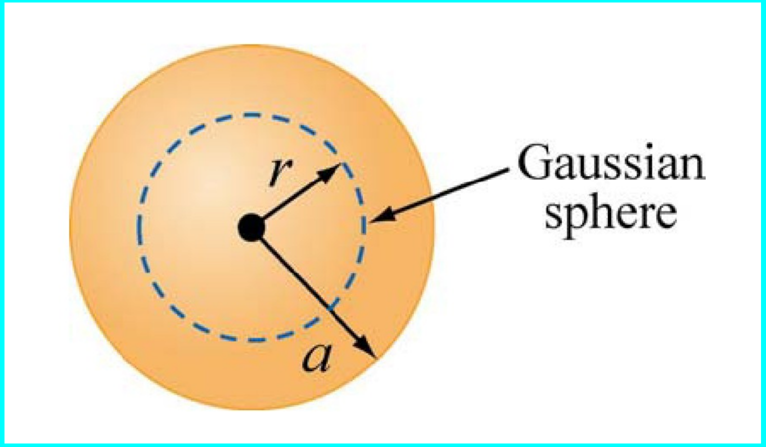
$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k} \qquad \vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r} \qquad \vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}$$



What about a solid sphere of charge?

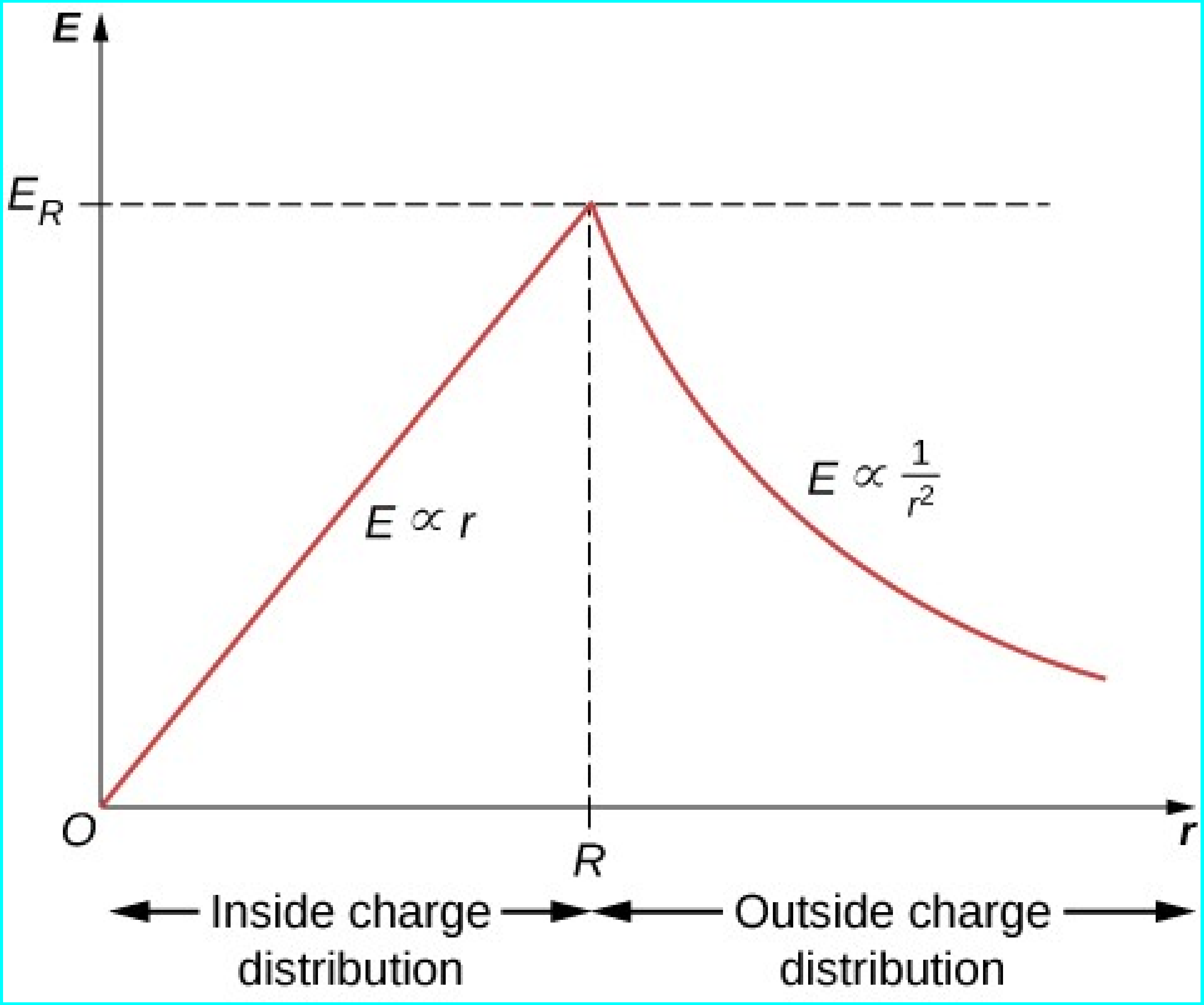


What about a solid sphere of charge?

