Lecture 08: 02/08/2024

- Announcements
  About the recitation problems

- Last Time
  Electric field lines
  Flux
  Gauss's law

- Today
  Gauss's law
  Field of symmetrical charge configurations

Key Equations

Coulomb's law

$$\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$$

Superposition of electric forces

$$\vec{\mathbf{F}}(r) = \frac{1}{4\pi\varepsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Electric force due to an electric field

$$\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$$

Electric field at point P

$$\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Field of an infinite wire

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{z} \hat{\mathbf{k}}$$

Field of an infinite plane

$$\vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0} \hat{\mathbf{k}}$$

Dipole moment

$$\vec{\mathbf{p}} = q\vec{\mathbf{d}}$$

Key Equations

Definition of electric flux, for uniform electric field	$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \to EA\cos\theta$
Electric flux through an open surface	$\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Electric flux through a closed surface	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Gauss's law	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{\text{enc}}}{\varepsilon_0}$
Gauss's Law for systems with symmetry	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E \oint_S dA = EA = \frac{q_{\text{enc}}}{\varepsilon_0}$
The magnitude of the electric field just outside the surface of a conductor	$E = \frac{\sigma}{\varepsilon_0}$

Gauss's law

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

icphysweb_field_line_simulator

https://icphysweb.z13.web.core.windows.net/simulation.html

academo_field_line_sim

https://academo.org/demos/electric-field-line-simulator/

electric_field_hockey

https://phet.colorado.edu/sims/cheerpj/electric-hockey/latest/electric-hockey.html?simulation=electric-hockey

Gauss's law

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

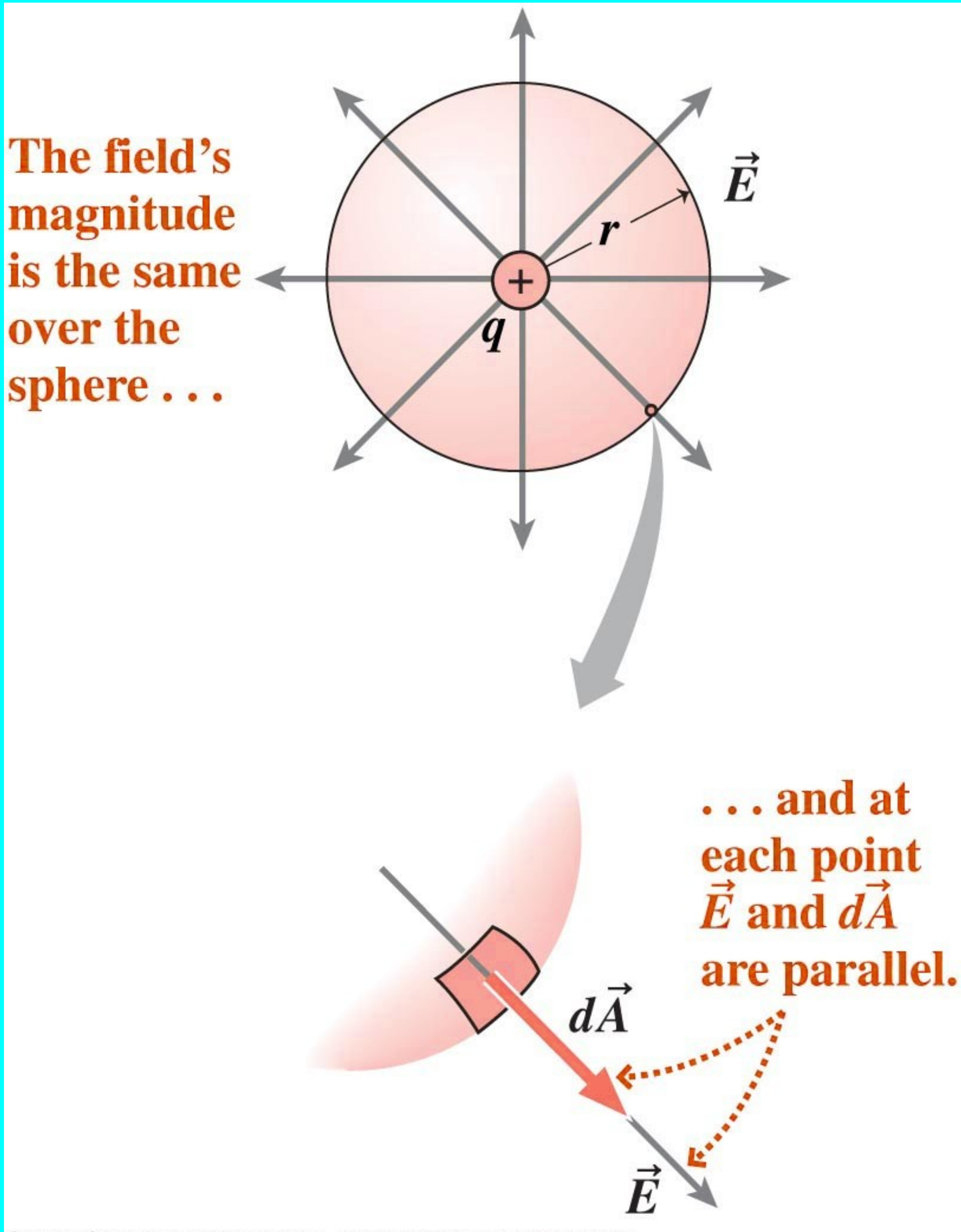
$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

Gauss's law for simple cases

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Gauss's law is a generalization of Coulomb's law



$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

Gauss's law is a generalization of Coulomb's law

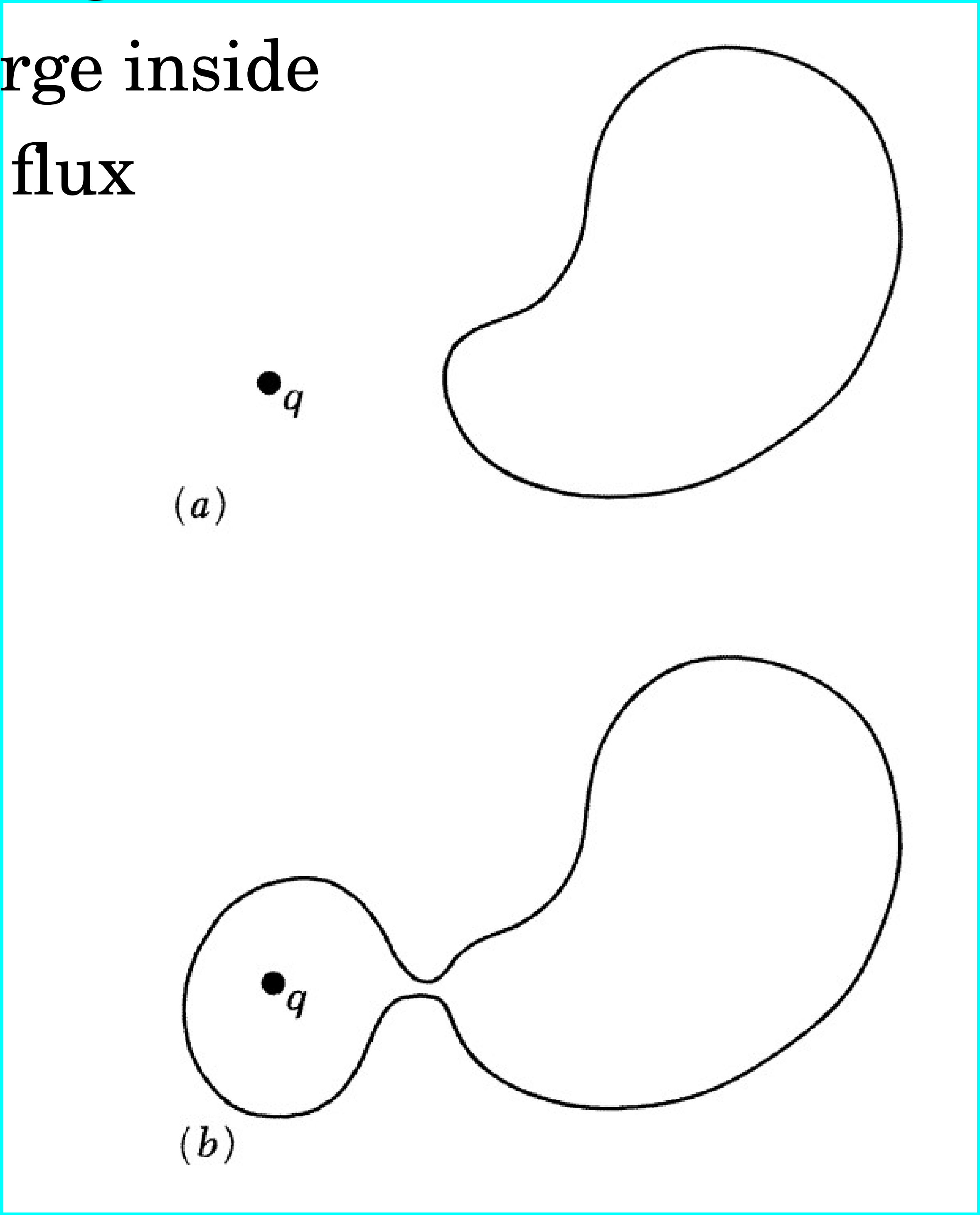
$$\Phi_{total} = \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

