

- Announcements
  - About the recitation problems
- Last Time
  - Electric field lines
  - Flux
  - Gauss's law
- Today
  - Gauss's law
  - Field of symmetrical charge configurations

## Key Equations

Coulomb's law

$$\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$$

Superposition of electric forces

$$\vec{\mathbf{F}}(r) = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^N \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Electric force due to an electric field

$$\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$$

Electric field at point  $P$

$$\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Field of an infinite wire

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} \hat{\mathbf{k}}$$

Field of an infinite plane

$$\vec{\mathbf{E}} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{k}}$$

Dipole moment

~~$$\vec{\mathbf{p}} = q\vec{\mathbf{d}}$$~~

## Key Equations

Definition of electric flux, for uniform electric field

$$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \rightarrow EA \cos \theta$$

Electric flux through an open surface

$$\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Electric flux through a closed surface

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Gauss's law

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \frac{q_{\text{enc}}}{\epsilon_0}$$

Gauss's Law for systems with symmetry

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = E \oint_S dA = EA = \frac{q_{\text{enc}}}{\epsilon_0}$$

The magnitude of the electric field just outside the surface of a conductor

$$E = \frac{\sigma}{\epsilon_0}$$

# Gauss's law

“The total flux through any closed surface is equal to the enclosed charge over epsilon naught”.

$$\Phi_{\text{total}} = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



icphysweb\_field\_line\_simulator

<https://icphysweb.z13.web.core.windows.net/simulation.html>

academo\_field\_line\_sim

<https://academo.org/demos/electric-field-line-simulator/>

electric\_field\_hockey

<https://phet.colorado.edu/sims/cheerpj/electric-hockey/latest/electric-hockey.html?simulation=electric-hockey>

# Gauss's law

“The total flux through any closed surface is equal to the enclosed charge over epsilon naught”.

$$\Phi_{\text{total}} = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



# Gauss's law for simple cases

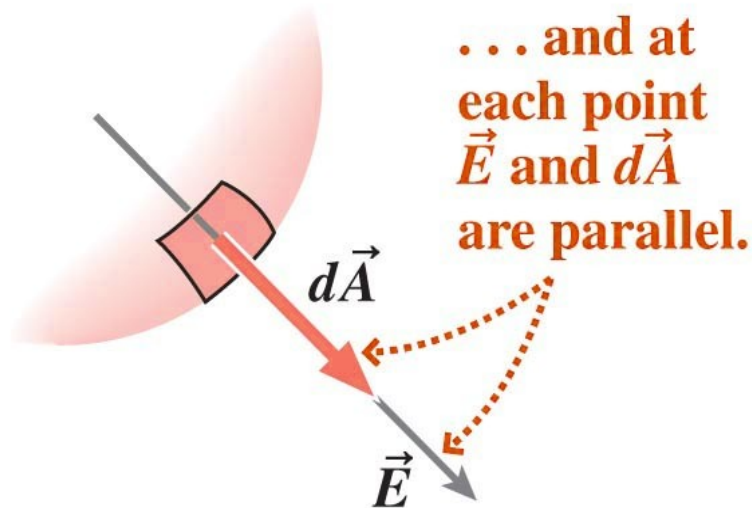
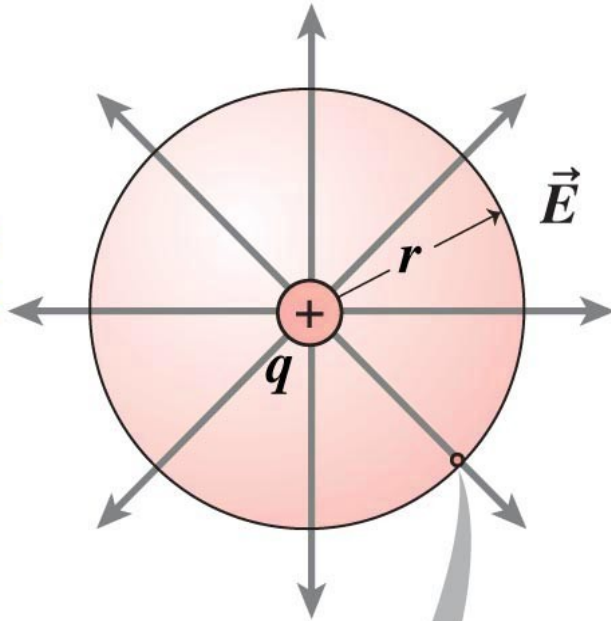
“The total flux through any closed surface is equal to the enclosed charge over epsilon naught”.

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



# Gauss's law is a generalization of Coulomb's law

The field's magnitude is the same over the sphere ...



$$\Phi_{\text{total}} = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



Gauss's law is a generalization of Coulomb's law

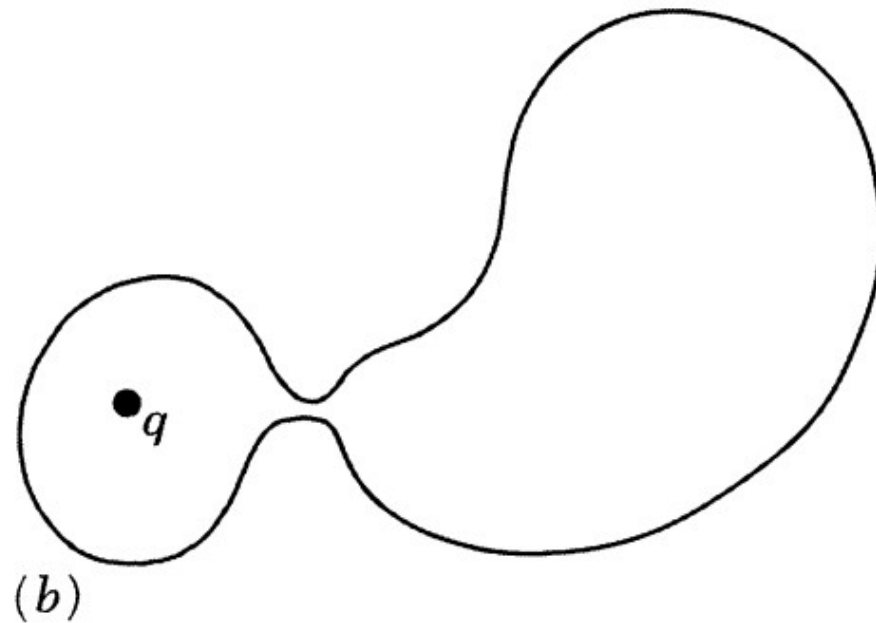
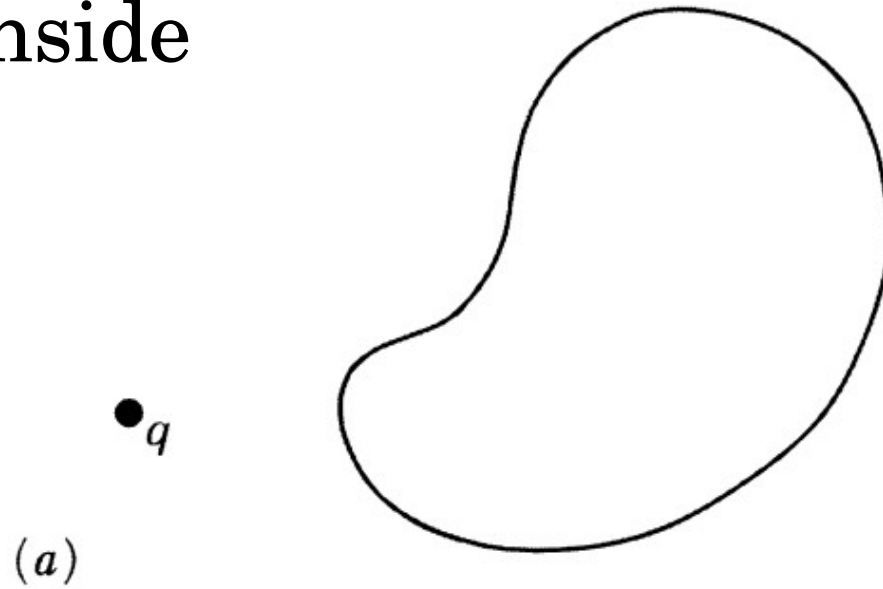
$$\Phi_{\text{total}} = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