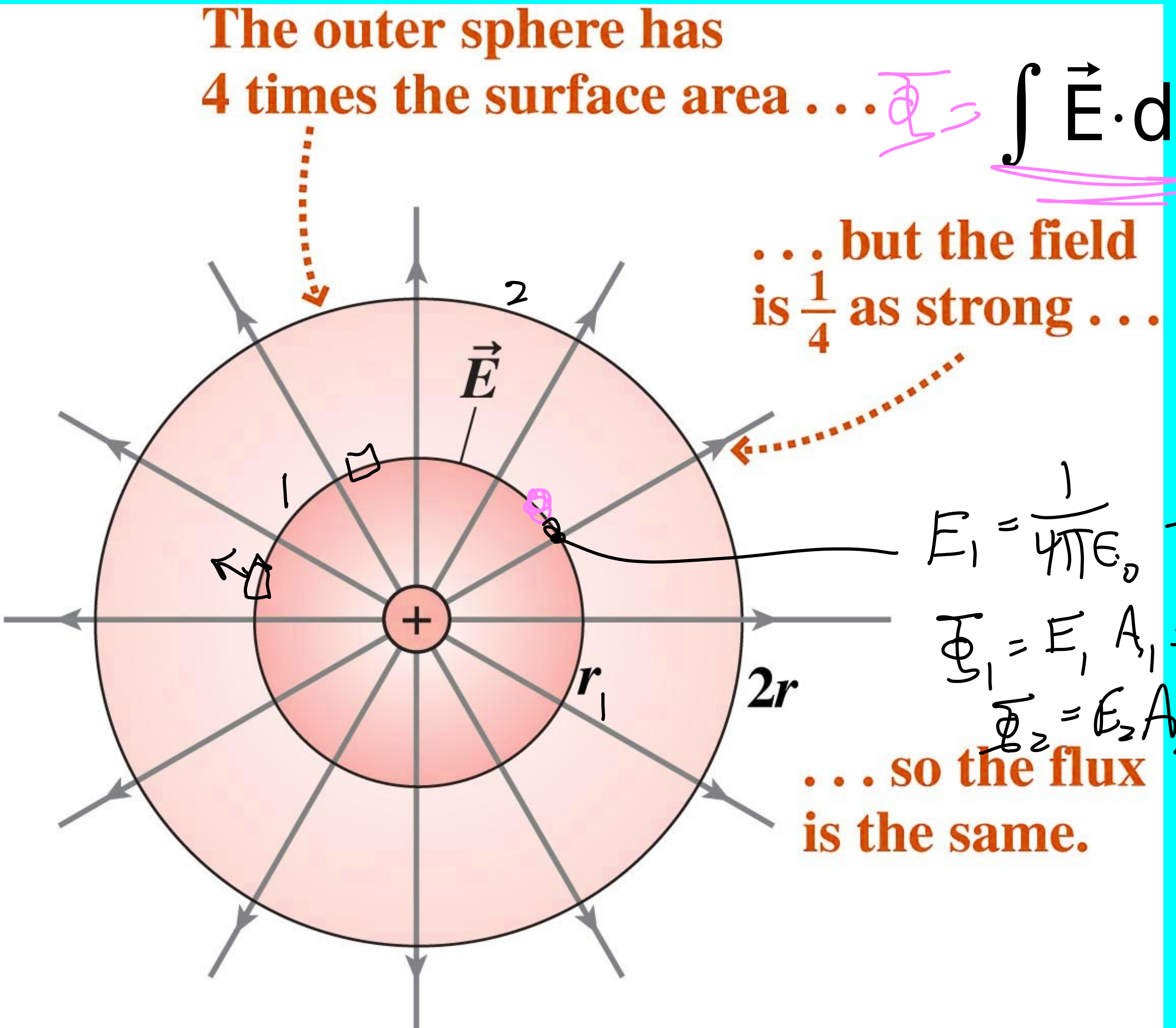


What about a solid sphere of charge?

What about a solid sphere of charge?