Gaussian logic trick

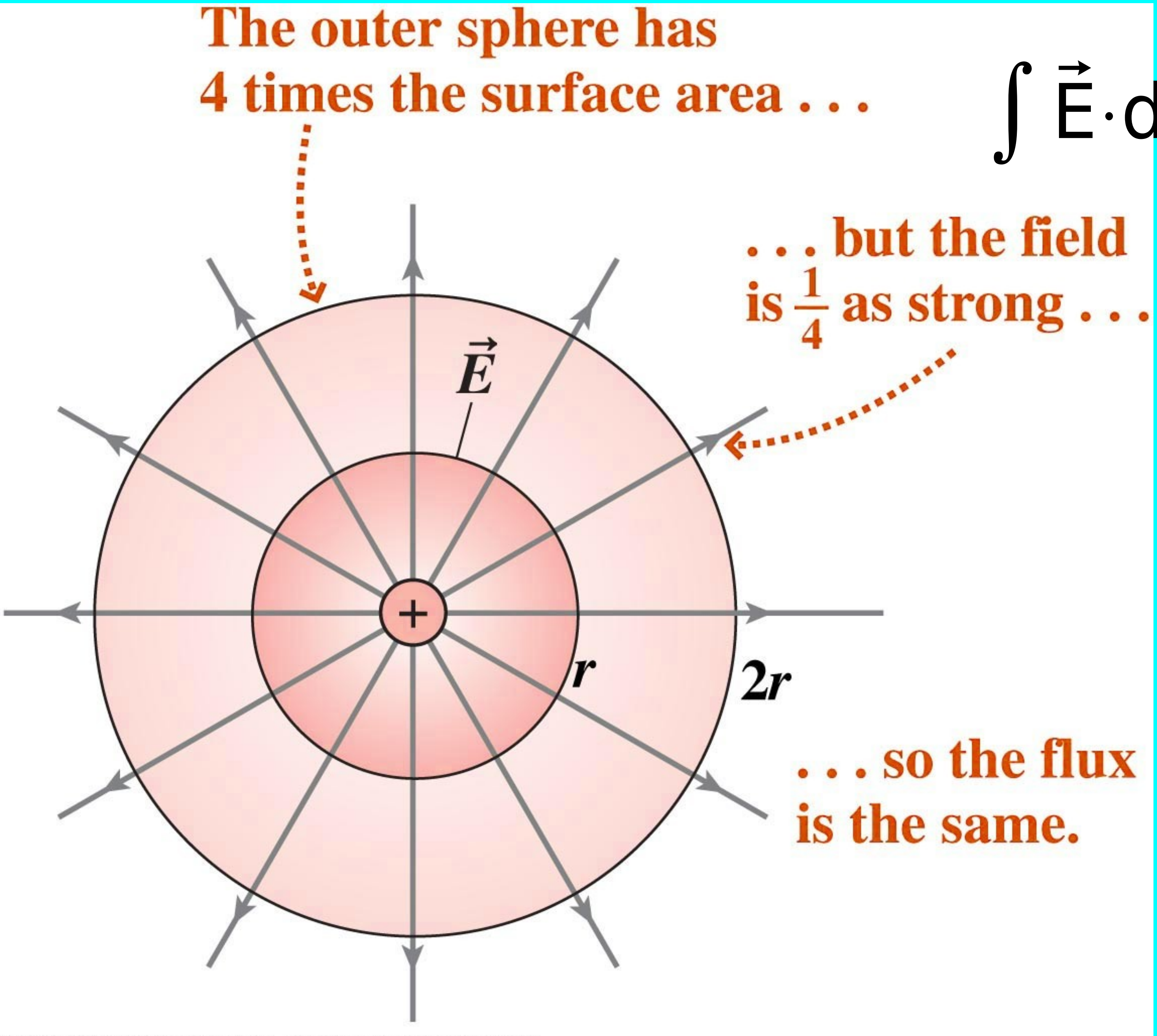
No charge inside

No net flux

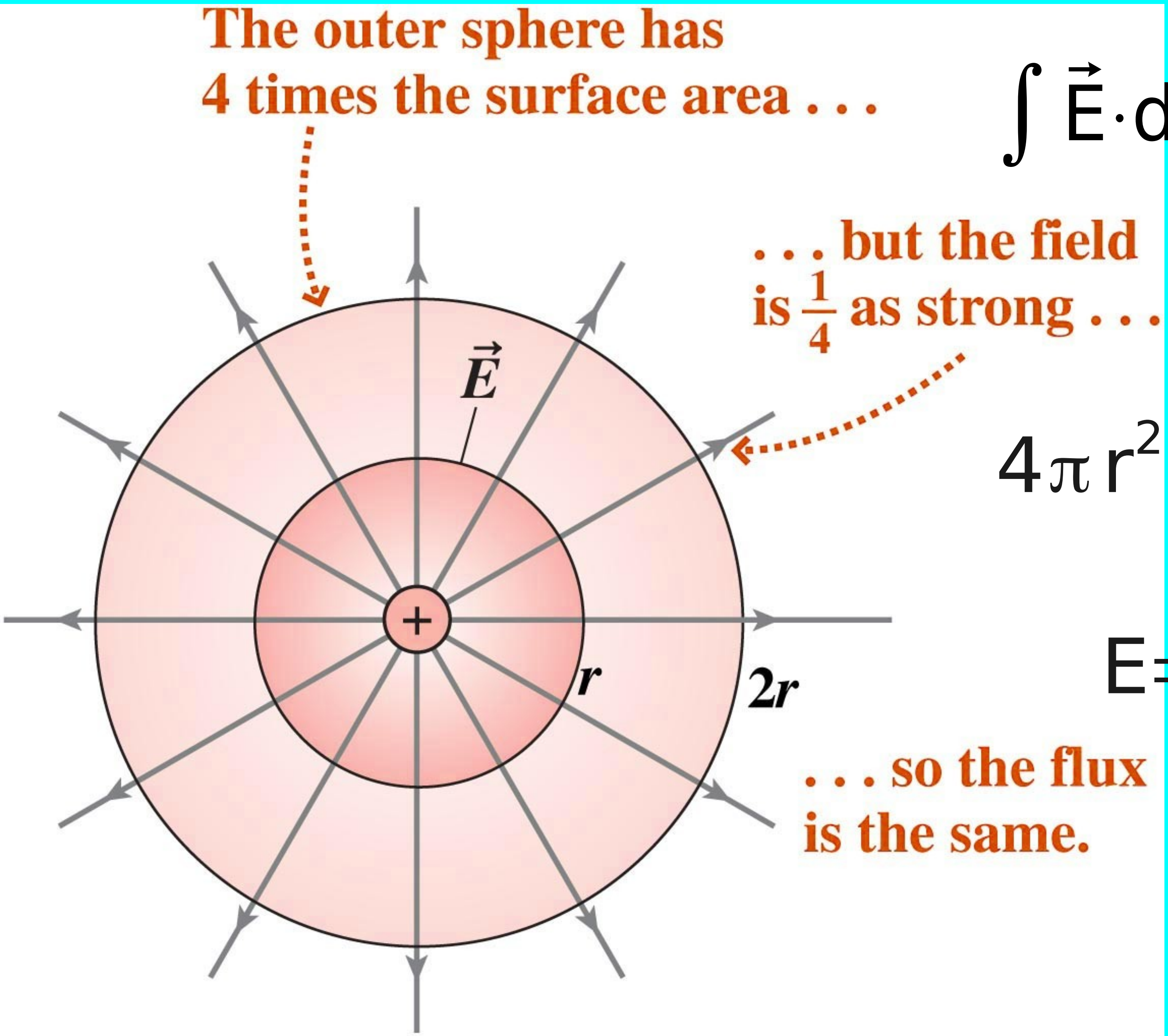


Gauss's law is a generalization of Coulomb's law

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$



Gauss's law is a generalization of Coulomb's law



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$4\pi r^2 E = \frac{q_{enclosed}}{\epsilon_0}$$

$$E = \frac{q_{enclosed}}{4\pi\epsilon_0}\frac{1}{r^2}$$

$$E = \frac{kq}{r^2}$$

Simple Case I: Long (infinite) Wire

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

Simple Case I: Long (infinite) Wire

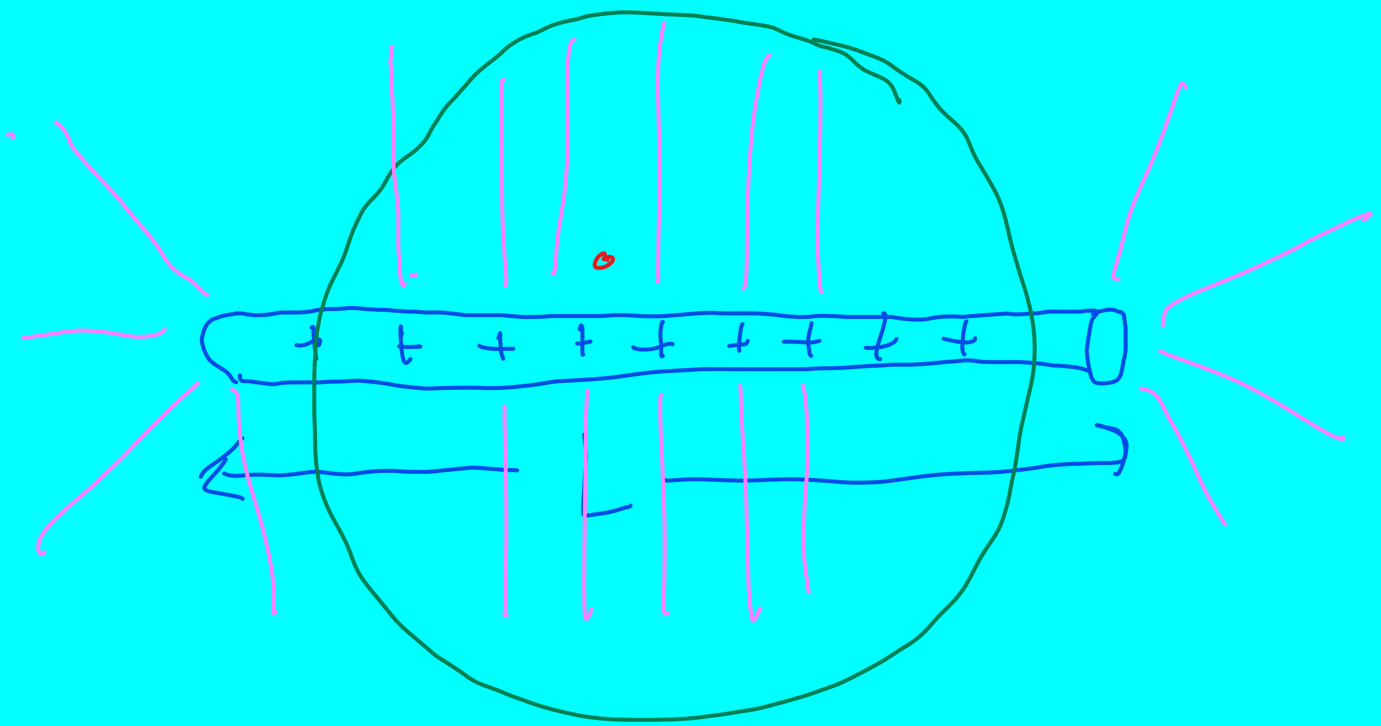
$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Simple Case I: Long (infinite) Wire

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

$\lambda = \frac{Q}{L}$ $\sigma = \frac{Q}{A}$

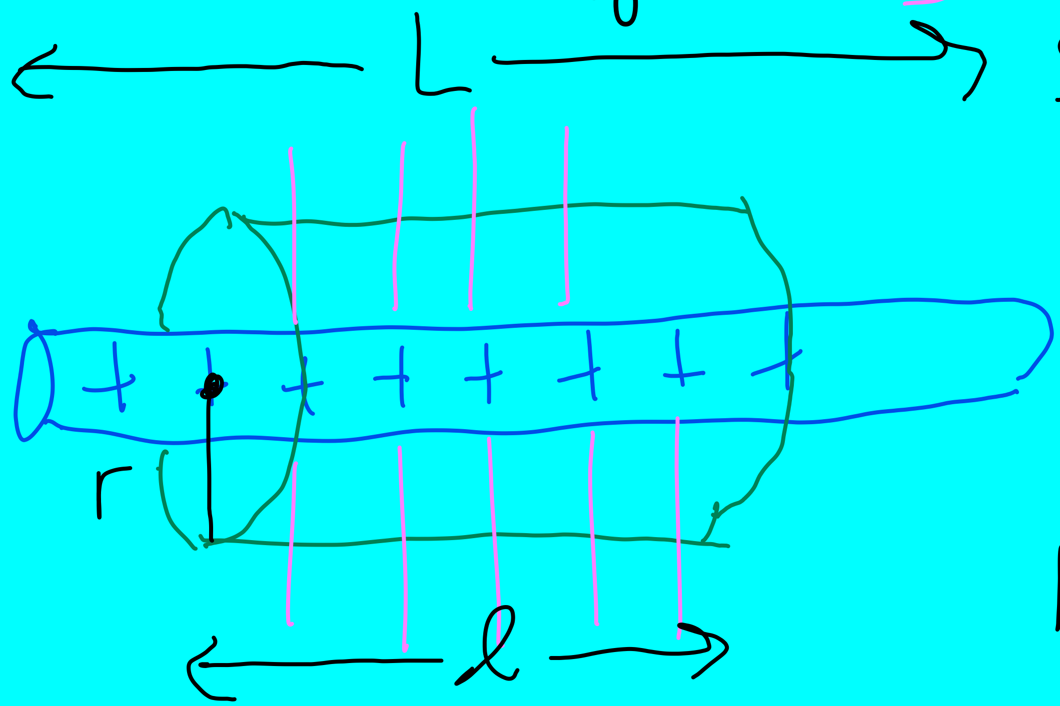
$\rho = \frac{Q}{Volume}$



Simple Case I: Long (infinite) Wire

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = SA \cdot E$$



$$SA = 2\pi r \ell + 2\pi r^2$$

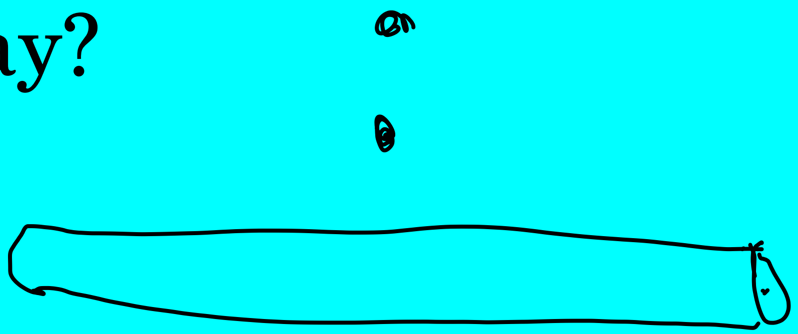
$$E\, 2\pi r \ell = \frac{q_{enc}}{\epsilon_0}$$

