## Lecture 05:

## 01/30/2024

- Announcements

Written HW\#2 due midnight tonight
Written HW\#2 due Friday
Special zoom review session at 7 pm tonight

- Last Time
- Coulomb's Law
- Solving problems by adding vectors geometrically
- Introduction to r-hat
- Today
- Coulomb's Law

Coulomb vector form and r-hat
Electric field
Electric field lines

## SCHEDULE

| $\#$ | Dates | Reading | Topic | Lab. |
| :--- | :--- | :---: | :---: | :---: |
| 1 | Jan 16 | B1Ch16 | Intro, Waves $(v=f \lambda, v=\sqrt{T / \mu})$ | no lab |
| 2 | Jan 18 |  | Superposition, Standing Waves |  |
| 3 | Jan 23 | B2Ch5 | ( $q_{1} q_{2} / r^{2} \hat{r}$, conductors/insulators <br> 4 <br> 4 | Jan 25 |

## Coulomb's Law

$$
\begin{aligned}
& \mathrm{F}=\mathrm{k} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}} \\
& \mathrm{k}=8.99 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}
\end{aligned}
$$

## Online Problem 2-9

$q_{1}=q_{2}=q \quad q_{3}=-q$
Force on $\mathrm{q}_{2}$ ?


$$
L \cos \theta \hat{\imath}+L \sin \theta \hat{\jmath}
$$



## Online Problem 2-9

$$
q_{1}=q_{2}=q \quad q_{3}=-q
$$

Force on $\mathrm{q}_{2}$ ?


Problem 9: Three charged particles lie in the $x y$ plane at an angle of $\theta$ relative to the $x$-axis. Charge $q_{1}$ is located at the origin, $q_{2}$ is a distance $r$ from $q_{1}$, and $q_{3}$ is a distance $3 r$ from $q_{1}$. The charges each have magnitude of $q$, but $q_{1}=q_{2}=+q$, and $q_{3}=-q$. Charges $q_{1}$ and $q_{3}$ are fixed, and $q_{2}$ can move. However, $q_{1}$ and $q_{2}$ are connected by an ideal, neutral spring of spring constant $k_{\mathrm{s}}$. The spring is initially not stretched. Let Coulomb's constant be $k_{\mathrm{e} .} . q_{1}$ and $q_{2}$ are positive and $q_{3}$ is negative.

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Part (a) Choose the best expression for the net electrostatic force on $q_{2}$, in terms of the given variables.

## Expression :

$\mathbf{F}=$ $\qquad$
Select from the variables below to write your expression. Note that all variables may not be required.
$\cos (\alpha), \cos (\varphi), \cos (\theta), \sin (\alpha), \sin (\varphi), \sin (\theta), \gamma,(), i, j,, k_{e}, \mathbf{n}, \mathbf{q}, r$
Part (b) Because the force on $q_{2}$ is nonzero, it will begin to move from rest. In which direction will it move?
MultipleChoice :

1) It will move toward $q_{3}$.
2) It will not move.
3) There is not enough information.
4) It will move out of the $x y$ plane.
5) It will move toward $q_{I}$.
6) It will move along the $+x$ direction.
7) It will move along the $+y$ direction.

Part (c) When $q_{2}$ begins to move, it will stretch the spring. Choose the equation for the force vector from the spring, $\mathbf{F}_{\mathrm{s}}$, due to stretching the spring a dis

## Coulomb's Law, Vector Form

$$
\overrightarrow{F_{12}}=k \frac{q_{1} q_{2}}{r_{12}^{2}} \hat{r}_{12}
$$



## Making friends with "r-hat"

$r$-hat points from other charges to 'your' charge.