Gauss's law is a generalization of Coulomb's law



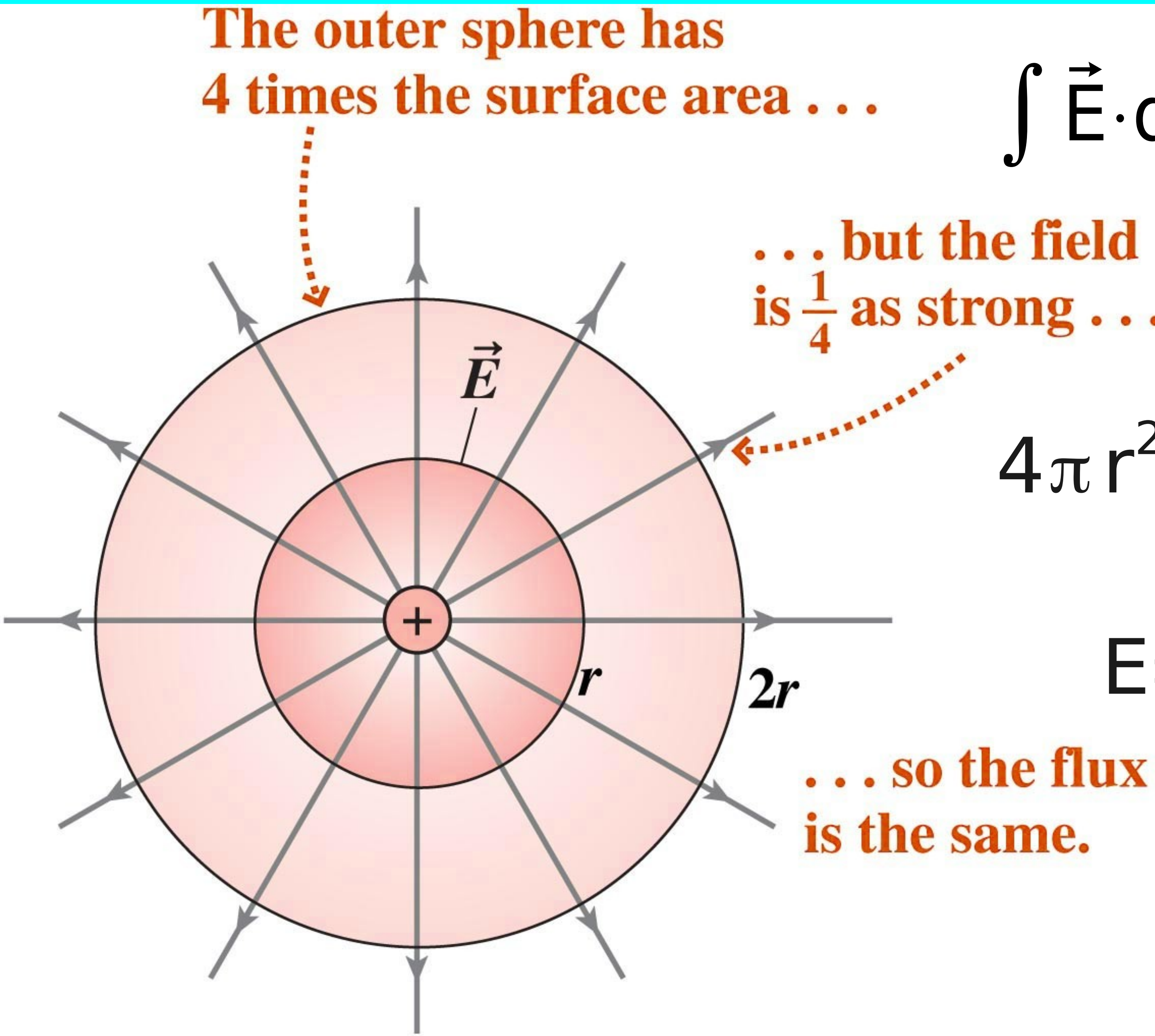
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1^2} \hat{r}$$