$$E\, 2\pi r \ell = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

Long Wire I

A wire is 10 meters long and you are 10 cm away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 20 cm away?



(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

Long Wire II

A wire is 10 meters long and you are 100 m away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 200 m away?

(A) 4 N/C

(B) 8 N/C

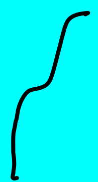
(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

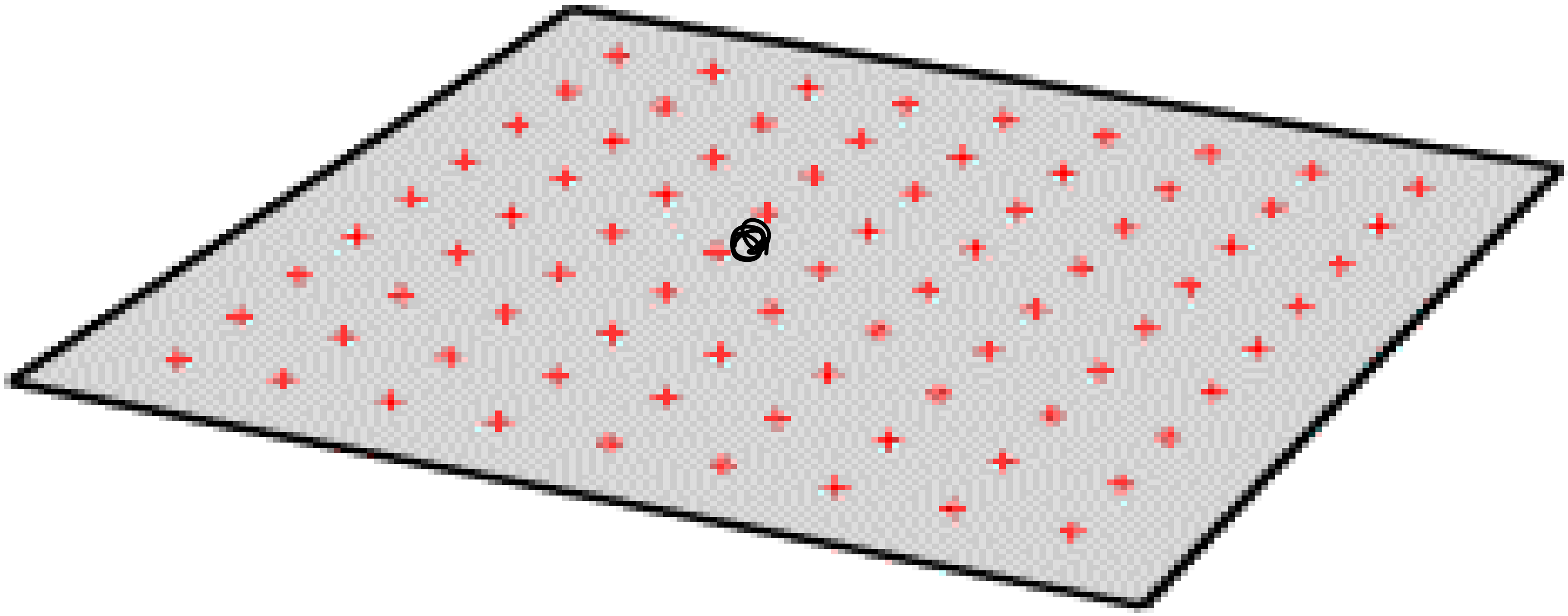
Simple Case II: Large (infinite) Plane

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



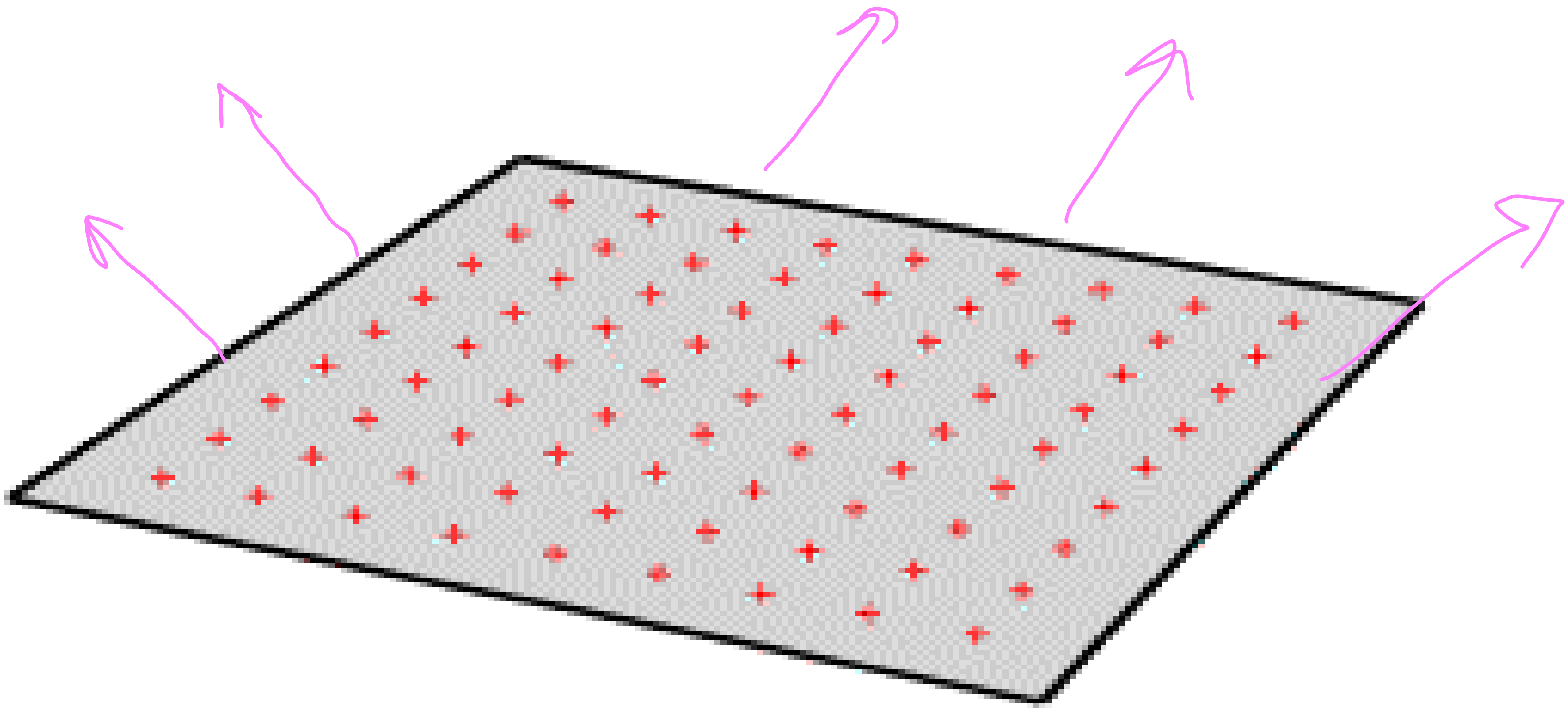
Relation between symmetry and Electric Field

Imagine an infinite plane of charge.



Relation between symmetry and Electric Field

Because you can't tell what direction you are facing, the field must be ONLY Perpendicular to the plane.

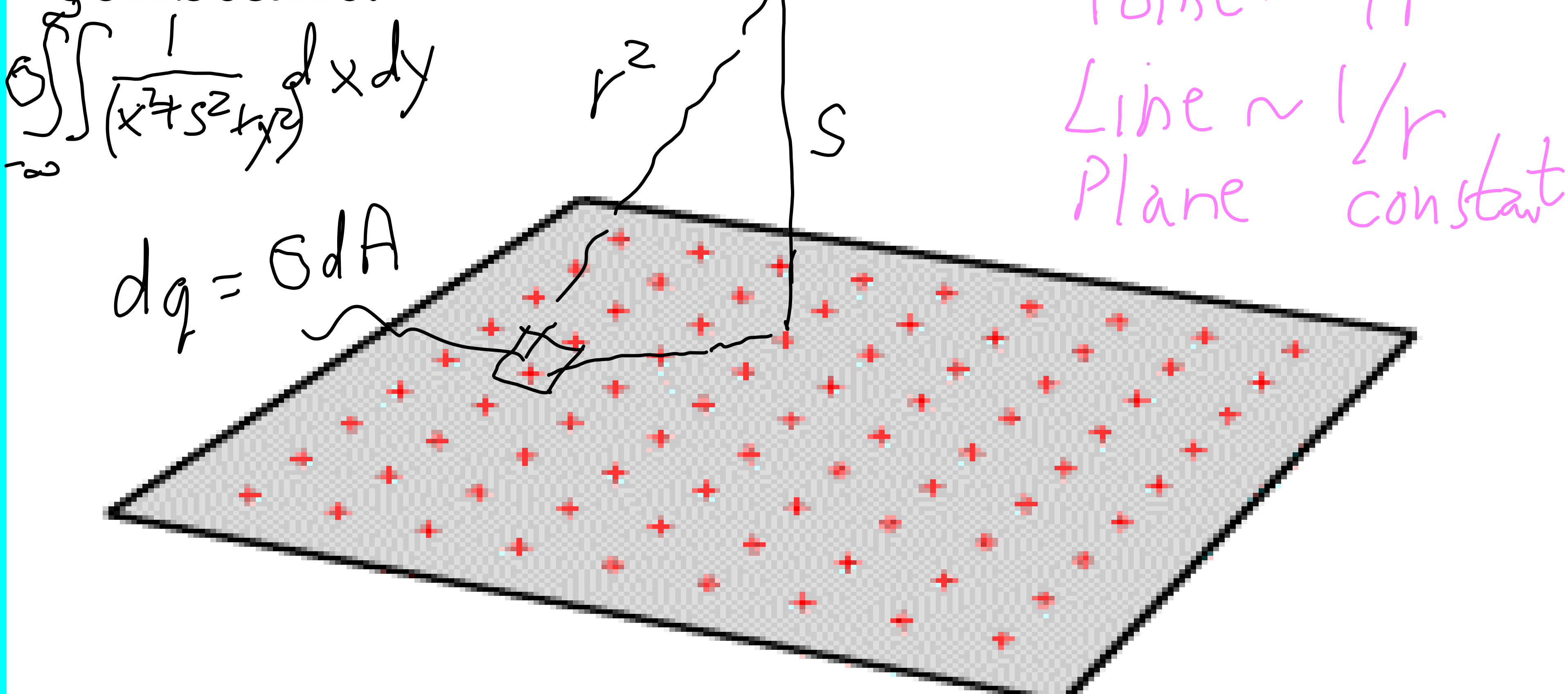


How large is this area?