# Gaussian logic trick

No charge inside

No net flux

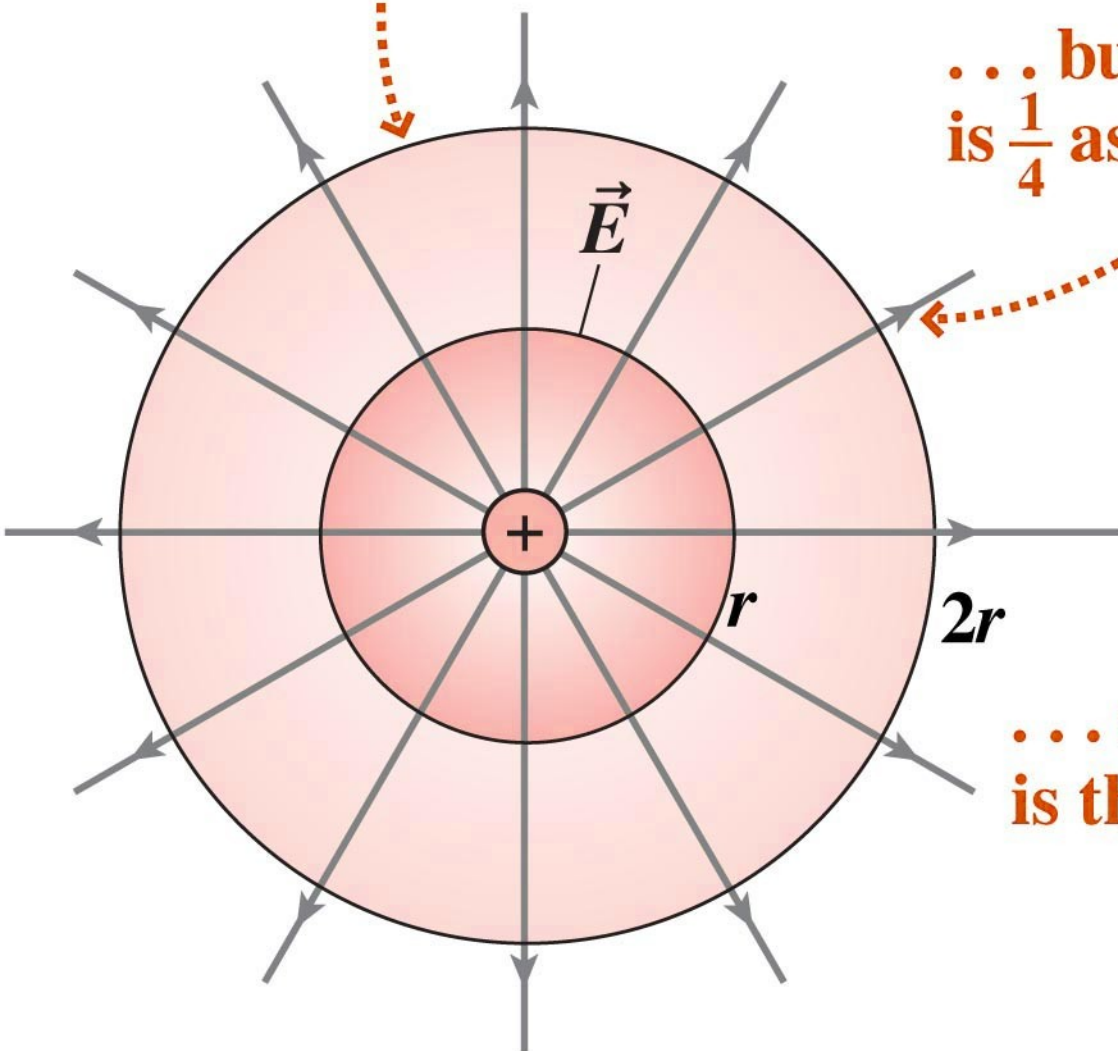


# Gauss's law is a generalization of Coulomb's law

The outer sphere has  
4 times the surface area ...

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

... but the field  
is  $\frac{1}{4}$  as strong ...



... so the flux  
is the same.

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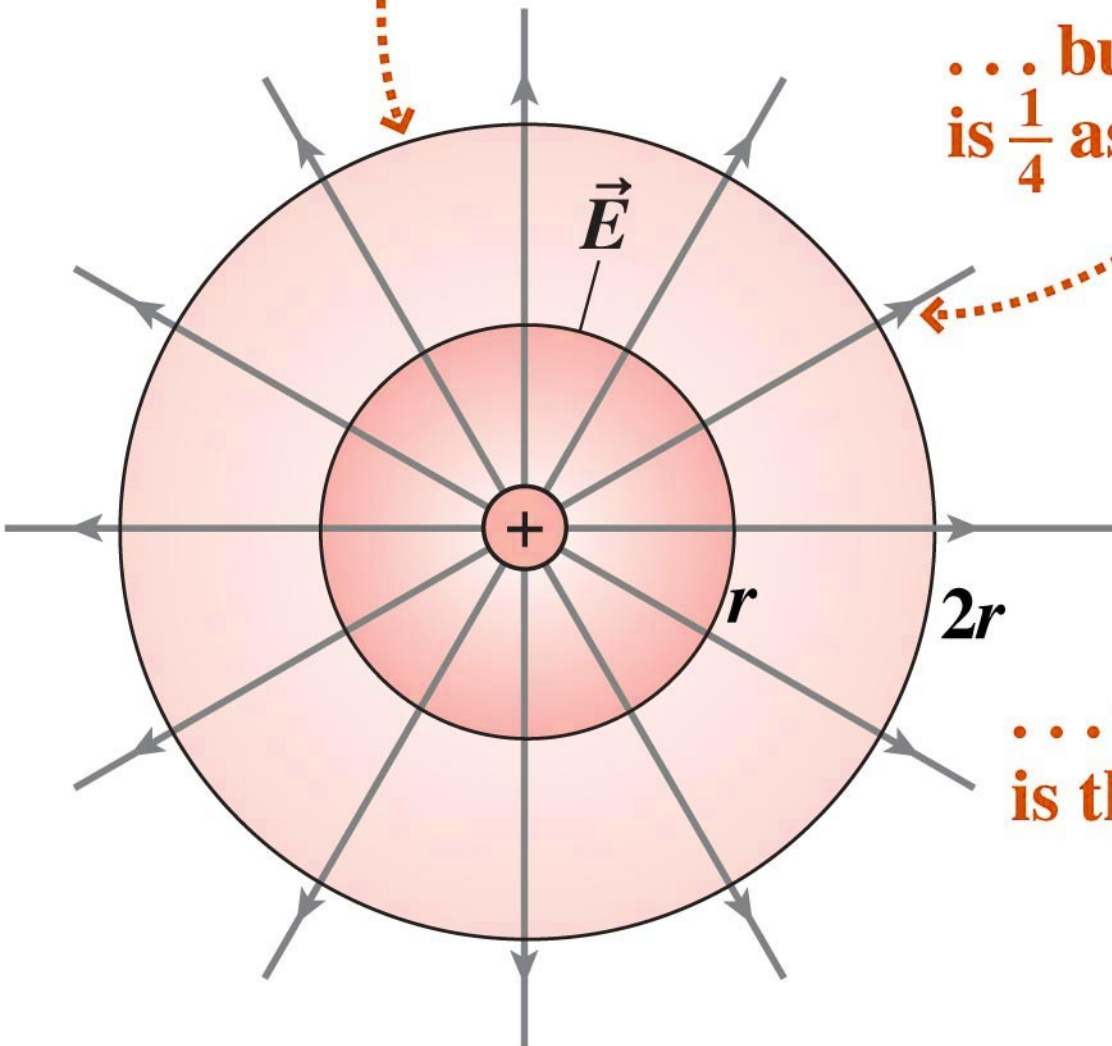
... but the field  
is  $\frac{1}{4}$  as strong ...

$$4\pi r^2 E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E = \frac{q_{\text{enclosed}}}{4\pi\epsilon_0} \frac{1}{r^2}$$

... so the flux  
is the same.

$$E = \frac{kq}{r^2}$$



# Simple Case I: Long (infinite) Wire

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



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$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



# Long Wire I

A wire is 10 meters long and you are 10 cm away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 20 cm away?

- (A) 4 N/C
- (B) 8 N/C
- (C) 12 N/C
- (D) 16 N/C
- (E) 32 N/C

## Long Wire II

A wire is 10 meters long and you are 100 m away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 200 m away?