$$\Phi_1 = E_1 A_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1^2} 4\pi r_1^2$$

$$\Phi_2 = E_2 A_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(2r_1)^2} 4\pi (2r_1)^2$$

Gauss's law is a generalization of Coulomb's law



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$4\pi r^2 E = \frac{q_{enclosed}}{\epsilon_0}$$

$$E = \frac{q_{enclosed}}{4\pi\epsilon_0} \frac{1}{r^2}$$

$$E = \frac{kq}{r^2}$$

Gauss's law is a generalization of Coulomb's law

$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

Next Class:

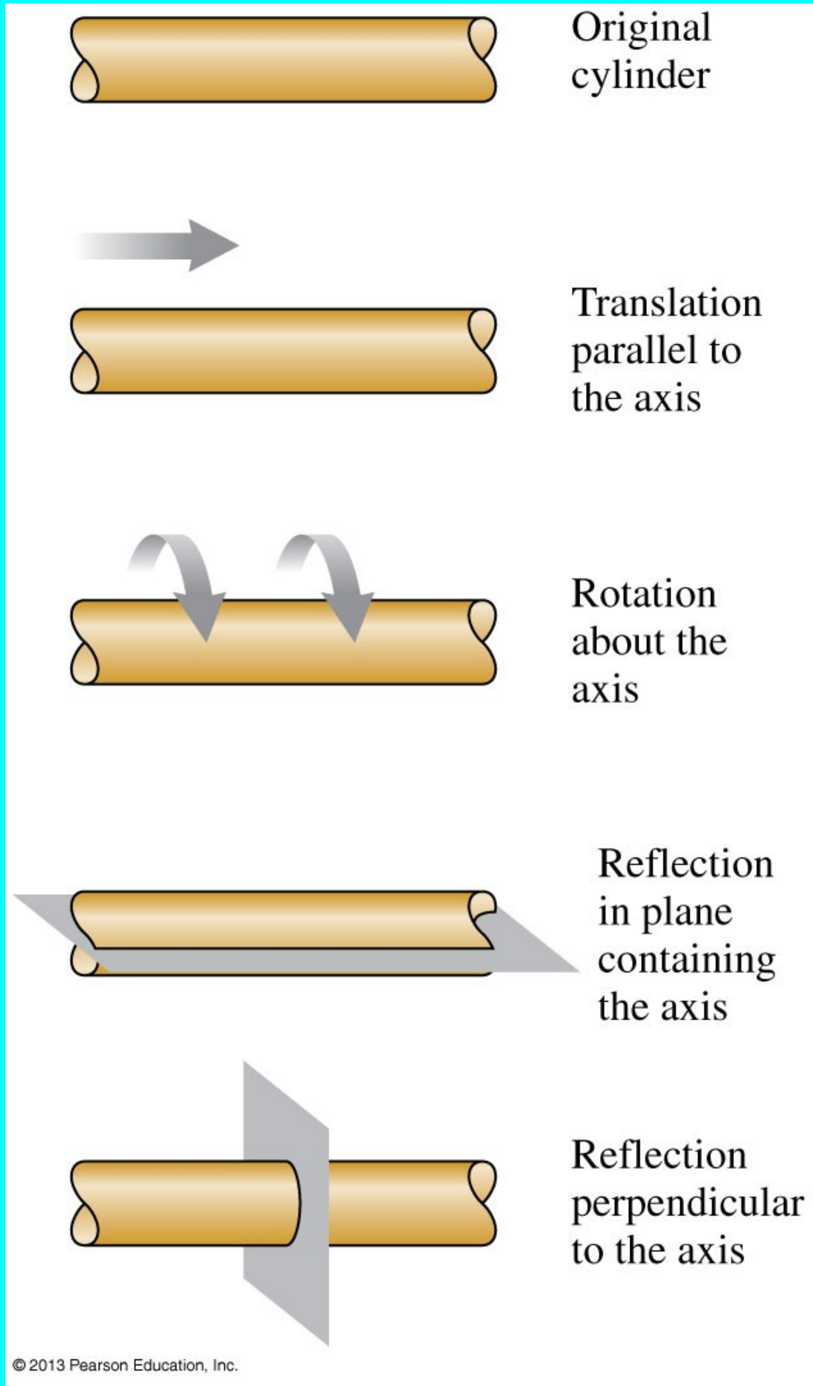
Electric potential … What's a volt anyway?