[A] Floor tiles (4'x6')
[B] Painting (12"x18")
[C] Warehouse (60'x90')
[D] Airfield (1000'x1500')
[E] Not enough Info, can't tell

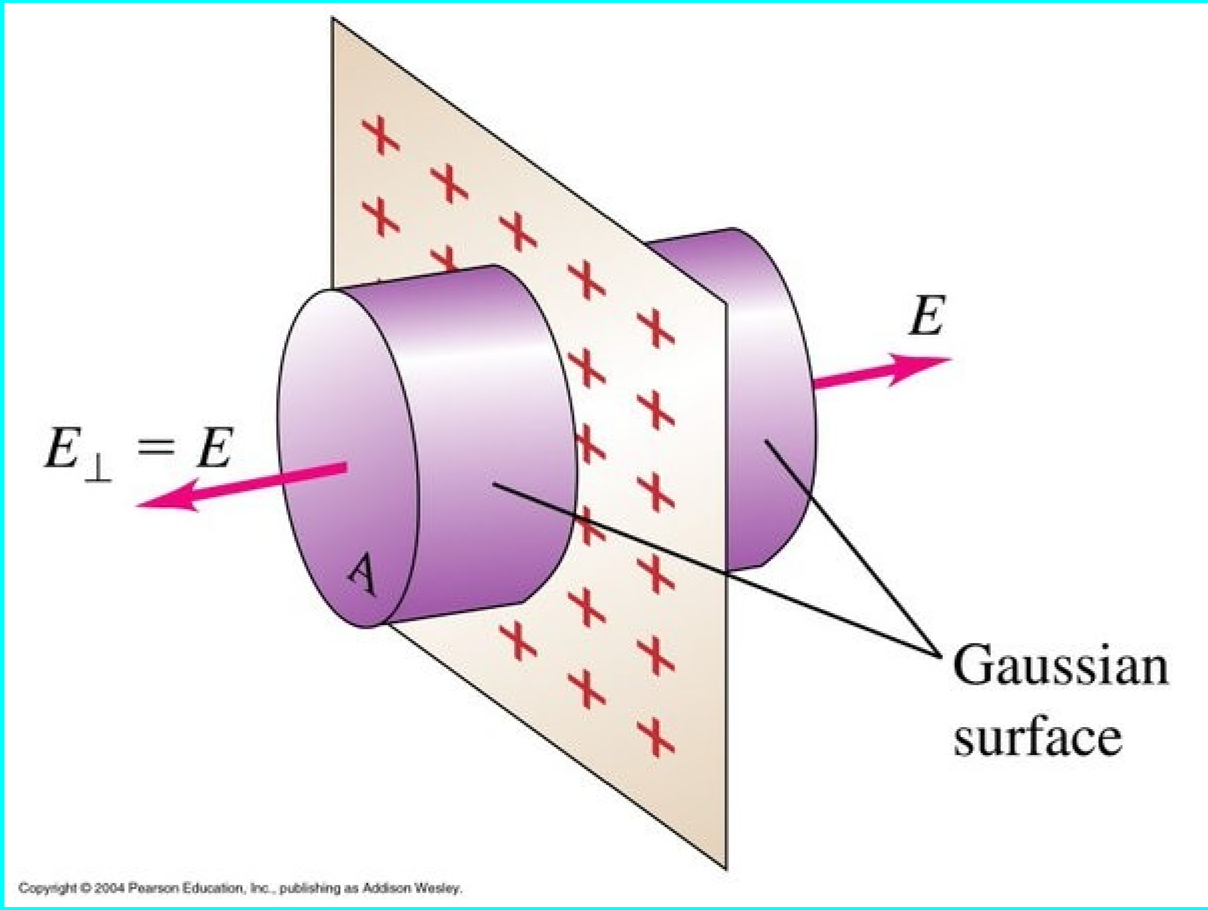
Electric field of a plane of charge

Because you ALSO can't tell how far away you are from the plane, the field cannot change magnitude. It must be constant.



Electric field of a plane of charge

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

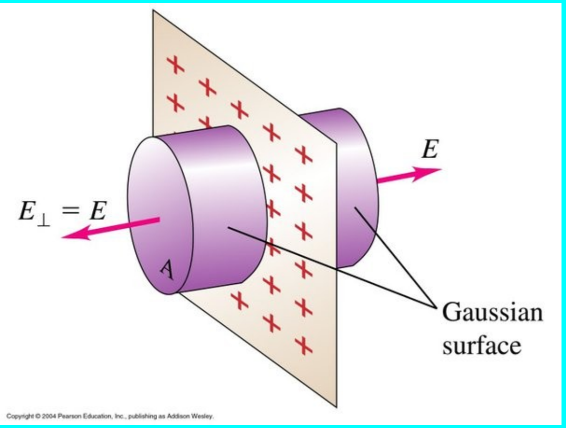


Electric field of a plane of charge

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\sigma = \frac{Q}{A}$$



Infinite Plane I

A square plate is 10 meters on a side and you are 10 cm away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 20 cm away?

$$E = \frac{\sigma}{2\epsilon_0}\hat{k}$$

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

Infinite Plane II

A square plate is 10 meters on a side and you are 100 m away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 200 m away?