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

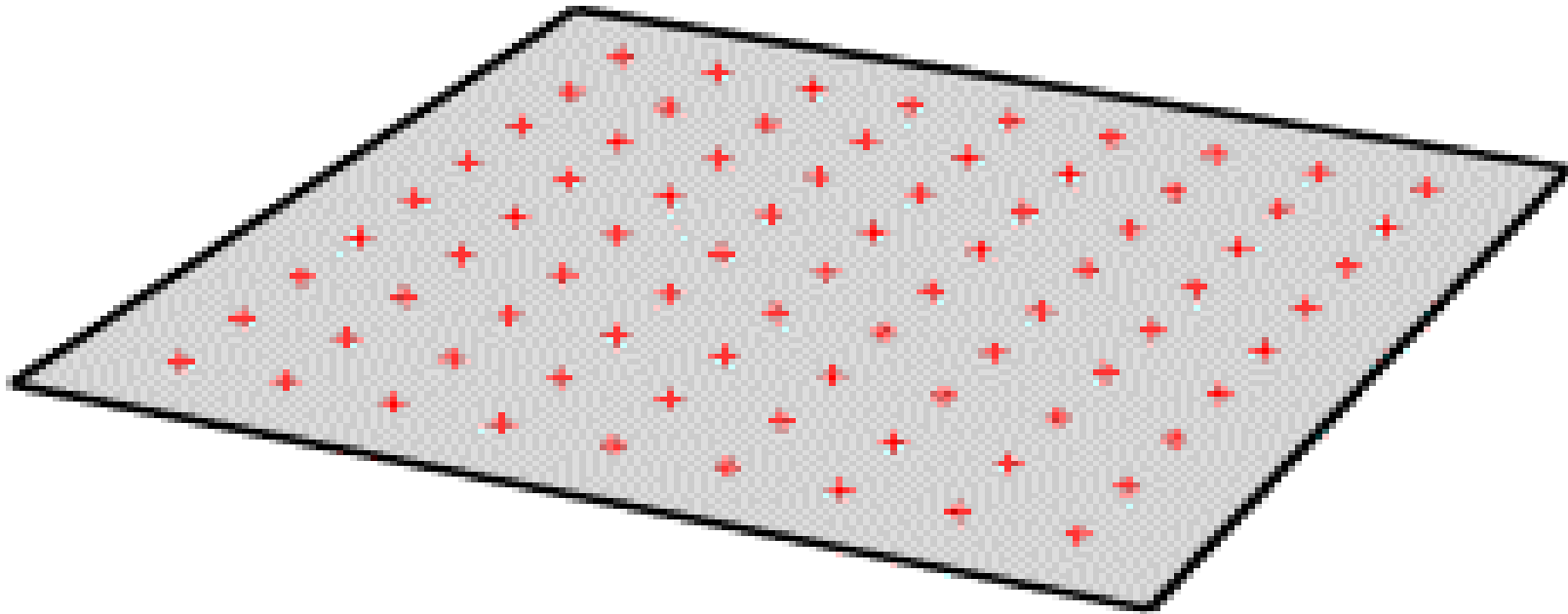
(E) 32 N/C

## Simple Case II: Large (infinite) Plane

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

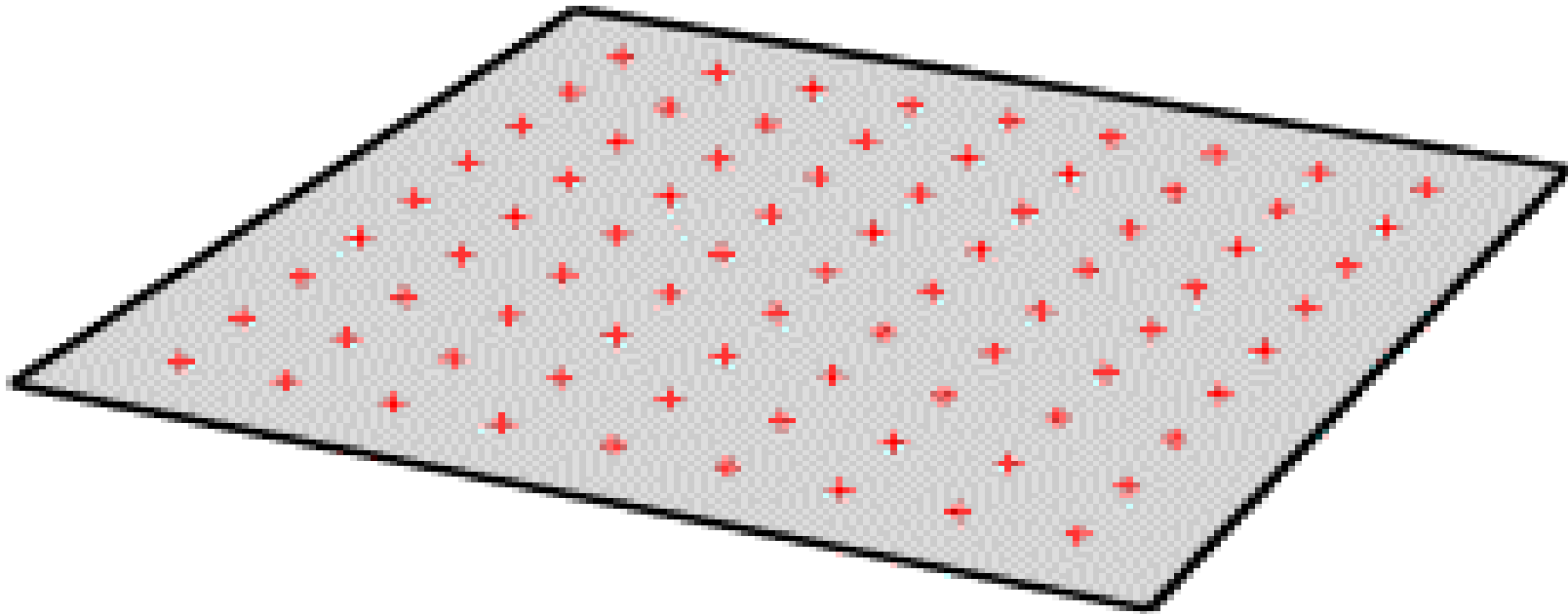
# Relation between symmetry and Electric Field

Imagine an infinite plane of charge.



# Relation between symmetry and Electric Field

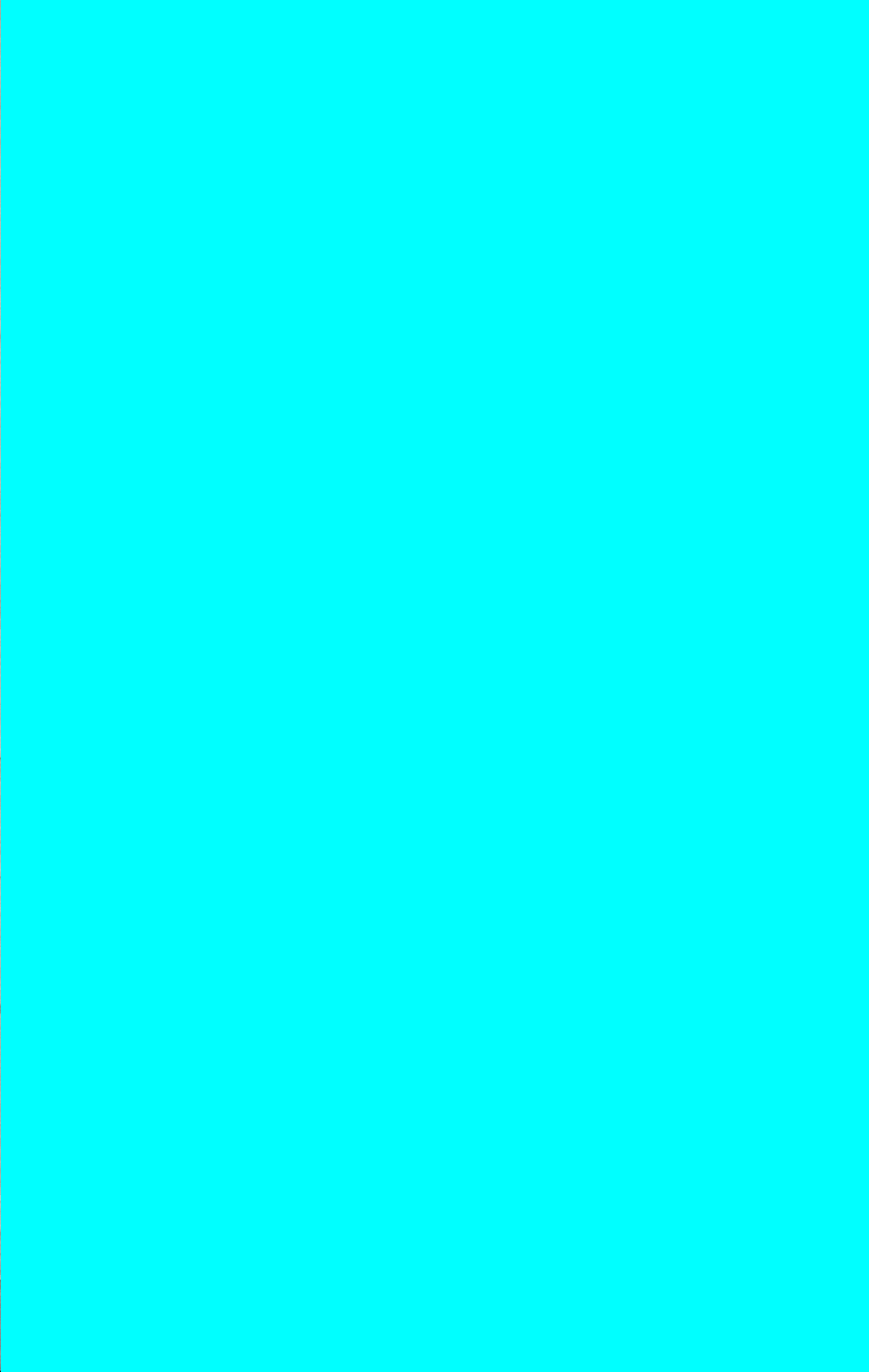
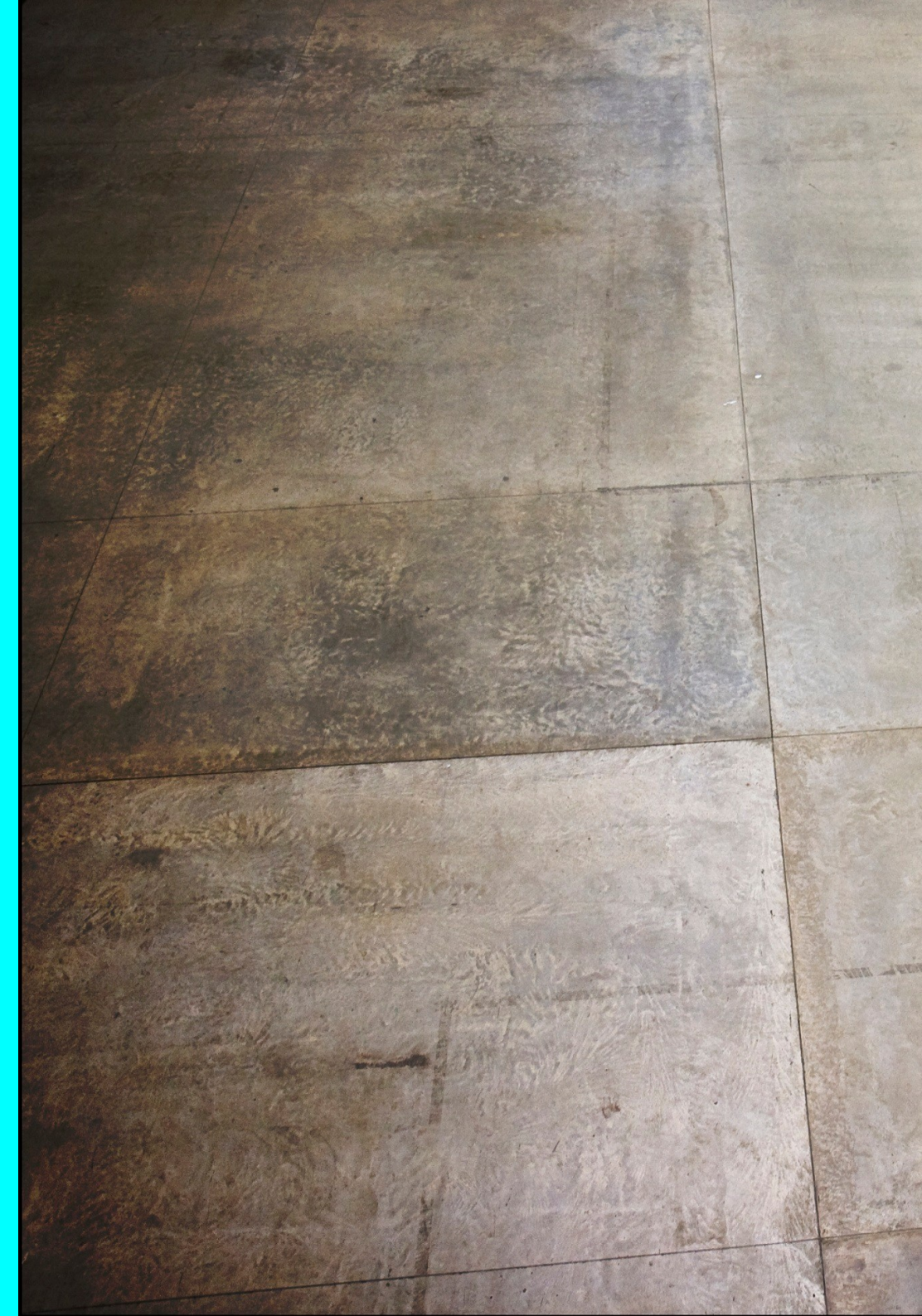
Because you can't tell what direction you are facing, the field must be **ONLY** Perpendicular to the plane.