Relation between symmetry and Electric Field

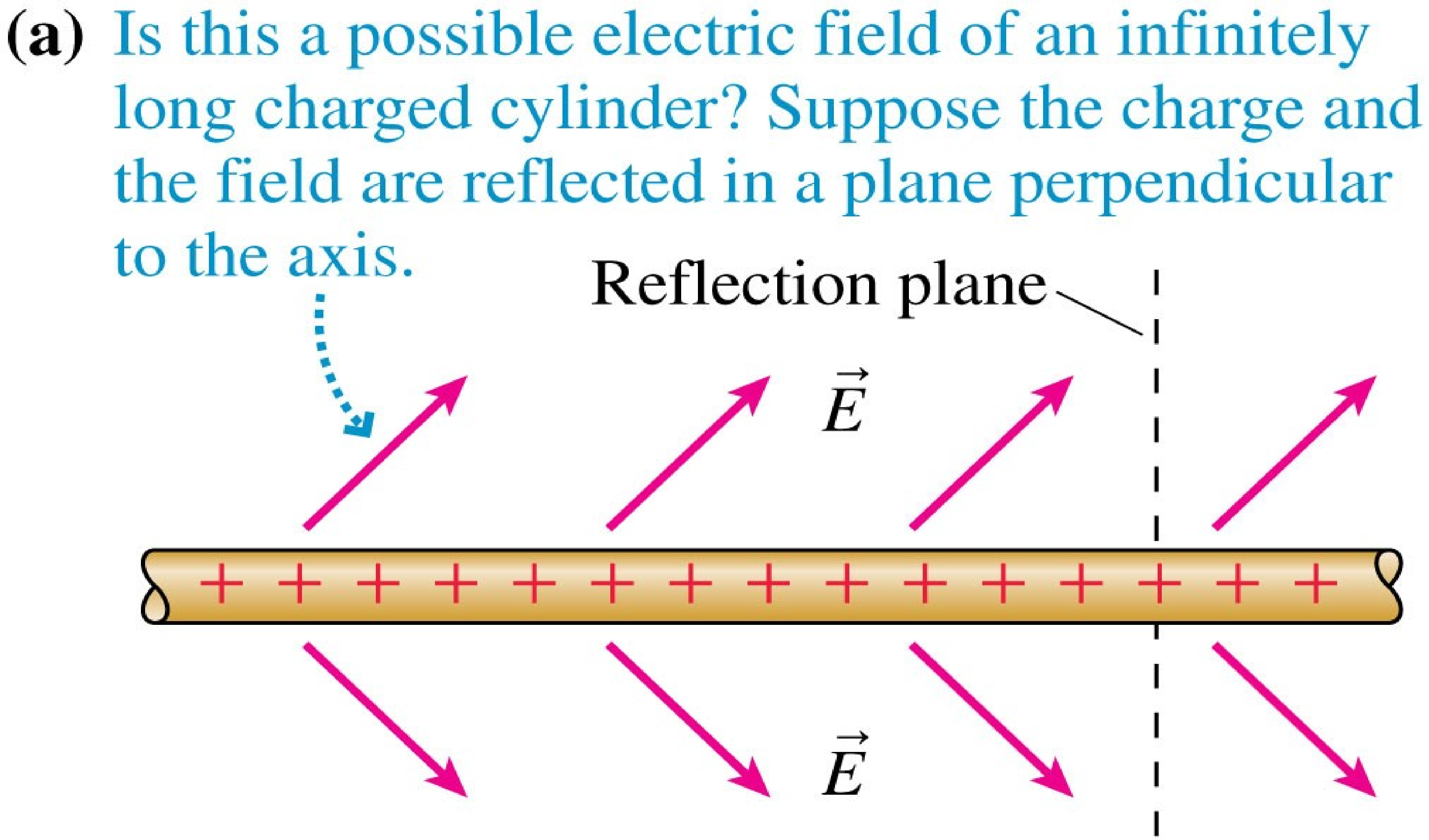
If you can't tell where you are with respect to a charge distribution

Then the electric field direction cannot give you a hint.