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

Infinite Plane III

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

(A) $8.85\times10^{-5}\ \text{N/C}$

(B) $4.43\times10^{-5}\ \text{N/C}$

(C) $5.00\times10^{6}\ \text{N/C}$

(D) $1.00\times10^{7}\ \text{N/C}$

(E) $1.00\times10^{8}\ \text{N/C}$

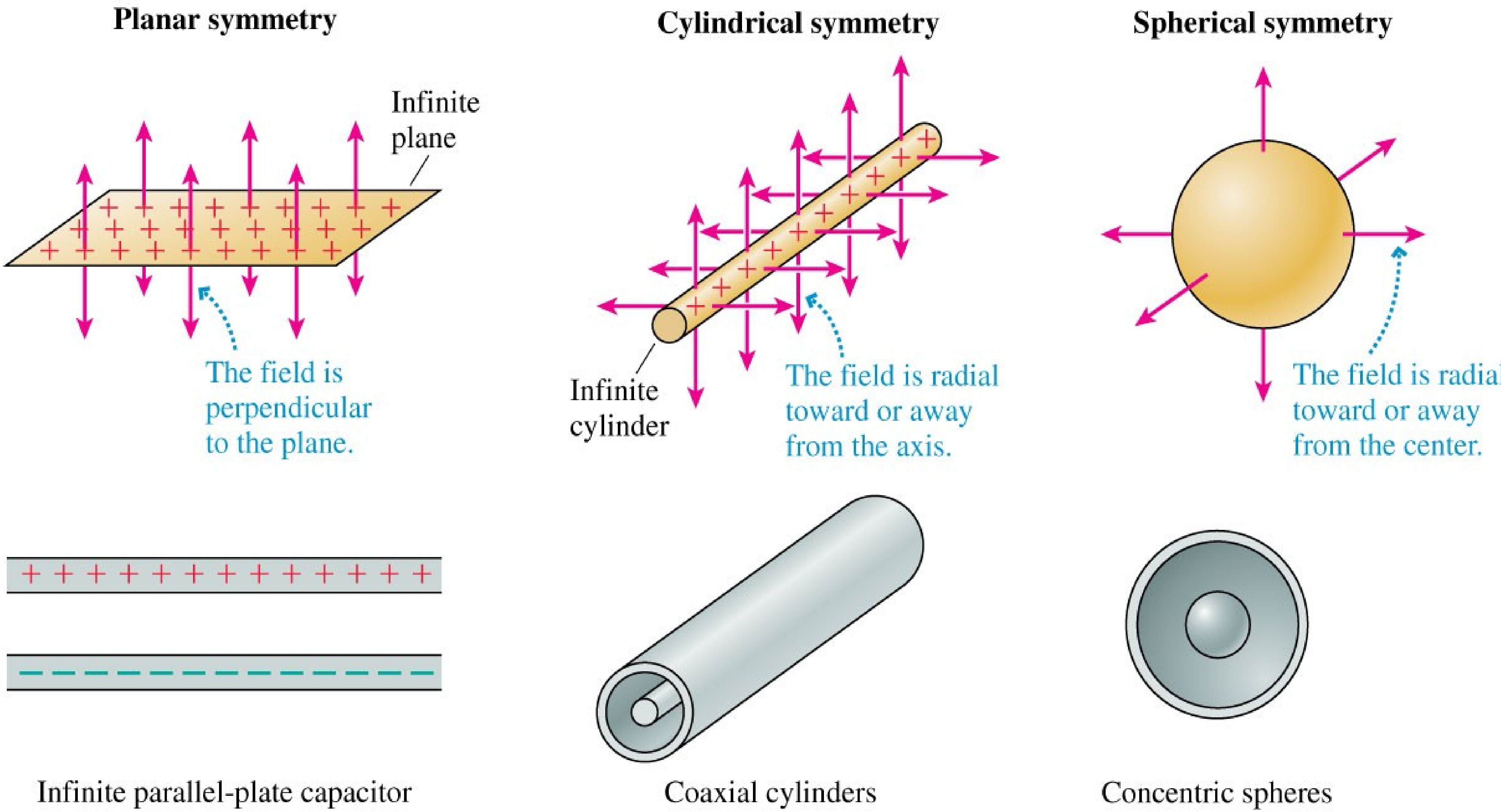
Infinite Plane III

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k}$$

$$\vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r}$$

$$\vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}$$

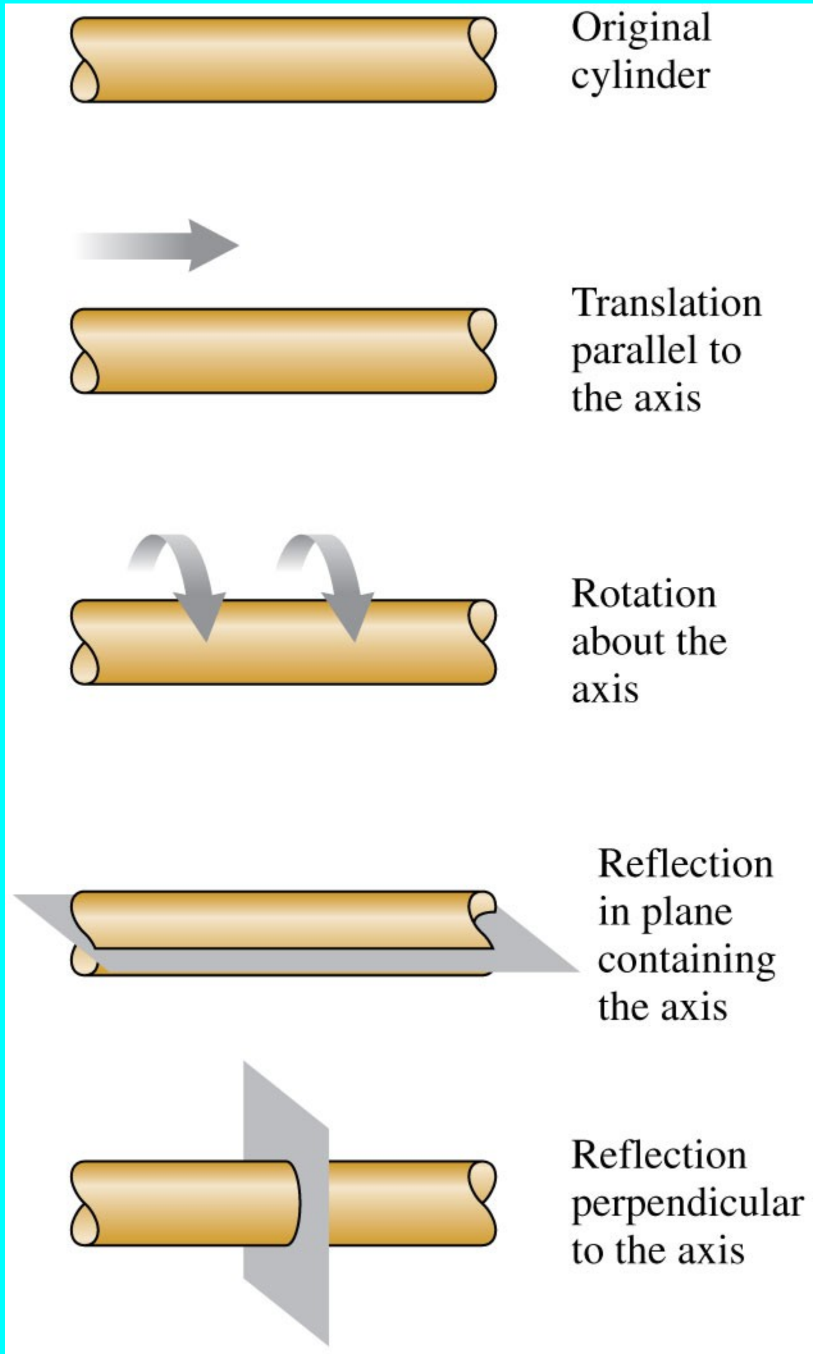


Relation between symmetry and Electric Field

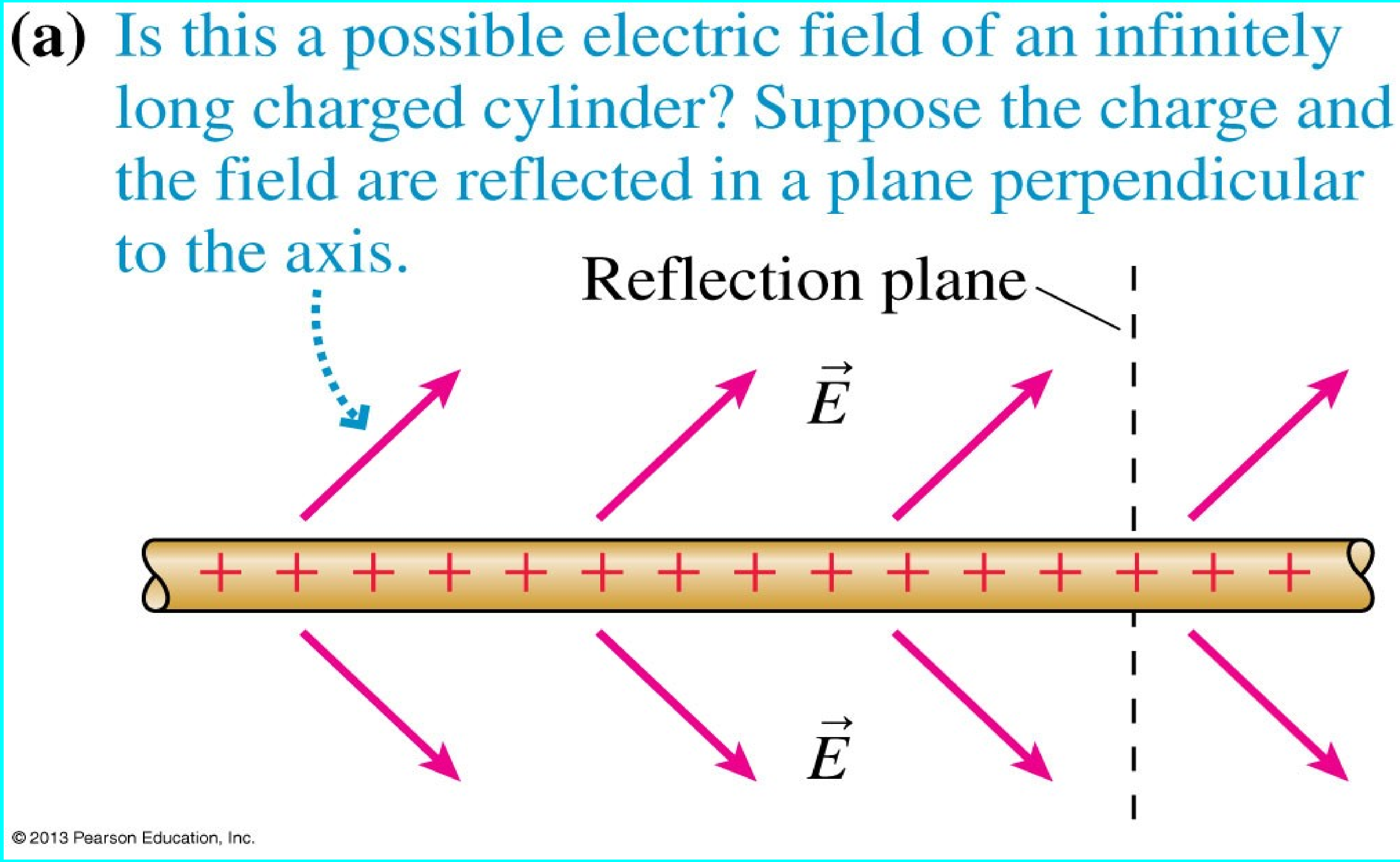
If you can't tell where you are with respect to a charge distribution

Then the electric field direction cannot give you a hint.

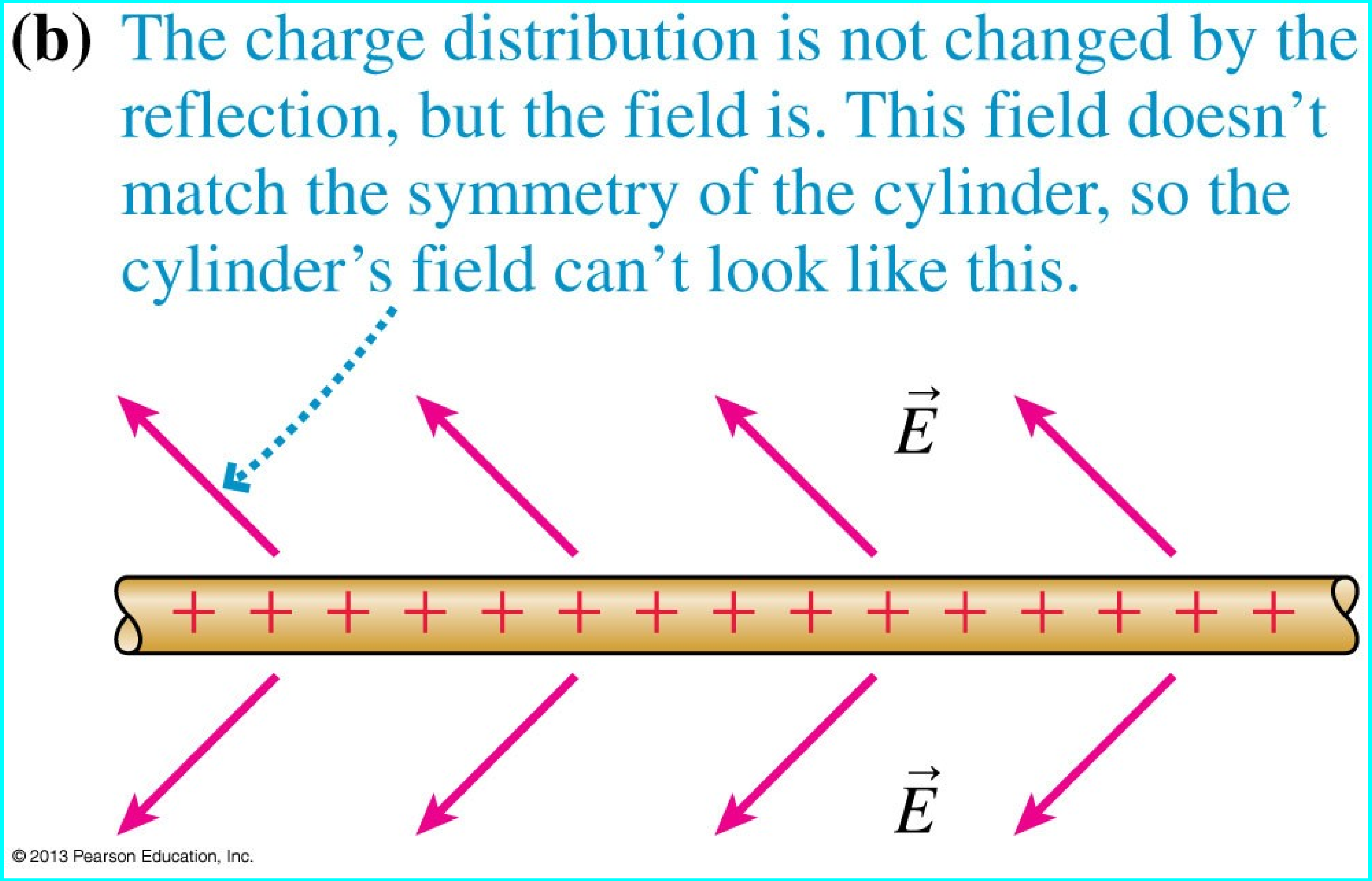
Relation between symmetry and Electric Field