**How large is this area?**

- [A] Floor tiles (4'x6')
- [B] Painting (12"x18")
- [C] Warehouse (60'x90')
- [D] Airfield (1000'x1500')
- [E] Not enough Info, can't tell



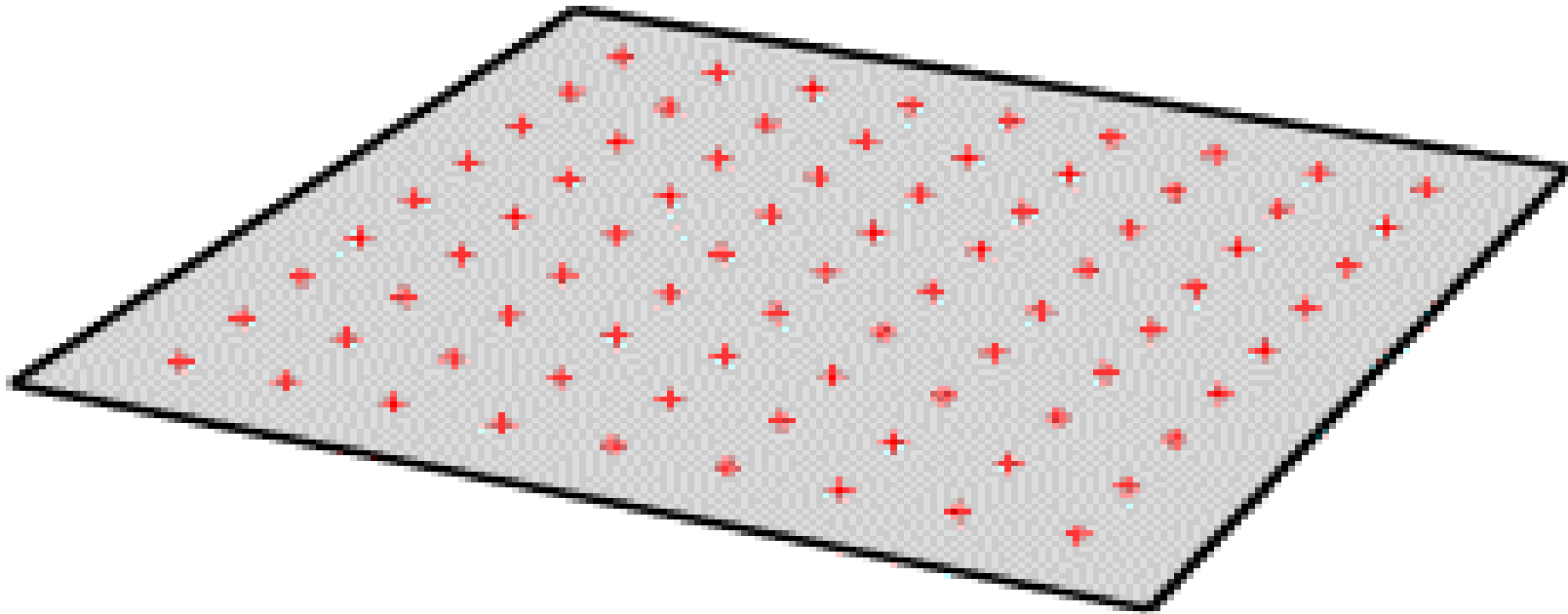






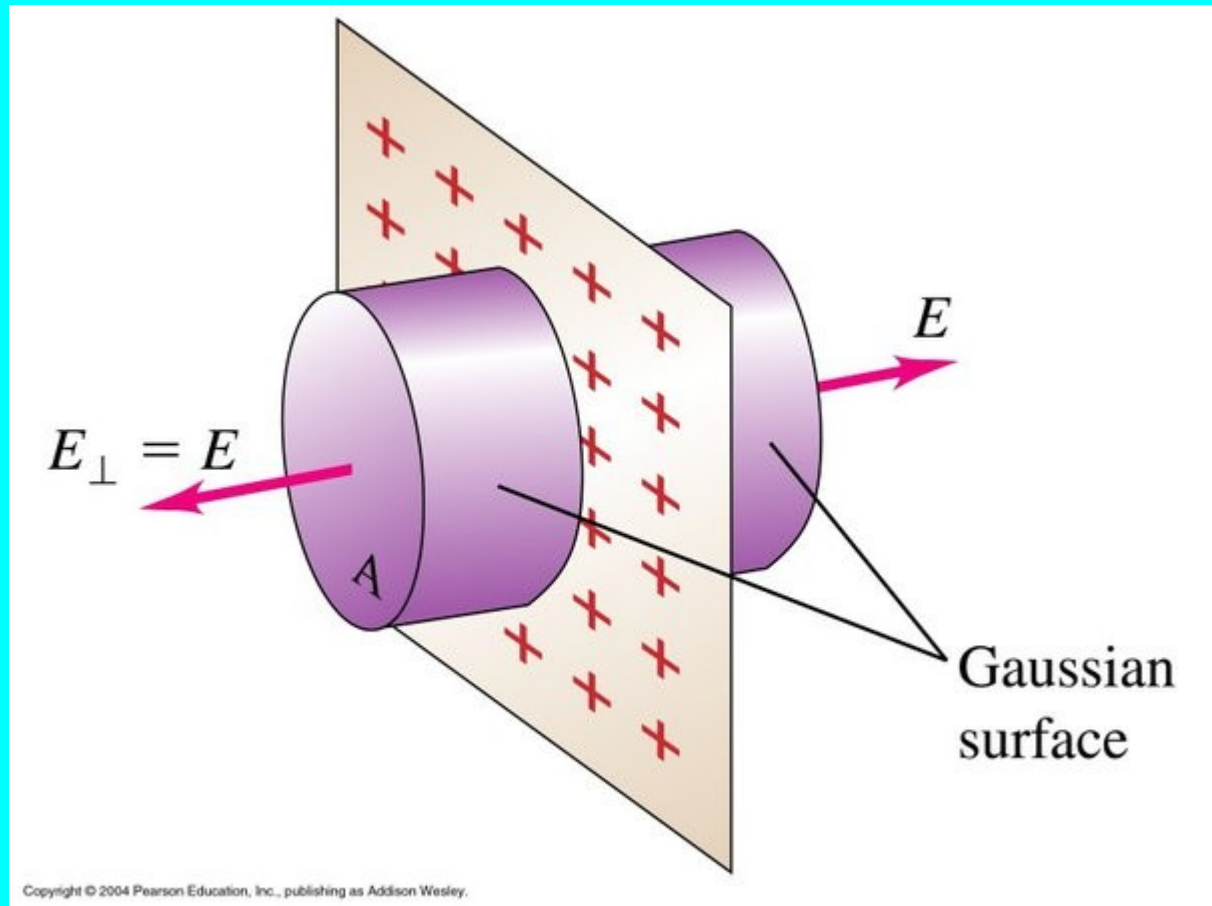
# Electric field of a plane of charge

Because you **ALSO** can't tell how far away you are from the plane, the field cannot change magnitude. It must be constant.



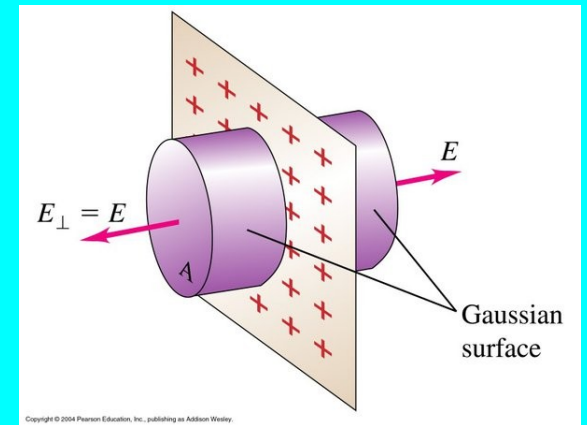
# Electric field of a plane of charge

$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



# Electric field of a plane of charge

$$E \times (\text{Surface Area}) = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$



# Infinite Plane I

A square plate is 10 meters on a side and you are 10 cm away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 20 cm away?

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

# Infinite Plane II

A square plate is 10 meters on a side and you are 100 m away from its middle. The electric field magnitude is 16 N/C. What is the approximate electric field if you move 200 m away?

(A) 4 N/C

(B) 8 N/C

(C) 12 N/C

(D) 16 N/C

(E) 32 N/C

# Infinite Plane III

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

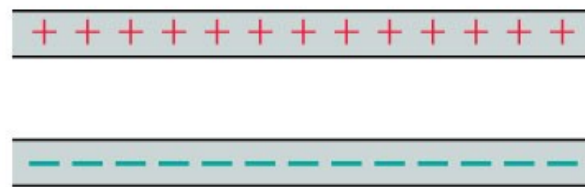
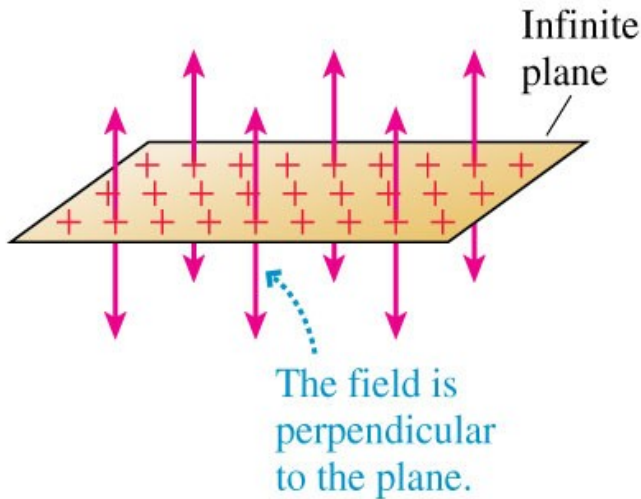
- (A)  $8.85 \times 10^{-5}$  N/C
- (B)  $4.43 \times 10^{-5}$  N/C
- (C)  $5.00 \times 10^6$  N/C
- (D)  $1.00 \times 10^7$  N/C
- (E)  $1.00 \times 10^8$  N/C

# Infinite Plane III

A square plate is 10 meters on a side and has a total charge of 8.85 mC. You are 1 cm away from its middle. What is the electric field magnitude?