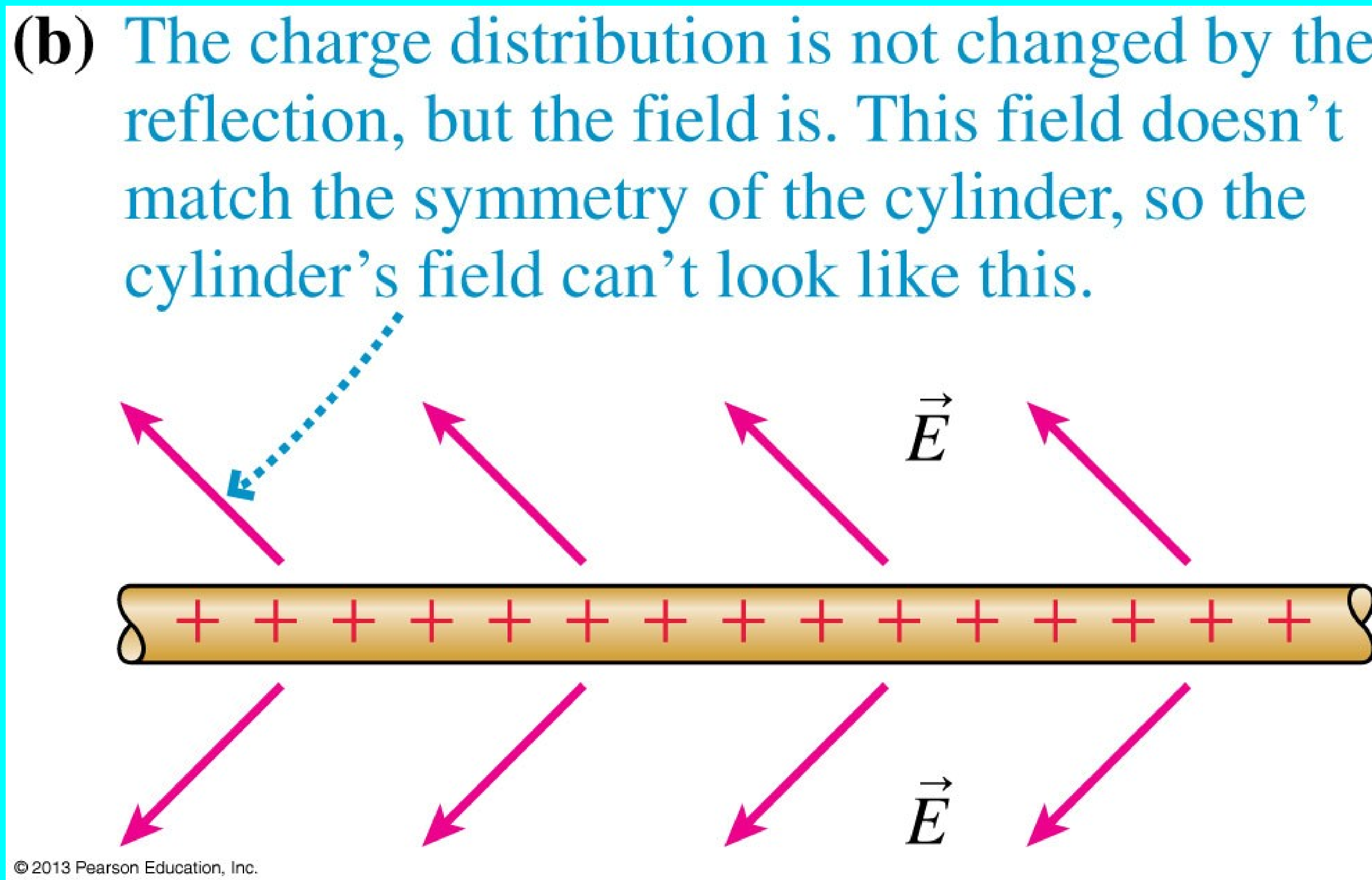
Relation between symmetry and Electric Field