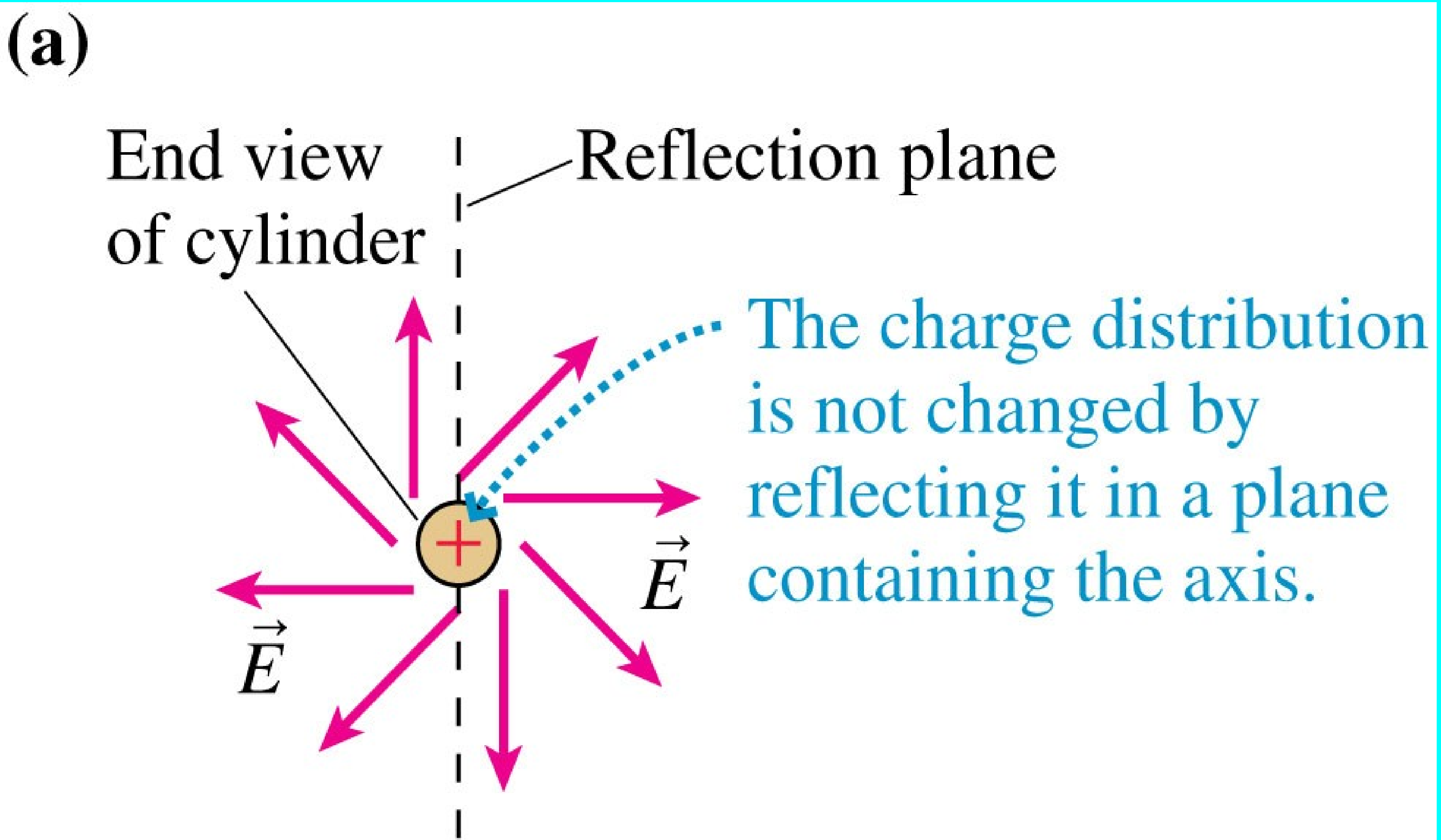
Relation between symmetry and Electric Field



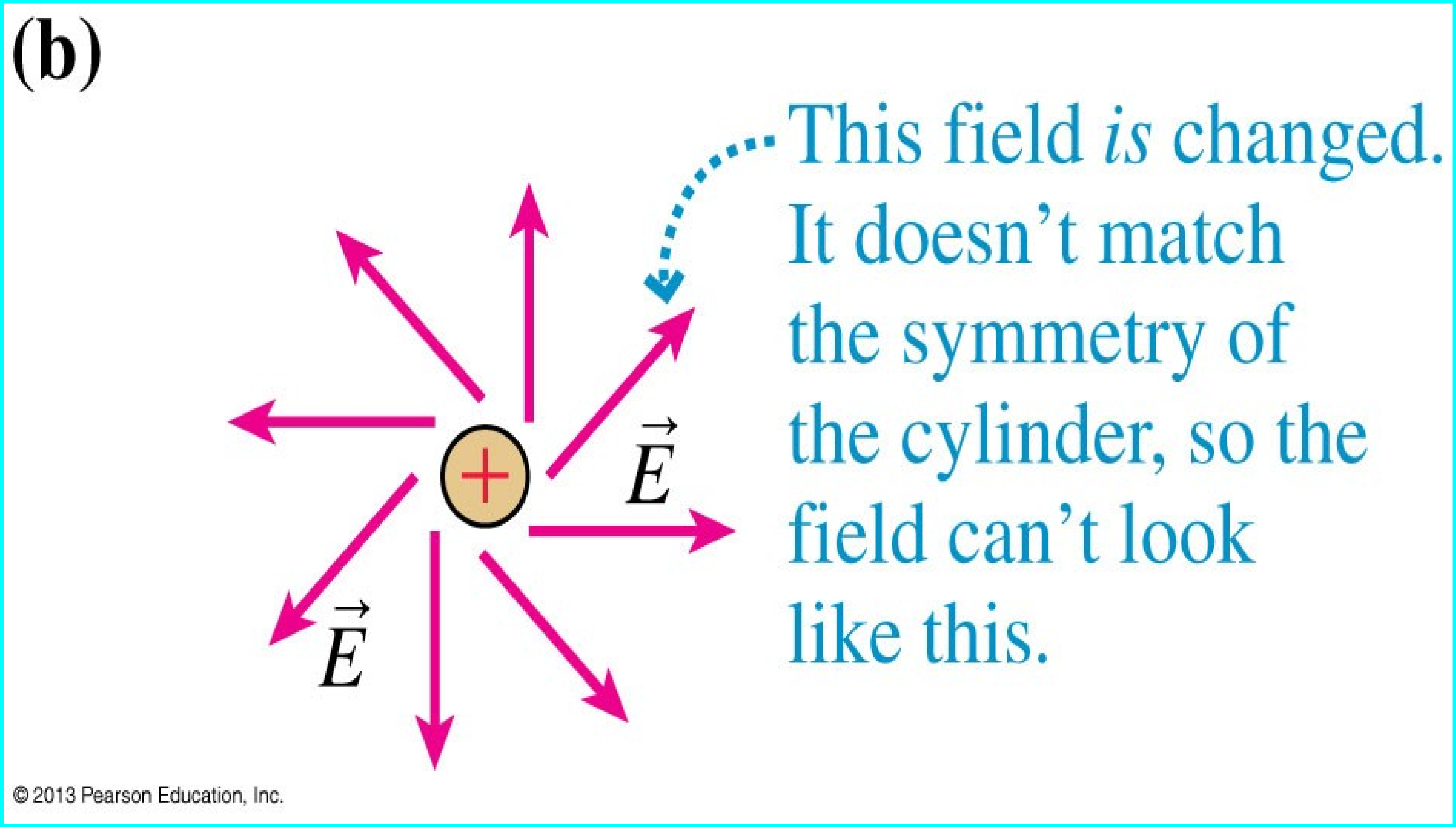
Relation between symmetry and Electric Field