$\hat{r}$ is a unit vector like
$\hat{i}, \hat{j}$, and $\hat{k}$
$\hat{r}$ points in different directions
for different charges

## Homework 5-63-ish



Find force on the q on top right corner

$$
\overrightarrow{F_{\text {net }}}=Q \sum_{n=1}^{N} k \frac{q_{n}}{r_{n}^{2}} \hat{r}_{n}
$$



$$
\begin{aligned}
& \quad \overrightarrow{F_{\text {net }}}=Q \sum_{n=1}^{N} k \frac{q_{n}}{r_{n}^{2}} \hat{r}_{n} \\
& F_{\text {Net }}=Q k\left[\frac{q_{1}}{r_{1}^{2}} \hat{r}_{1}+\frac{q_{2}}{r_{2}^{2}} \hat{r_{2}}+\frac{q_{3}}{r_{3}^{2}} \hat{r}_{3}\right] \\
& F_{\text {net }}=Q \dot{q} k\left[\frac{\hat{r}_{1}}{r_{1}^{2}}+\frac{\hat{r}_{2}}{r_{2}^{2}}+\frac{\hat{r}_{3}}{r_{3}^{2}}\right]
\end{aligned}
$$

$$
r_{1}^{2}=8 a^{2}
$$

$$
F_{1}=q Q k \frac{\hat{r}_{1}}{r_{1}^{2}} \rightarrow \frac{q Q k}{2 a^{2}}(\cos \theta \hat{\imath}+\sin \theta \hat{\jmath})
$$



## $\overrightarrow{F_{\text {net }}}=Q \sum_{n=1}^{N} k \frac{q_{n}}{r_{n}^{2}} \hat{r}_{n}$

What is $r_{1}^{2}$ ?
(A) $\frac{\sqrt{2}}{2}$ a
(B) $\sqrt{2} \mathrm{a}$

(C) $\sqrt{2} a^{2}$
(D) 2 a
(E) $2 a^{2}$

## What is $\vec{r}_{1}$ ?

$(\partial \hat{\imath}+\hat{a} \hat{j}) \cdot\left(a \hat{l}+a^{\hat{j}}\right)$
$2^{2} \hat{\imath} \cdot \hat{\imath}+\lambda^{2} \hat{j} \cdot \hat{i}+2^{2} \hat{\imath} \hat{\jmath}+a^{2 \lambda} \hat{j} \cdot \hat{j}$
(A) a
(B) $a \hat{i}$

(C) $a \hat{i}+a \hat{j}$
(D) $\sqrt{2} a \hat{i}+\sqrt{2} a \hat{j}$
(E) $\frac{\sqrt{2}}{2} a \hat{i}+\frac{\sqrt{2}}{2} a \hat{j}$

$\left|r_{1}\right|=\sqrt{2 a^{2}}=\sqrt{2} a$

## What is $\hat{r_{1}}$ ?

$$
\vec{r}_{1}=a \hat{l}+a \hat{\jmath}
$$

(A) a

(B) $\hat{i}$
(C) $\hat{i}+\hat{j}$
(D) $\sqrt{2} \hat{i}+\sqrt{2} \hat{j}$
(E) $\frac{\sqrt{2}}{2} \hat{i}+\frac{\sqrt{2}}{2} \hat{j}$



## From Coulomb's Law to Electric field

$$
\begin{aligned}
\overrightarrow{F_{n e t}}=Q \sum_{n=1}^{N} k \frac{q_{n}}{r_{n}^{2}} \hat{r}_{n} & \vec{E}=\sum_{n=1}^{N} k \frac{q_{n}}{r_{n}^{2}} \hat{r}_{n} \\
\vec{E}=\frac{Q}{A} E_{0} & \overrightarrow{F_{n e t}}=Q \vec{E}
\end{aligned}
$$



## Homework 5-63-ish with field

Find $\overrightarrow{\mathrm{E}}$-field at top right corner in absence of Q

$$
\vec{E}=k \sum_{n} \frac{\hat{r}_{i_{n}} q_{n}}{r_{n}^{2}}
$$

$$
\begin{array}{lll}
0 & p & \vec{E}_{p}=k \sum_{n} \frac{q_{n} \hat{r}_{n}}{r_{n}^{2}} \\
& \vec{E}=k q\left(\frac{\hat{\imath}}{x^{2}}+\frac{\hat{\jmath}}{a^{2}}+\frac{\sqrt{2}}{2} \frac{\hat{\imath}}{2 a^{2}}+\frac{\sqrt{2}}{2} \frac{\hat{\jmath}}{2 a^{2}}\right) \\
0 & 0 & 0
\end{array}
$$

## Superposition (Force)

The net force on -Q is

$$
+q \quad-q
$$

A. Up.
B. Down.
C. Left.
D. Right.
E. The force on $-q$ is zero.

## Superposition (Field)

The net field at the position " P " is
A. Up.
B. Down.
C. Left.
D. Right.
E. The force on $-q$ is zero.