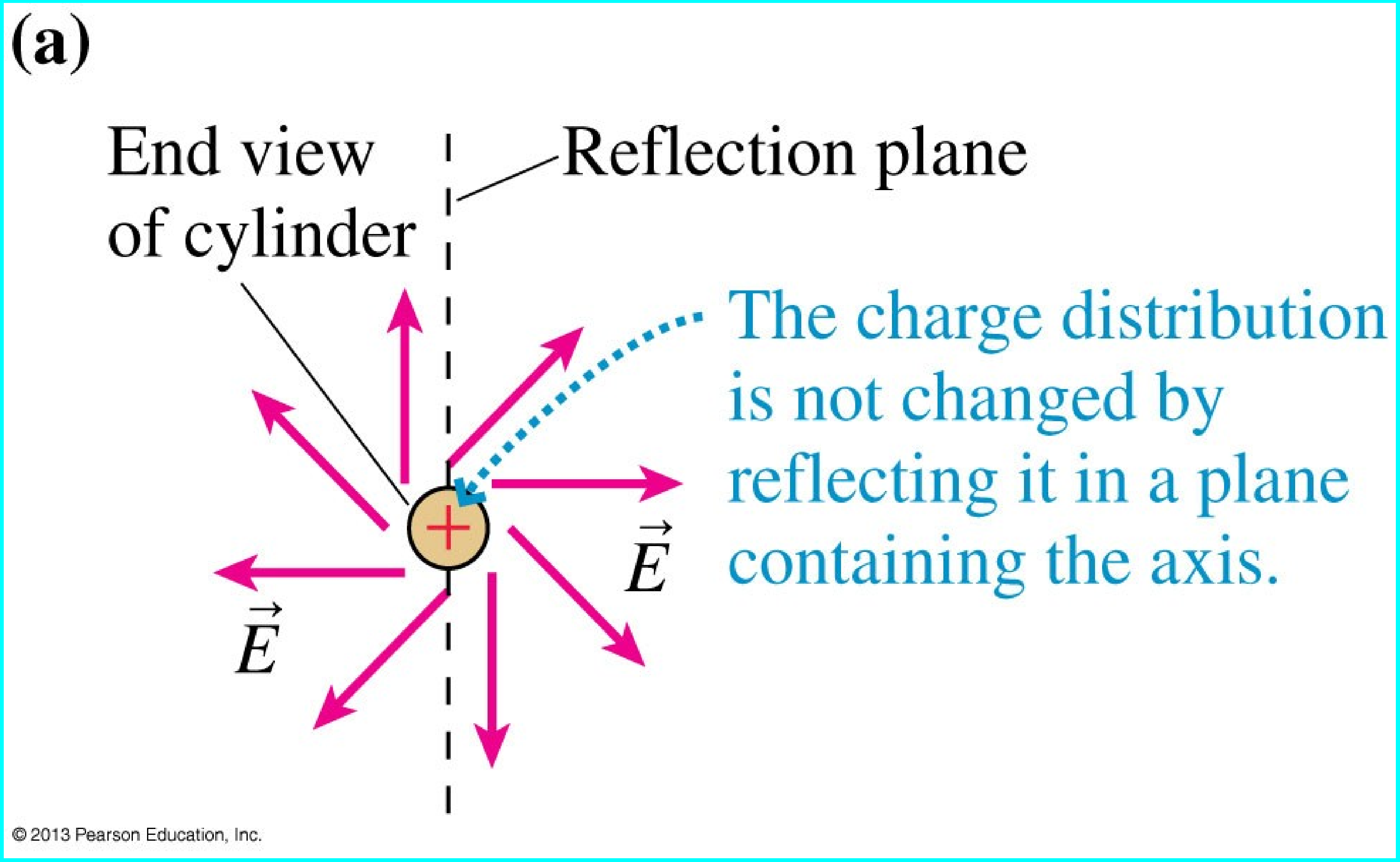
Relation between symmetry and Electric Field