Relation between symmetry and Electric Field

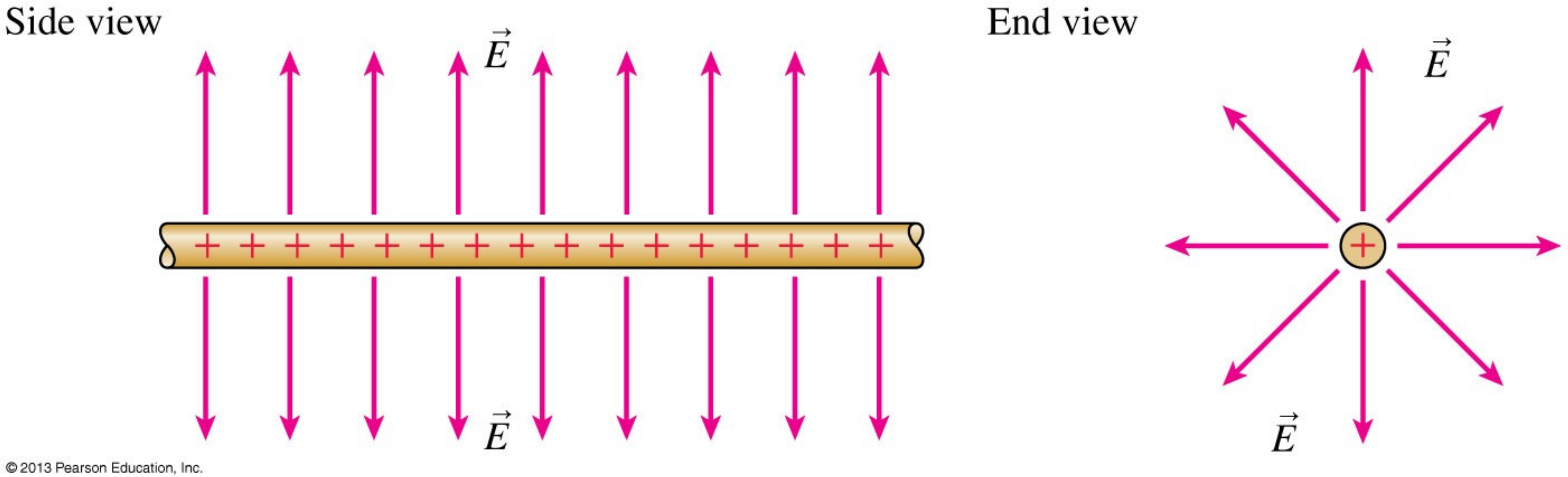


Relation between symmetry and Electric Field

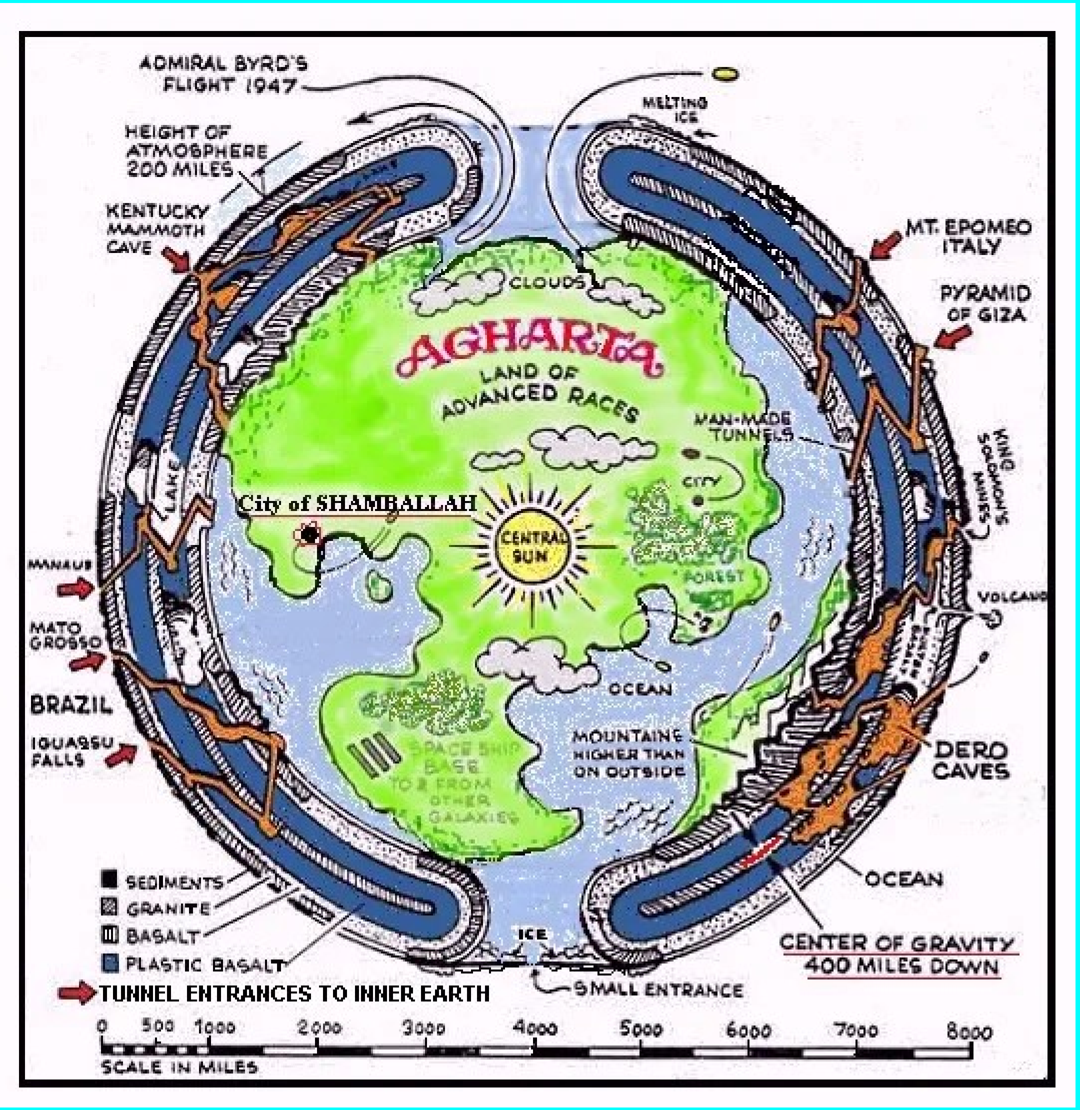


Relation between symmetry and Electric Field

The ONLY field consistent with symmetry of an infinitely long cylinder points radially outward.



What about a hollow sphere?



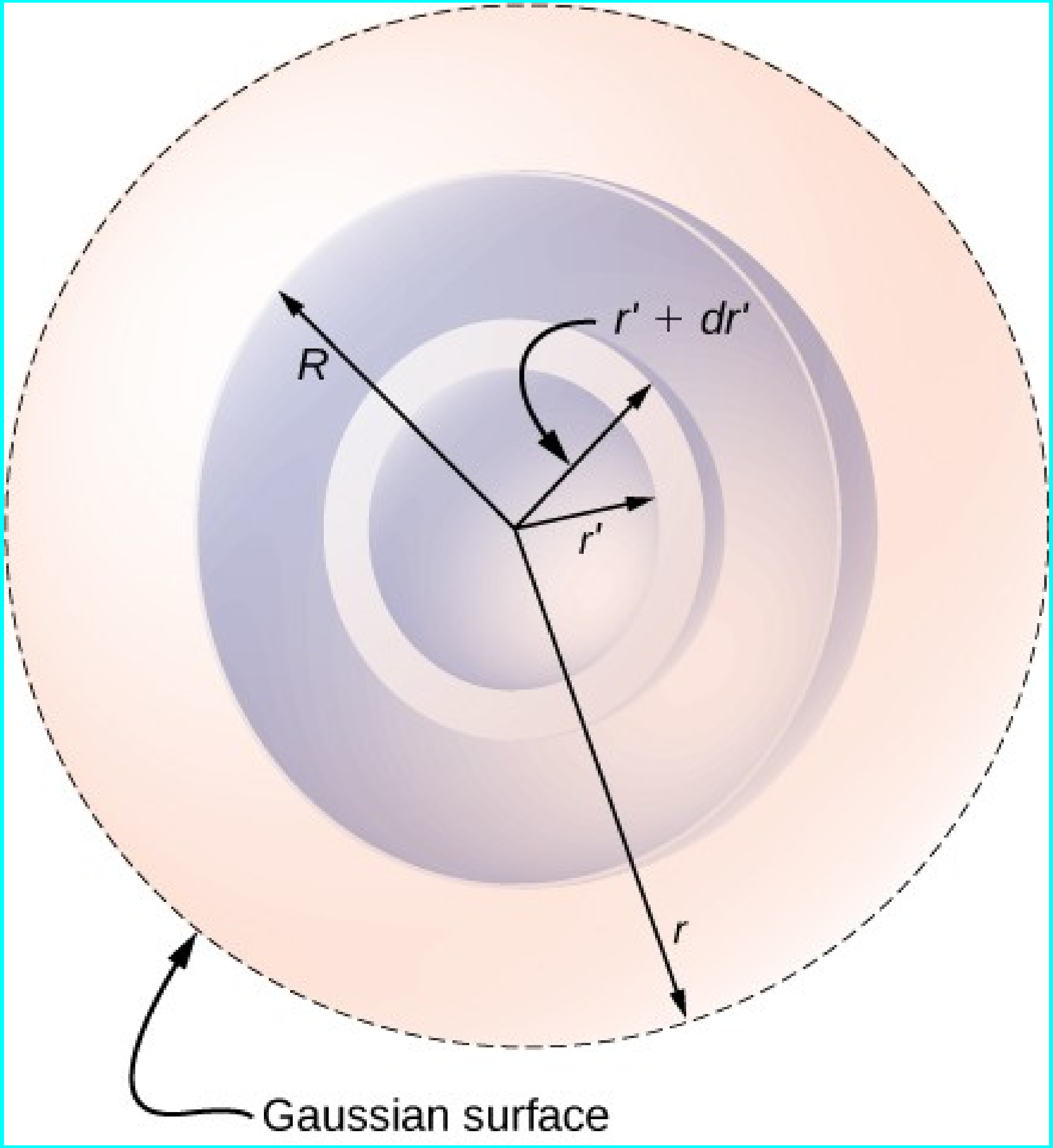
$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$E \times (\text{Surface Area}) = \frac{q_{enclosed}}{\epsilon_0}$$

What about a hollow sphere of charge?

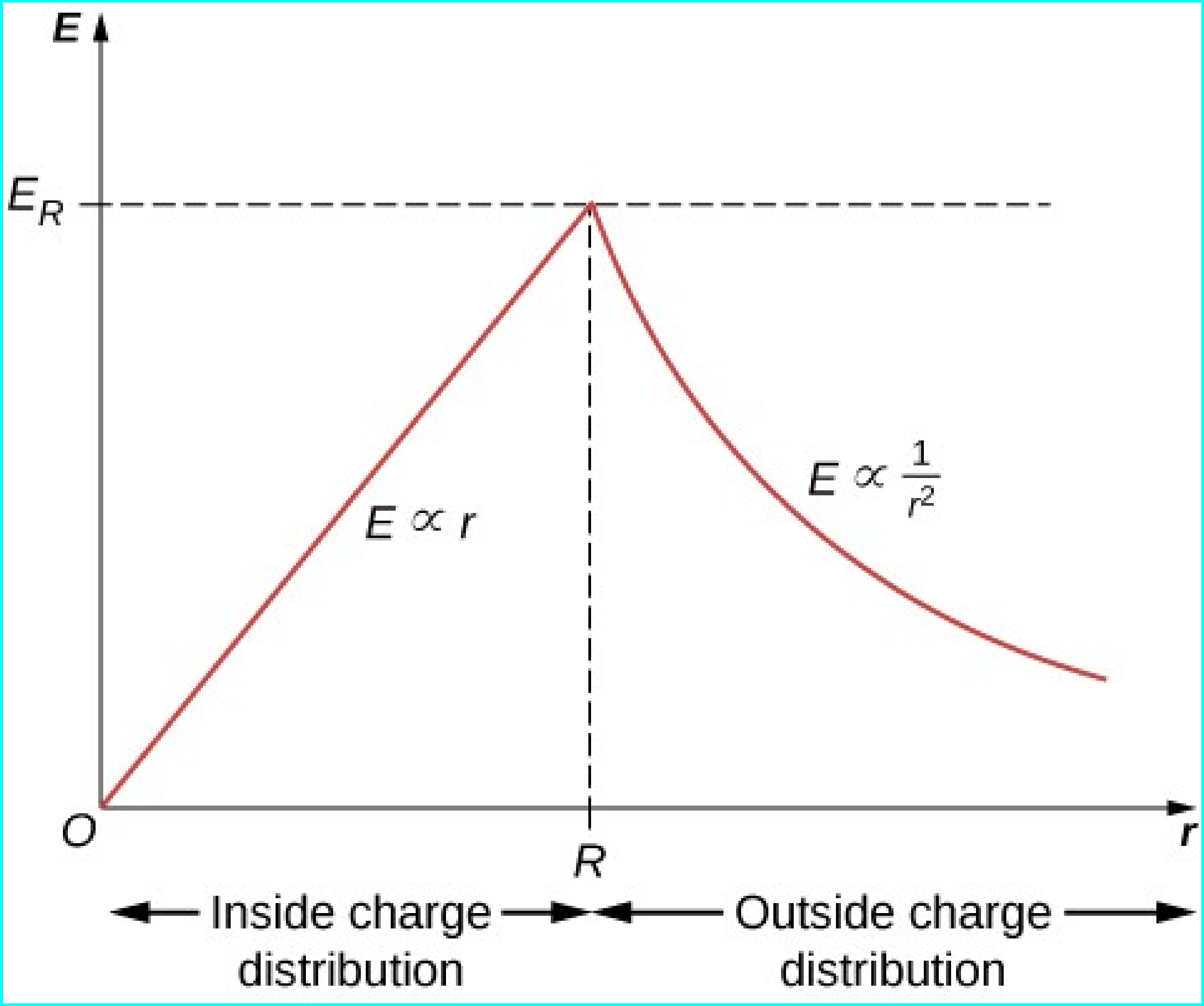
What about a hollow sphere?

What about a solid sphere of charge?