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{k}$$

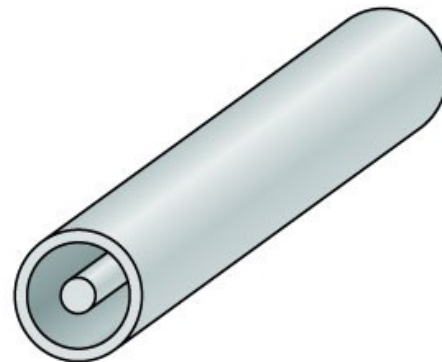
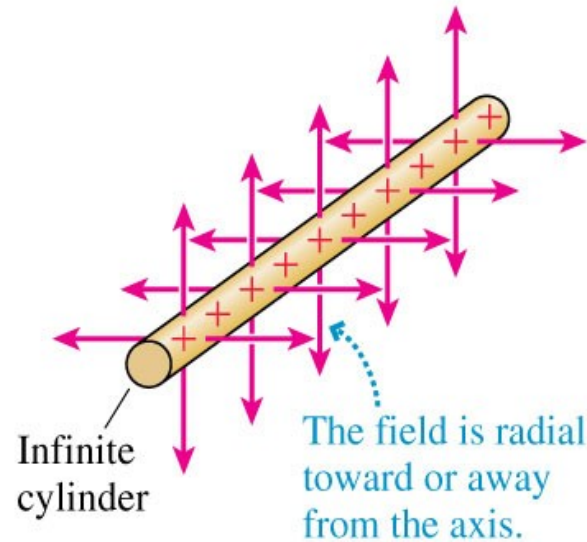
Planar symmetry



Infinite parallel-plate capacitor

$$\vec{E} = \frac{\lambda}{2\pi r \epsilon_0} \hat{r}$$

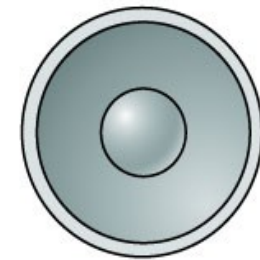
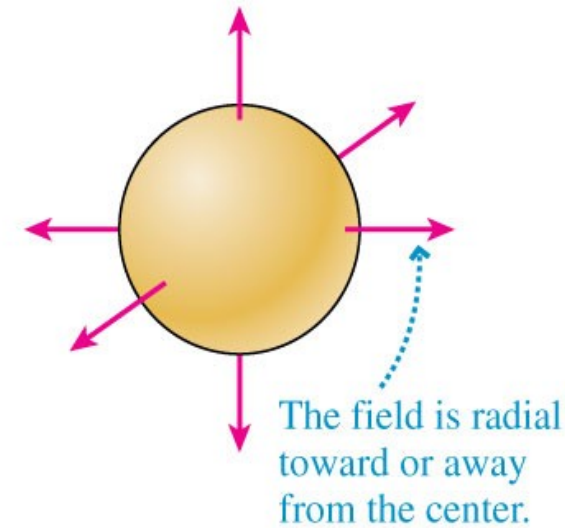
Cylindrical symmetry



Coaxial cylinders

$$\vec{E} = \frac{Q}{4\pi r^2 \epsilon_0} \hat{r}$$

Spherical symmetry



Concentric spheres



# Relation between symmetry and Electric Field

If you can't tell where you are with respect to a charge distribution

Then the electric field direction cannot give you a hint.

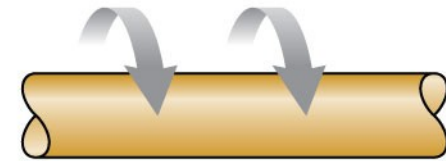
# Relation between symmetry and Electric Field



Original cylinder



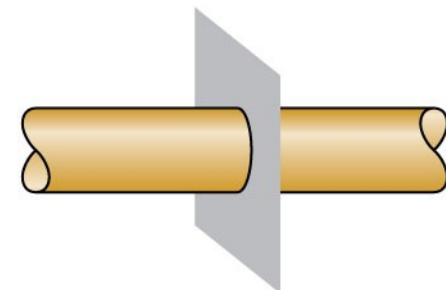
Translation parallel to the axis



Rotation about the axis



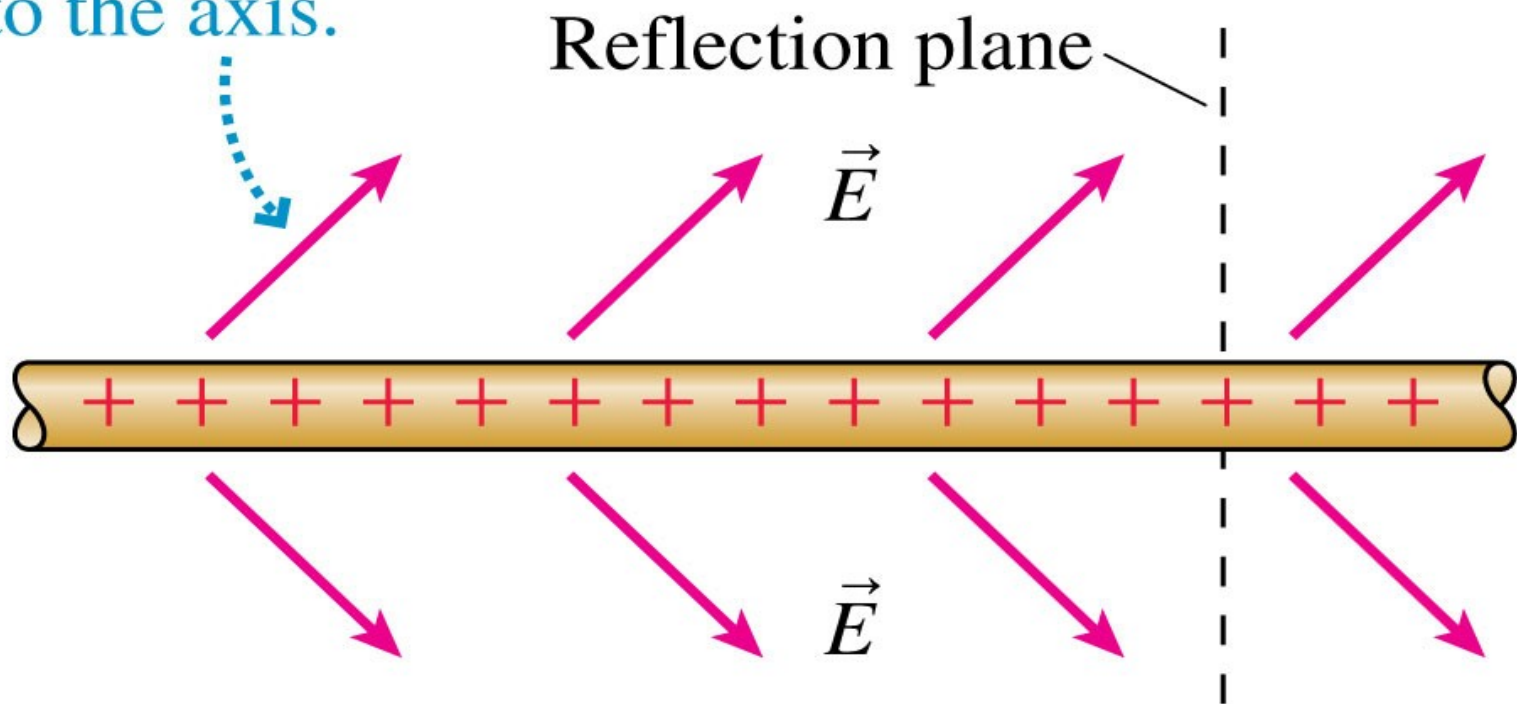
Reflection in plane containing the axis



Reflection perpendicular to the axis

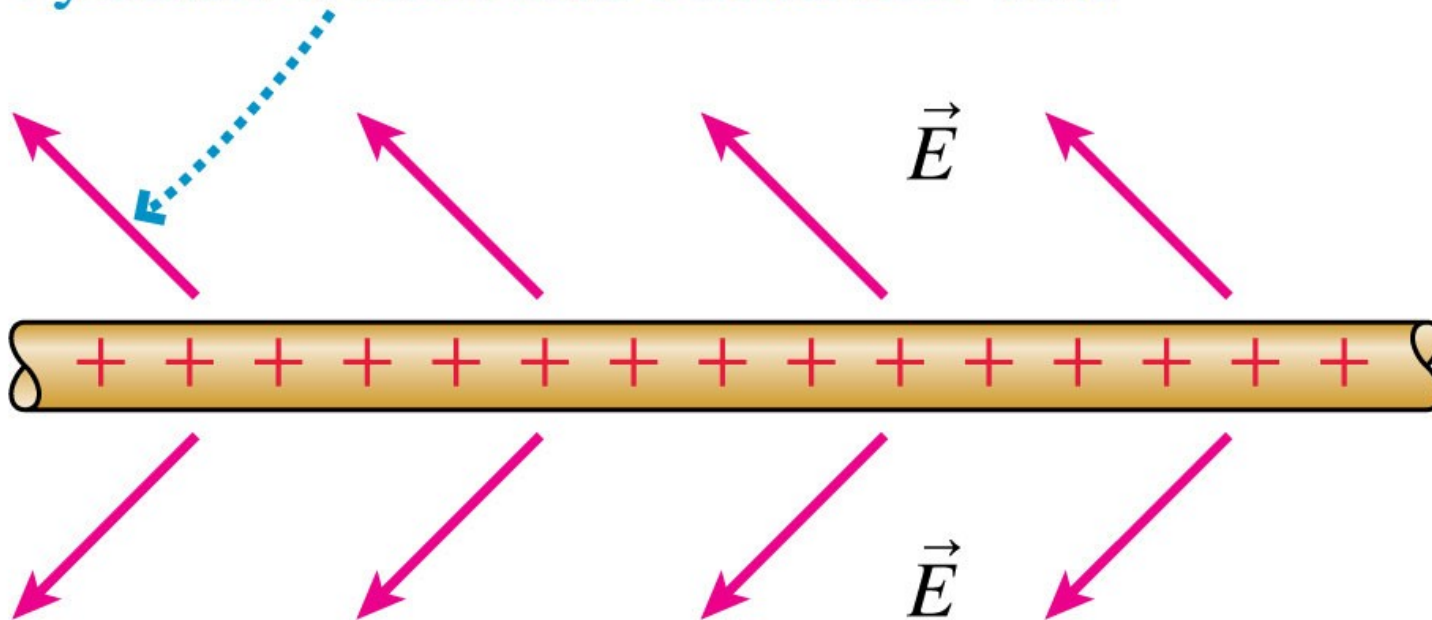
# Relation between symmetry and Electric Field

- (a) Is this a possible electric field of an infinitely long charged cylinder? Suppose the charge and the field are reflected in a plane perpendicular to the axis.