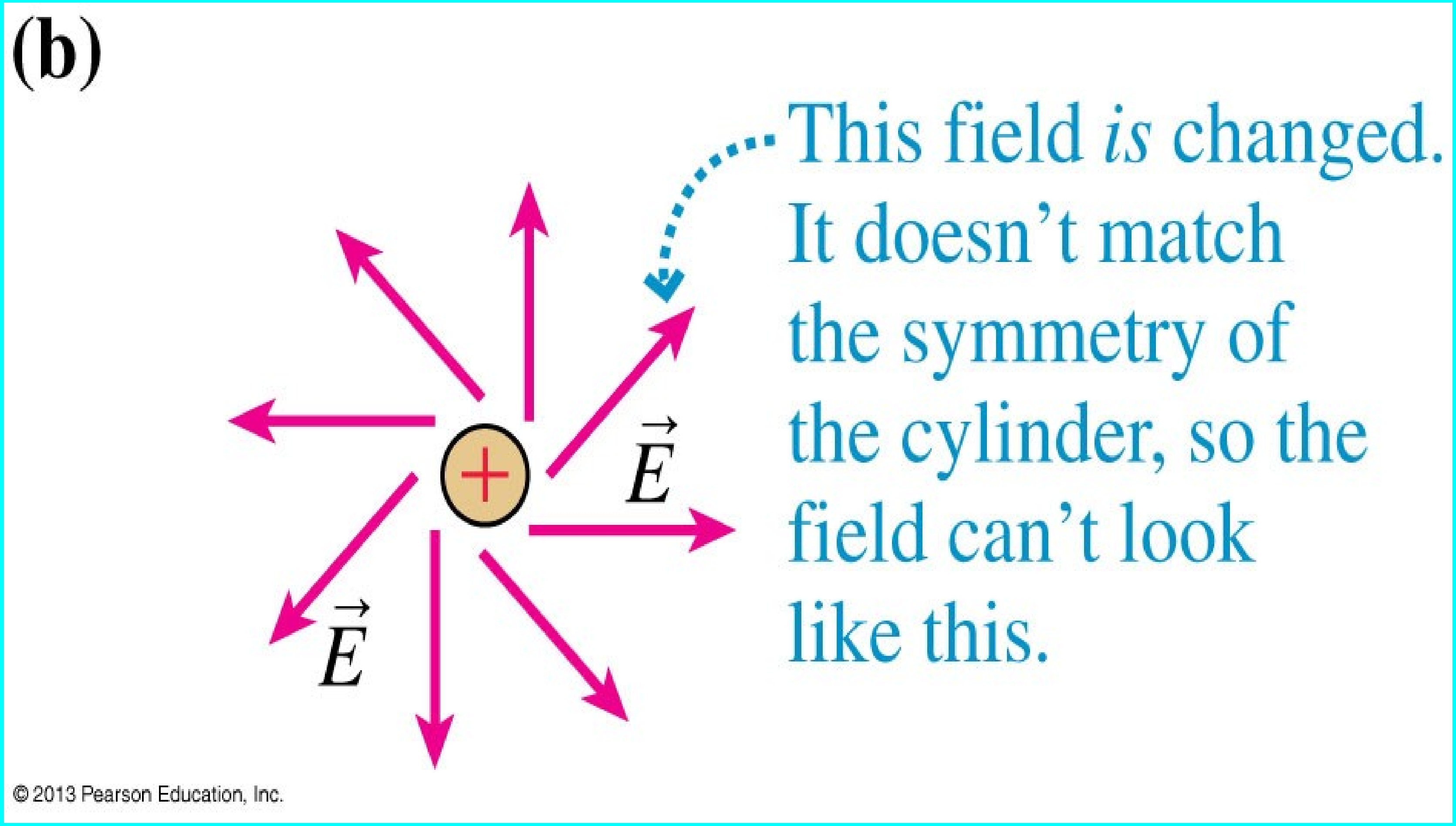
Relation between symmetry and Electric Field



Relation between symmetry and Electric Field



Relation between symmetry and Electric Field



Relation between symmetry and Electric Field

The ONLY field consistent with symmetry of an infinitely long cylinder points radially outward.