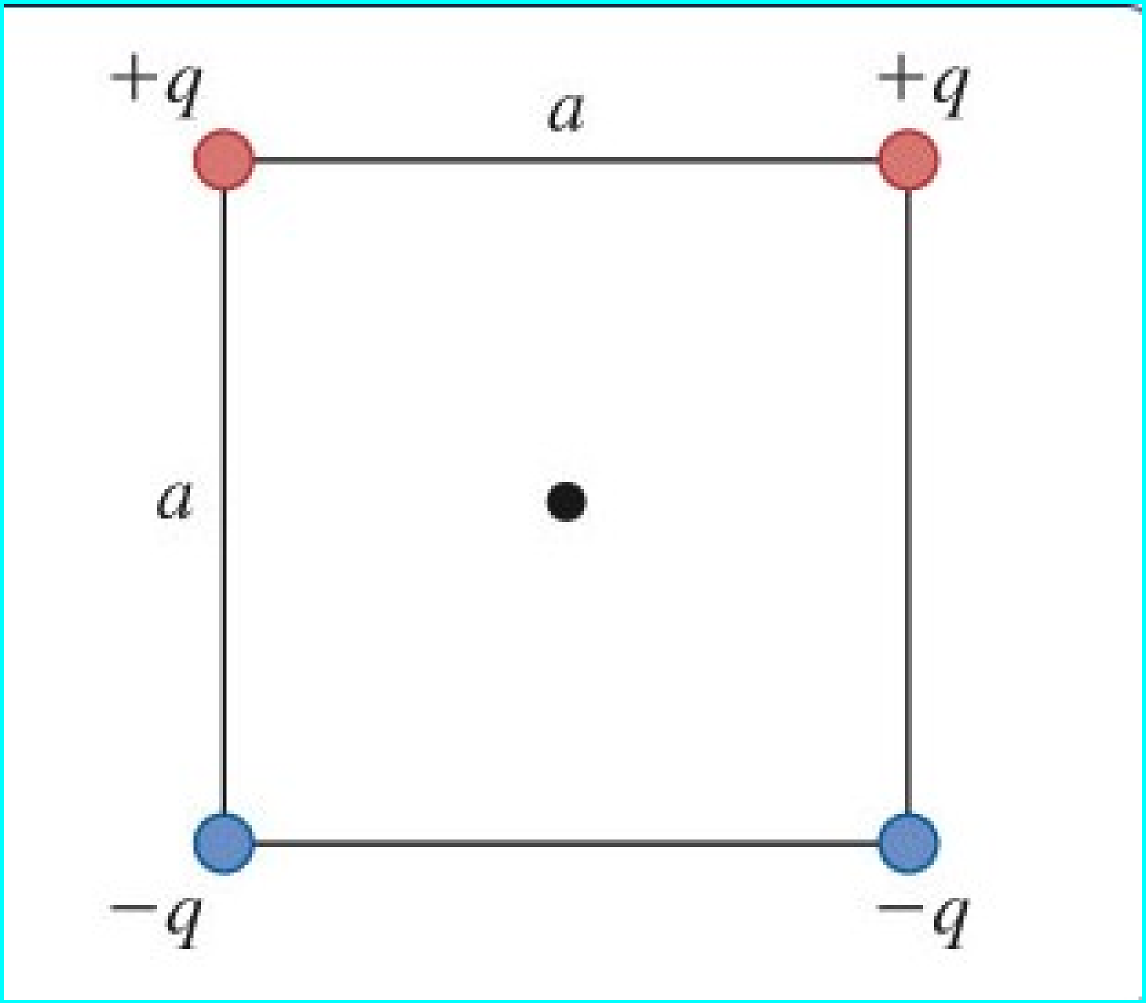


What about a solid sphere of charge?

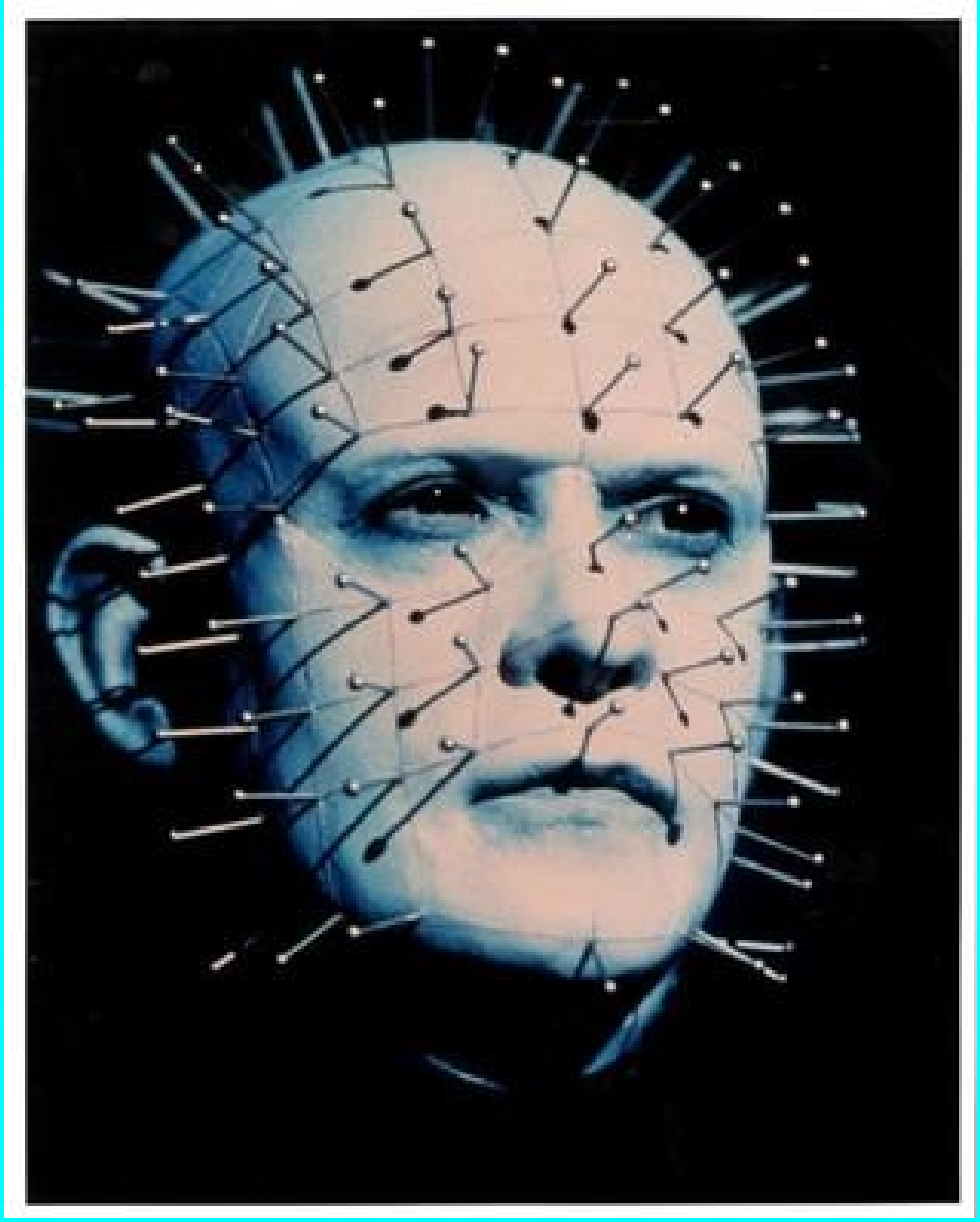
What about a solid sphere of charge?



Electric Field Superposition



Given four identical charges at corners of a square, find direction of field in the center of the square, and in the middle of each side.

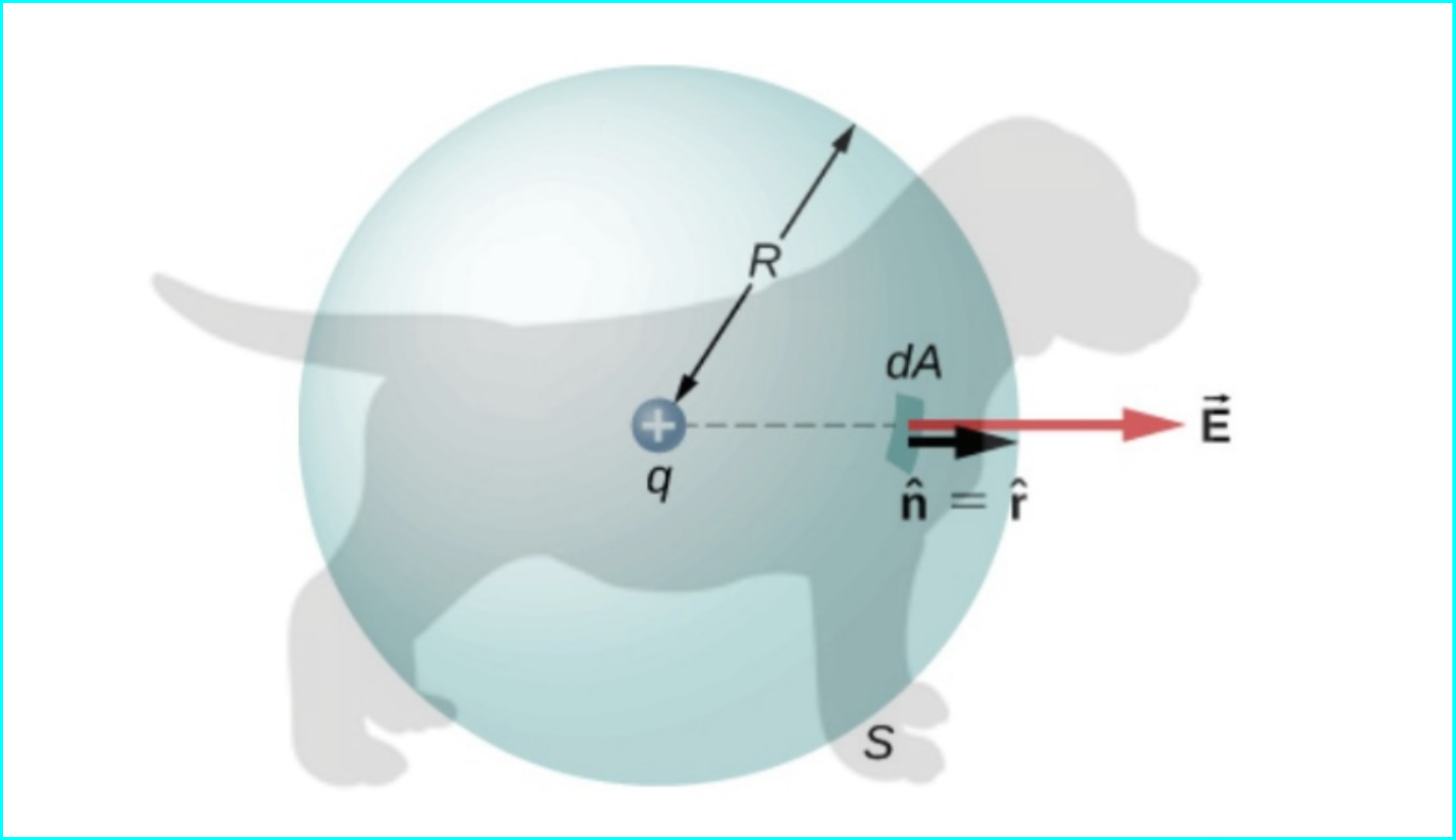


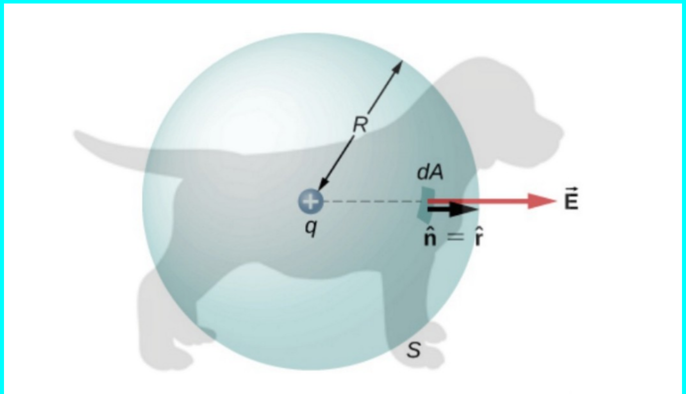
Gauss's law

"The total flux through any closed surface is equal to the enclosed charge over epsilon naught".

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

WTF?


R
dA
$\vec{E}$
q
$\hat{n} = \hat{r}$
S


R
dA
q
$\vec{E}$
$\hat{n} = \hat{r}$
S

Next Class:

Electric potential … What's a volt anyway?