# Relation between symmetry and Electric Field

(b) The charge distribution is not changed by the reflection, but the field is. This field doesn't match the symmetry of the cylinder, so the cylinder's field can't look like this.

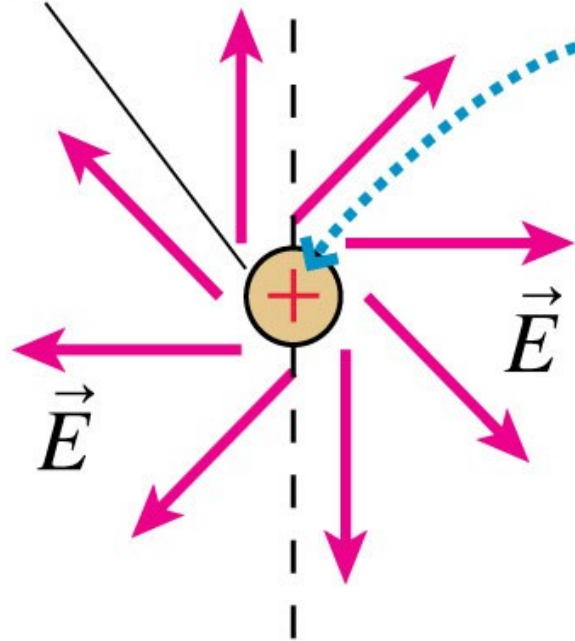


# Relation between symmetry and Electric Field

(a)

End view  
of cylinder

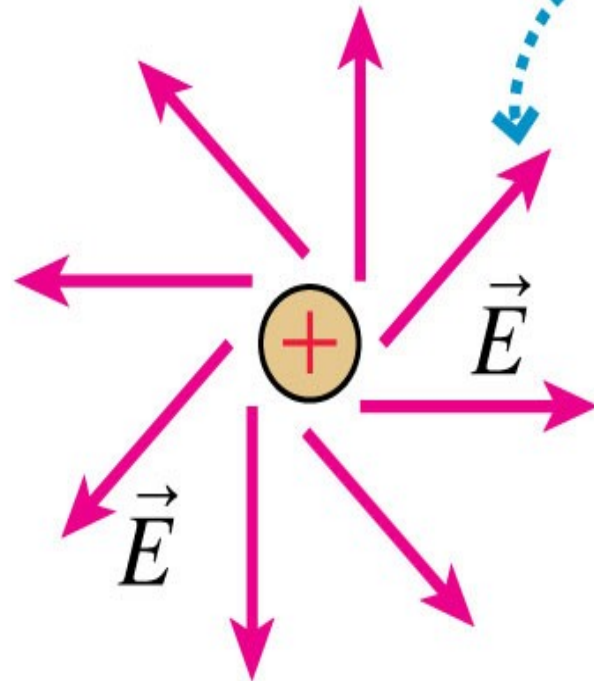
Reflection plane



The charge distribution is not changed by reflecting it in a plane containing the axis.

# Relation between symmetry and Electric Field

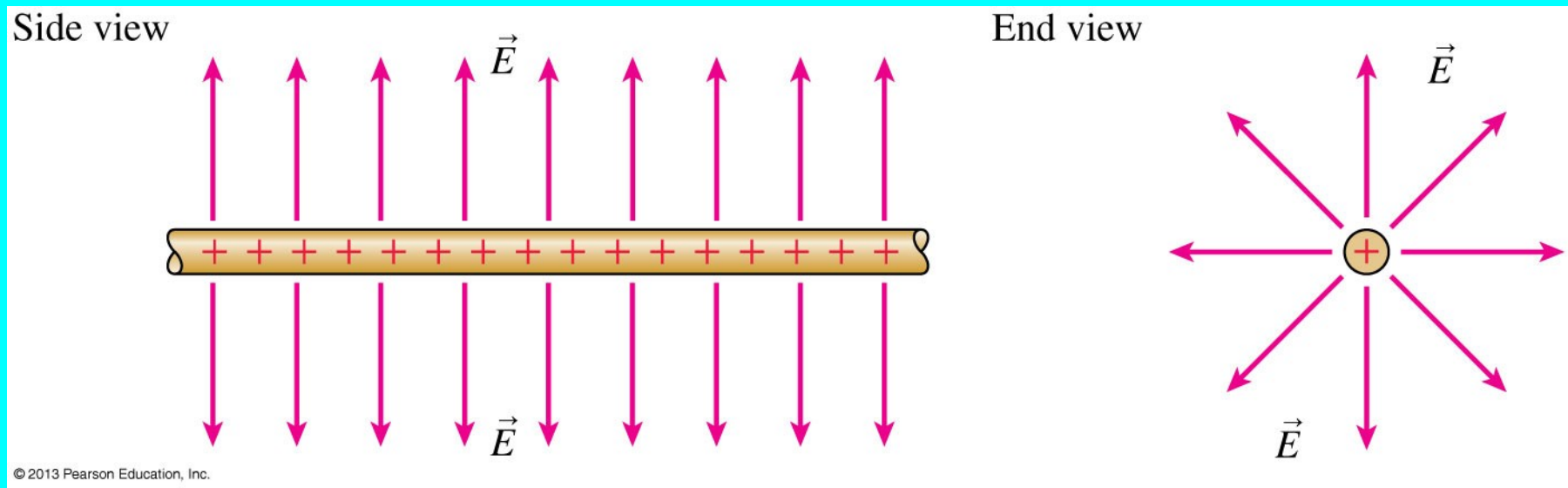
(b)



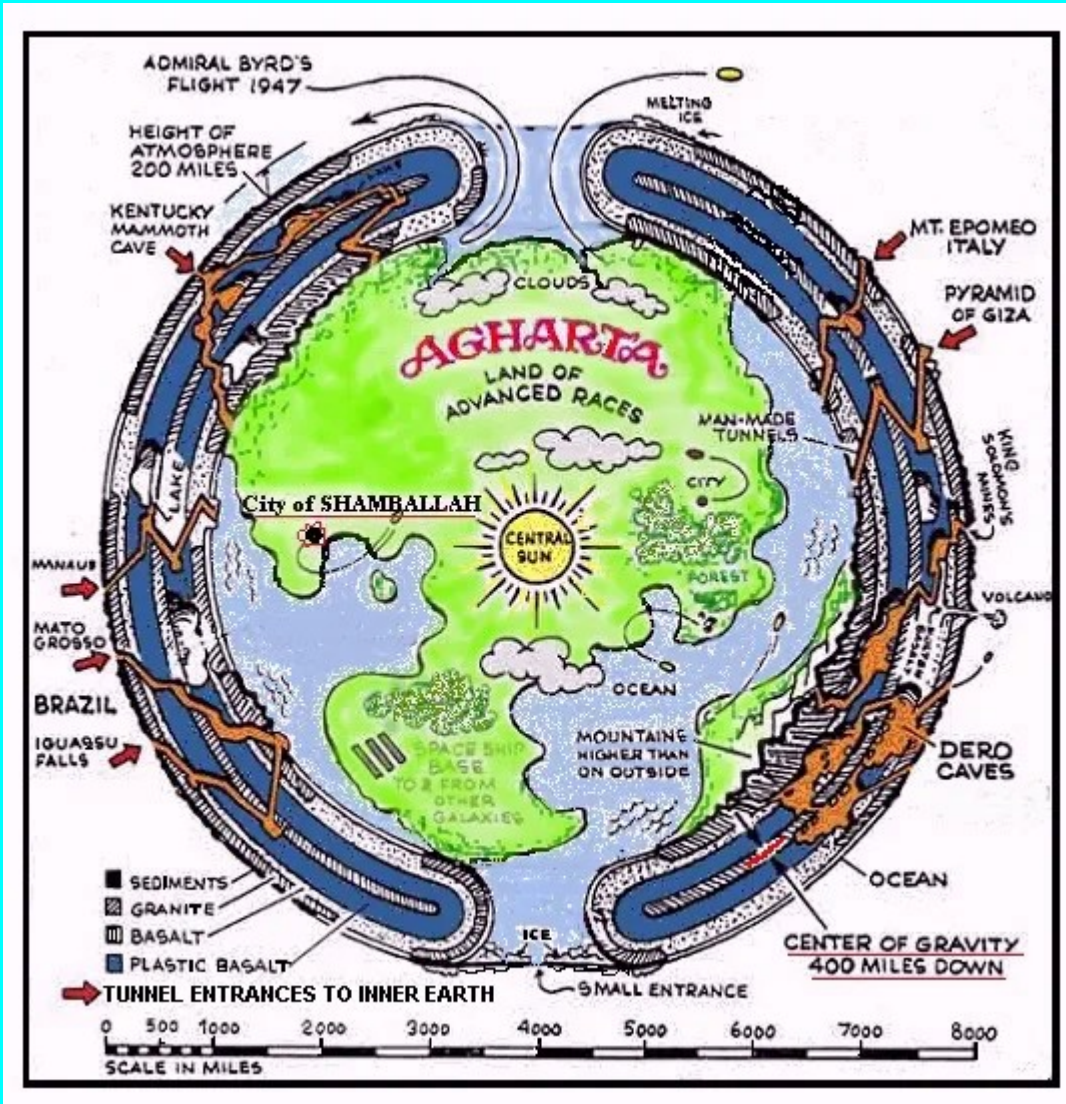
This field *is* changed.  
It doesn't match  
the symmetry of  
the cylinder, so the  
field can't look  
like this.

# Relation between symmetry and Electric Field

The **ONLY** field consistent with symmetry of an infinitely long cylinder points radially outward.