## $-6 \mathrm{nC}$



## The Field Model

- The photos show the patterns that iron filings make when sprinkled around a magnet.
- These patterns suggest that space itself around the magnet is filled with magnetic influence.

- This is called the magnetic field.
- The concept of such a "field" was first introduced by Michael Faraday in 1821.


## The Field Model

- A field is a function that assigns a vector to every point in space.
- The alteration of space around a mass is called the gravitational field.
- Similarly, the space around a charge is altered to create the electric field.


In Faraday's view, A alters the space around it. (The wavy lines are poetic license. We don't know what the alteration looks like.)

Particle B then responds to the altered space. The altered space is the agent that exerts the force on B.

## The Electric Field

- If a probe charge (or test charge) "q" experiences an electric force at a point in space, we say that there is an electric field $\vec{E}$ at that point causing the force.

$$
\vec{E}(x, y, z) \equiv \frac{\vec{F}_{\text {on } q} \text { at }(x, y, z)}{q}
$$



The units of the electric field are N/C. The magnitude $E$ of the electric field is called the electric field strength.



## Electric Field Lines

-A way of getting intuition for the fields caused by a few charges (without calculating)
-Positive charges "emit" field lines.
-Negative charges "absorb" field lines.
-Field lines begin at + charge and end at infinity or negative charge.
-The tangent to an electric field line gives direction of force
-Electric field lines do not cross

## E-field of a + point charge



## E-field of a pair of opposite point charges



(a)
so 16 lines begin on $+2 q \ldots$

(b)

(c)

Eight lines begin on each $+q$.

(d)

Eight lines begin on $+q$ and eight end on $-q$.

(e)

Eight lines begin on $+q$. Four go to infinity and four end on $-q / 2$.

(f)

## Field line views



## More field line views



PheT ...
Charges and fields Electric field of dreams

## Which set of field lines matches the charges shown?



## Key Equations

Coulomb's law

$$
\overrightarrow{\mathbf{F}}_{12}(r)=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r_{12}^{2}} \widehat{\mathbf{r}}_{12}
$$

Superposition of electric forces

$$
\overrightarrow{\mathbf{F}}(r)=\frac{1}{4 \pi \varepsilon_{0}} Q \sum_{i=1}^{N} \frac{q_{i}}{r_{i}^{2}} \widehat{\mathbf{r}}_{i}
$$

Electric force due to an electric field
$\overrightarrow{\mathbf{F}}=Q \overrightarrow{\mathbf{E}}$

Electric field at point $P$

Field of an infinite wire

$$
\overrightarrow{\mathbf{E}}(P) \equiv \frac{1}{4 \pi \varepsilon_{0}} \sum_{i=1}^{N} \frac{q_{i}}{r_{i}^{2}} \widehat{\mathbf{r}}_{i}
$$

Field of an infinite plane

$$
\overrightarrow{\mathbf{E}}(z)=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 \lambda}{z} \widehat{\mathbf{k}}
$$

$$
\overrightarrow{\mathbf{E}}=\frac{\sigma}{2 \varepsilon_{0}} \widehat{\mathbf{k}}
$$

Dipole moment


## Coulomb's Law and Gravitation

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{E}}=\mathrm{k} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}} \\
& \mathrm{k}=8.99 \times 10^{9} \frac{\mathrm{~N} \mathrm{~m}^{2}}{\mathrm{C}^{2}}
\end{aligned}
$$

$$
\mathrm{F}_{\mathrm{G}}=\mathrm{G} \frac{\mathrm{~m}_{1} \mathrm{~m}_{2}}{\mathrm{r}^{2}}
$$

$$
\mathrm{G}=6.674 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}
$$

## Why do masses attract?

Why do charges attract/repel?


## Next Class:

Electric field and flux