# What about a hollow sphere?



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

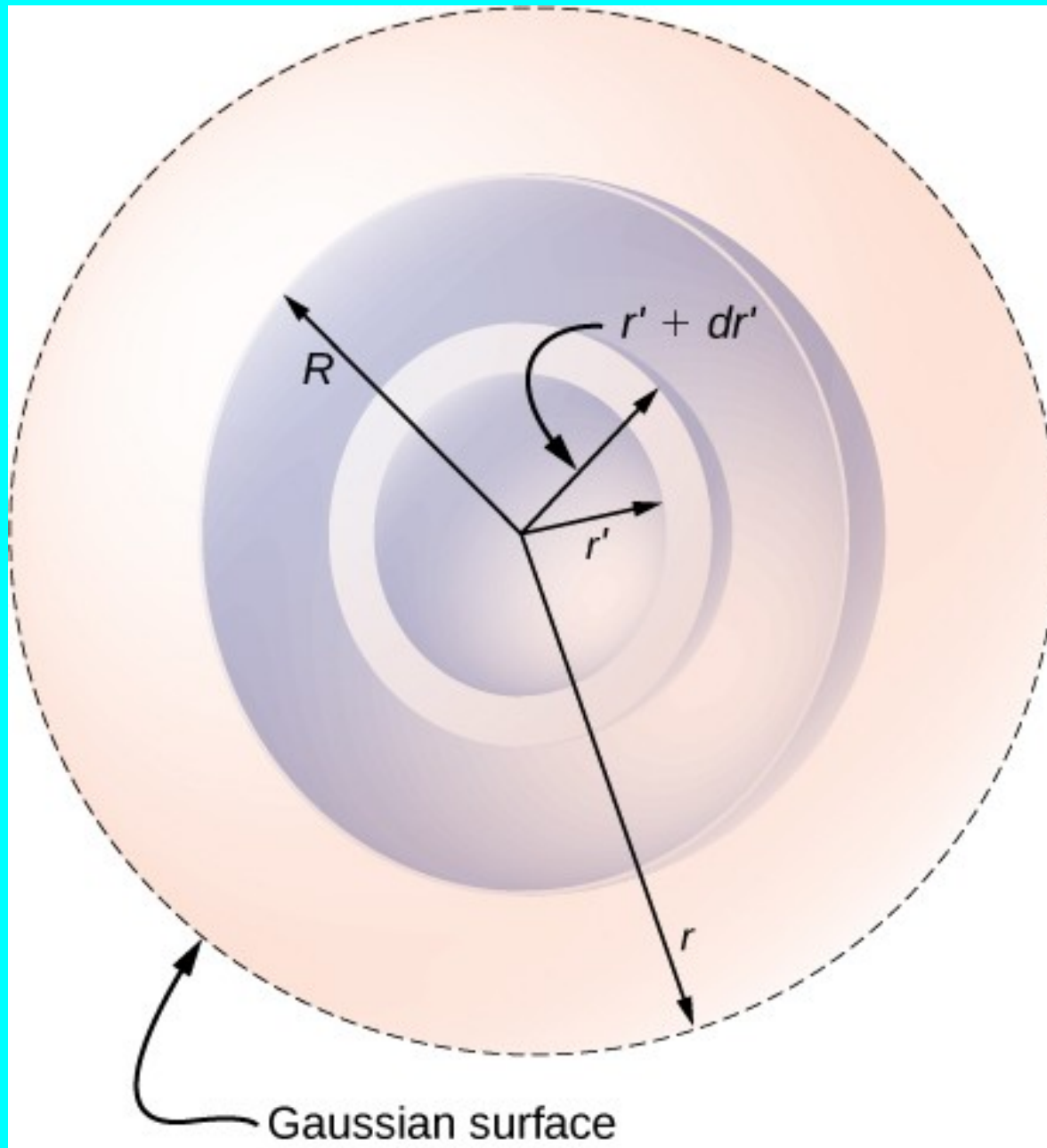
$$E \times (\text{Surface Area}) = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



What about a hollow sphere of charge?

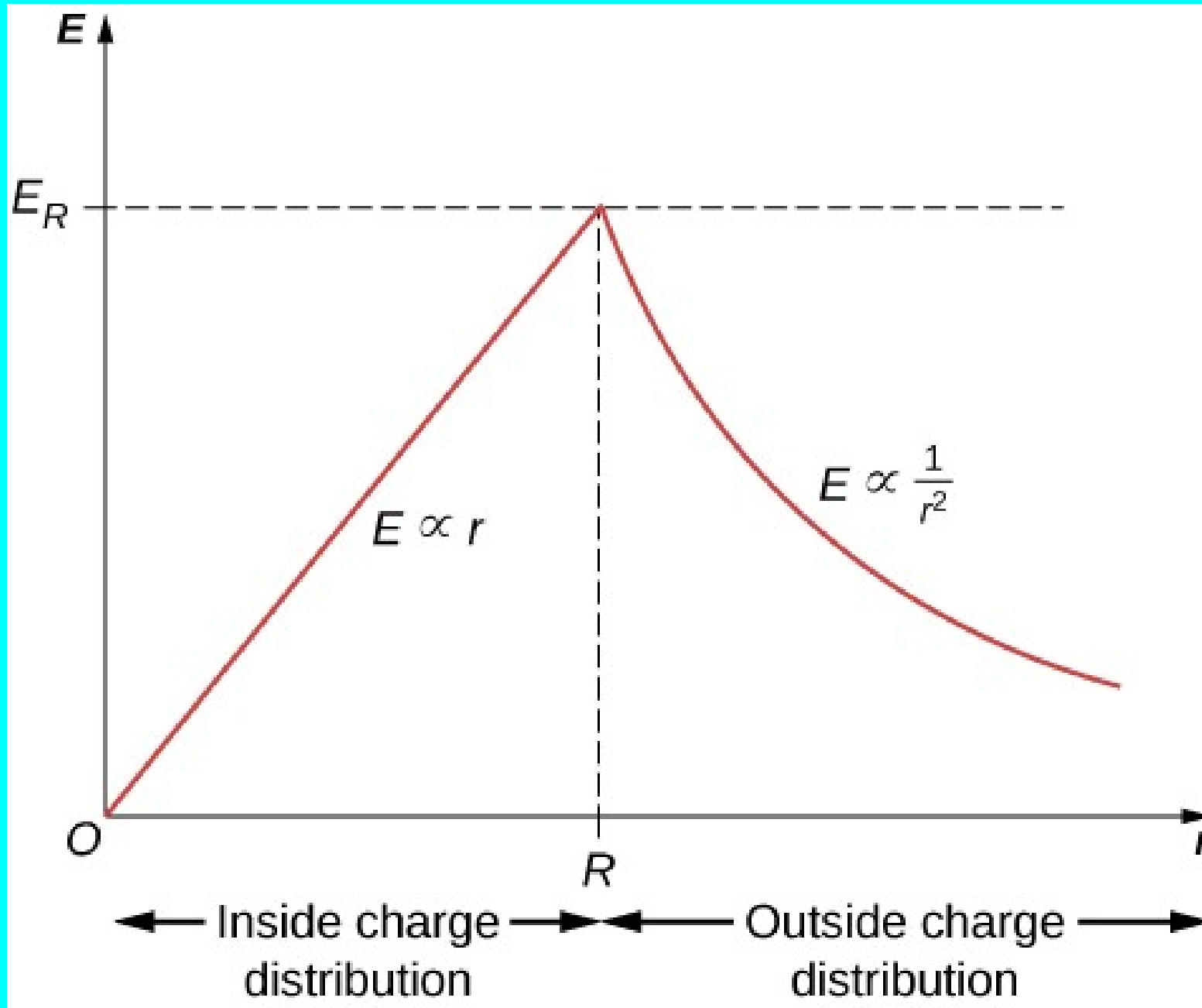
What about a hollow sphere?

# What about a solid sphere of charge?

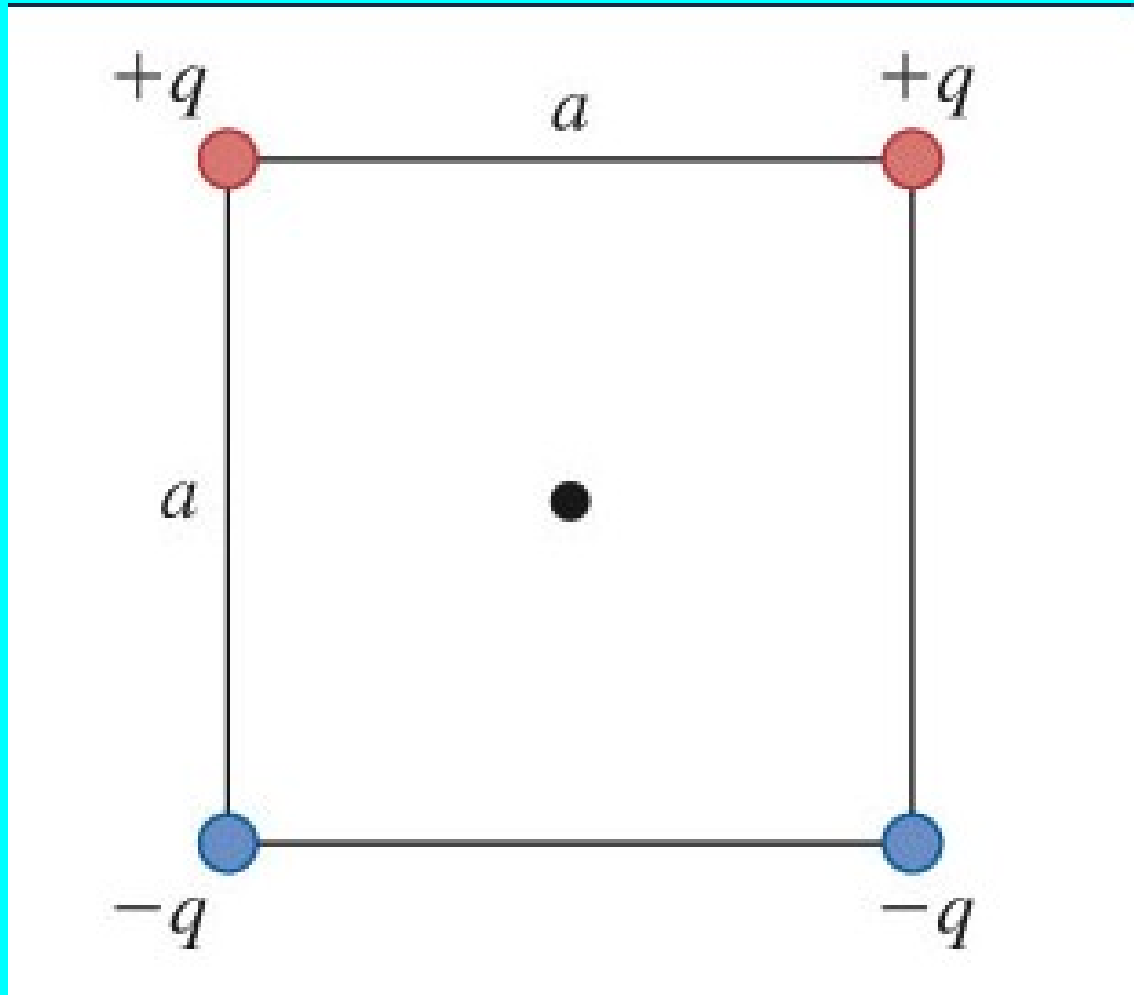


What about a solid sphere of charge?

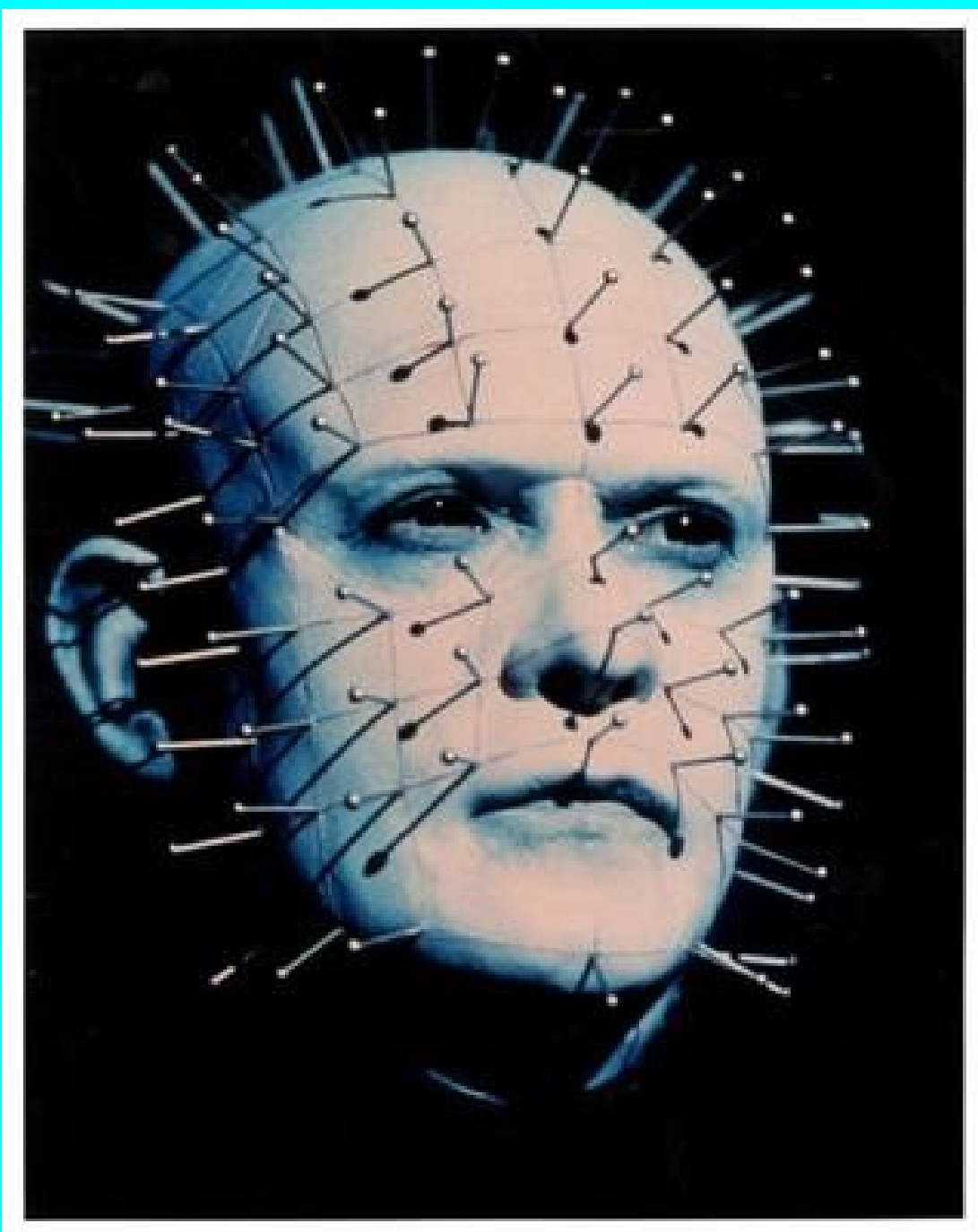
# What about a solid sphere of charge?



# Electric Field Superposition



Given four identical charges at corners of a square, find direction of field in the center of the square, and in the middle of each side.



# Gauss's law

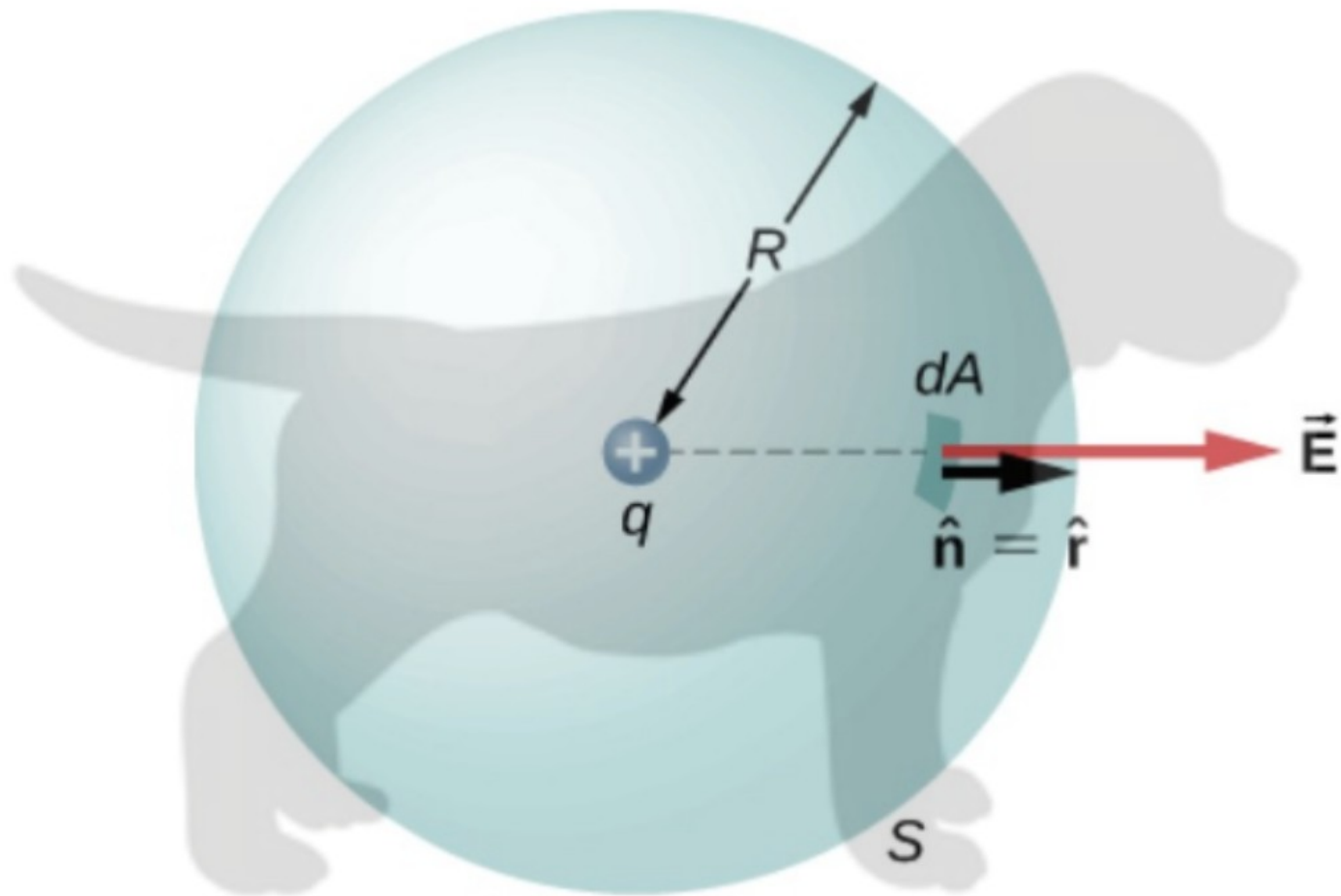
“The total flux through any closed surface is equal to the enclosed charge over epsilon naught”.

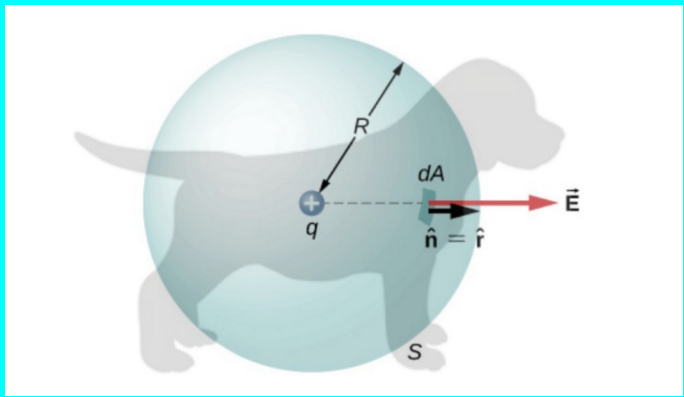
$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

**WTF?**













## **Next Class:**

Electric potential ... What's a volt anyway?