Lecture 04: 01/25/2024

- Announcements
  Written HW#1 due midnight tonight
  Written HW#2 date corrected (next Thursday)
  Online HW#2 next Tuesday

- Last Time
  - Types of charge
  - Tribocharging
  - Insulators and Conductors

- Today
  - Coulomb's Law
    Coulomb vector form and r-hat
    Superposition

iClicker

- We had 45 clickers last time. There are 51 students.
- Has everyone got a clicker? Please see me if have issues.

Coulomb's Law

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = 8.99 \times 10^9 \frac{N\,m^2}{C^2}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$$

Forces in Hydrogen atom

A hydrogen atom is composed of a proton and an electron with equal charges. The proton has roughly 1800 times the electron mass.
Compare the forces on electron and proton.

(A) The electron feels a greater force because it orbits the nucleus.

$$F = k\frac{q_1 q_2}{r^2}\hat{r}$$

(B) The proton feels a greater force because it is larger and has a larger surface area.

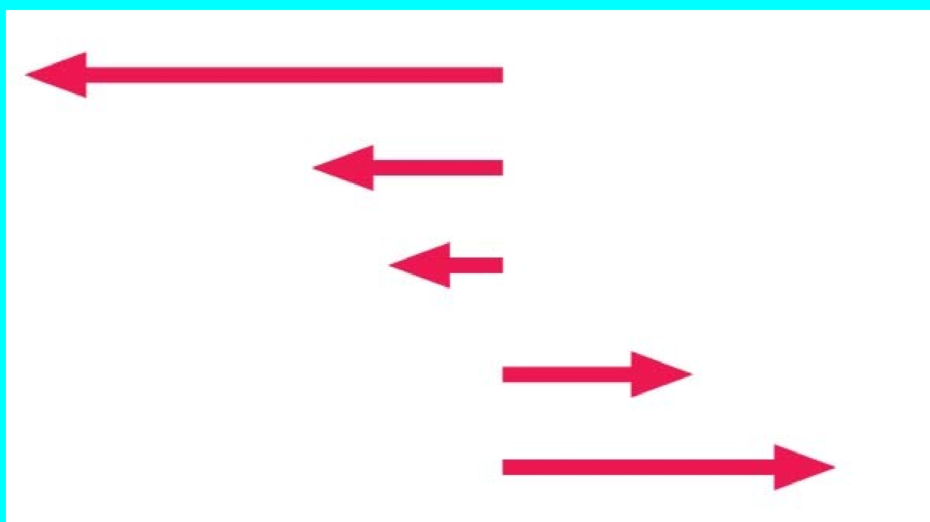
(C) Depends on whether the atom is in a molecule.

(D) The electron and proton feel the same force because coulomb's law is symmetrical.

Coulomb's Law for Unequal charges

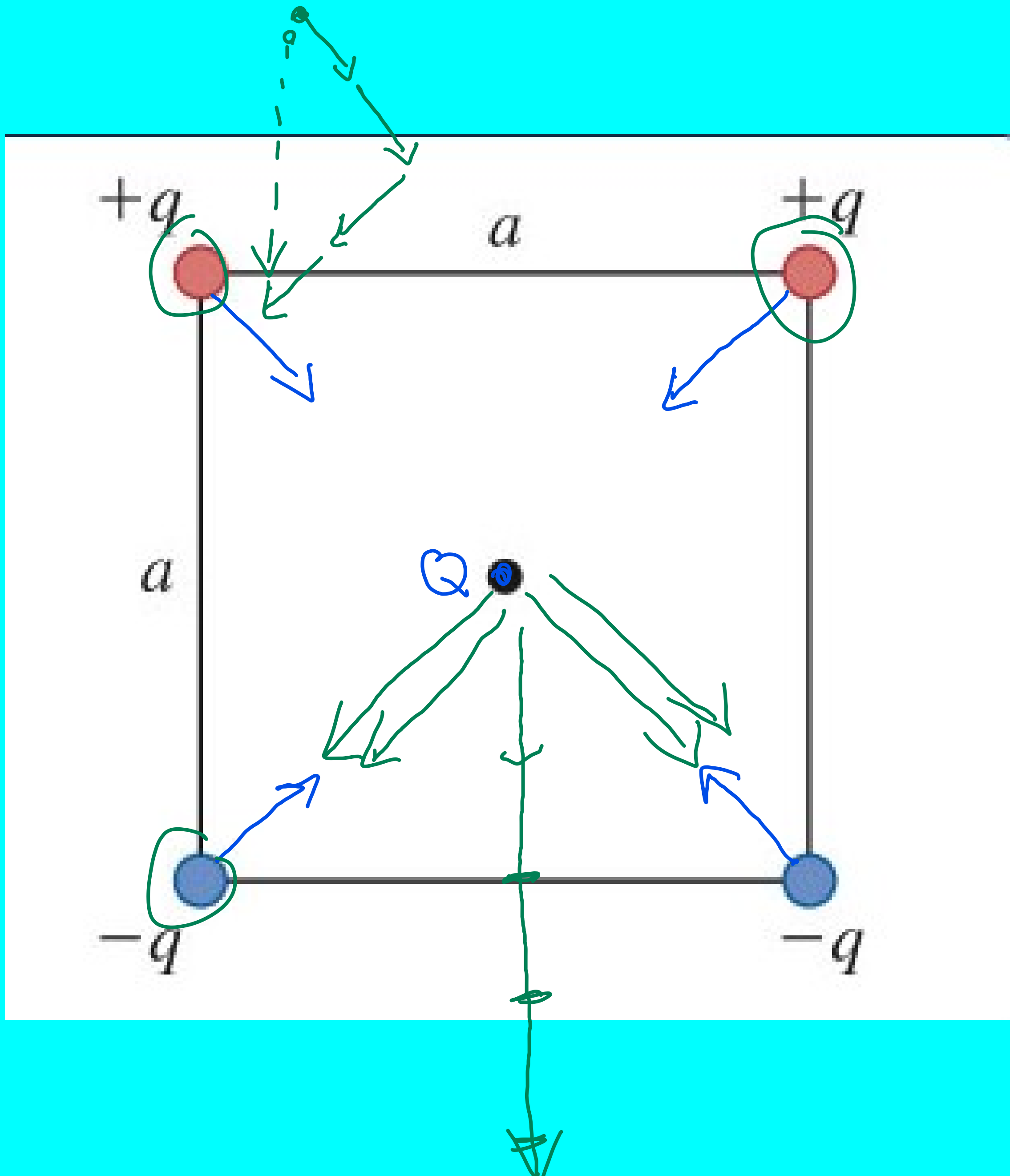
The charge of sphere 2 is twice that of sphere 1. Which vector below shows the force of 2 on 1?

A.
B.
C.
D.
E.



$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Electric Field Superposition



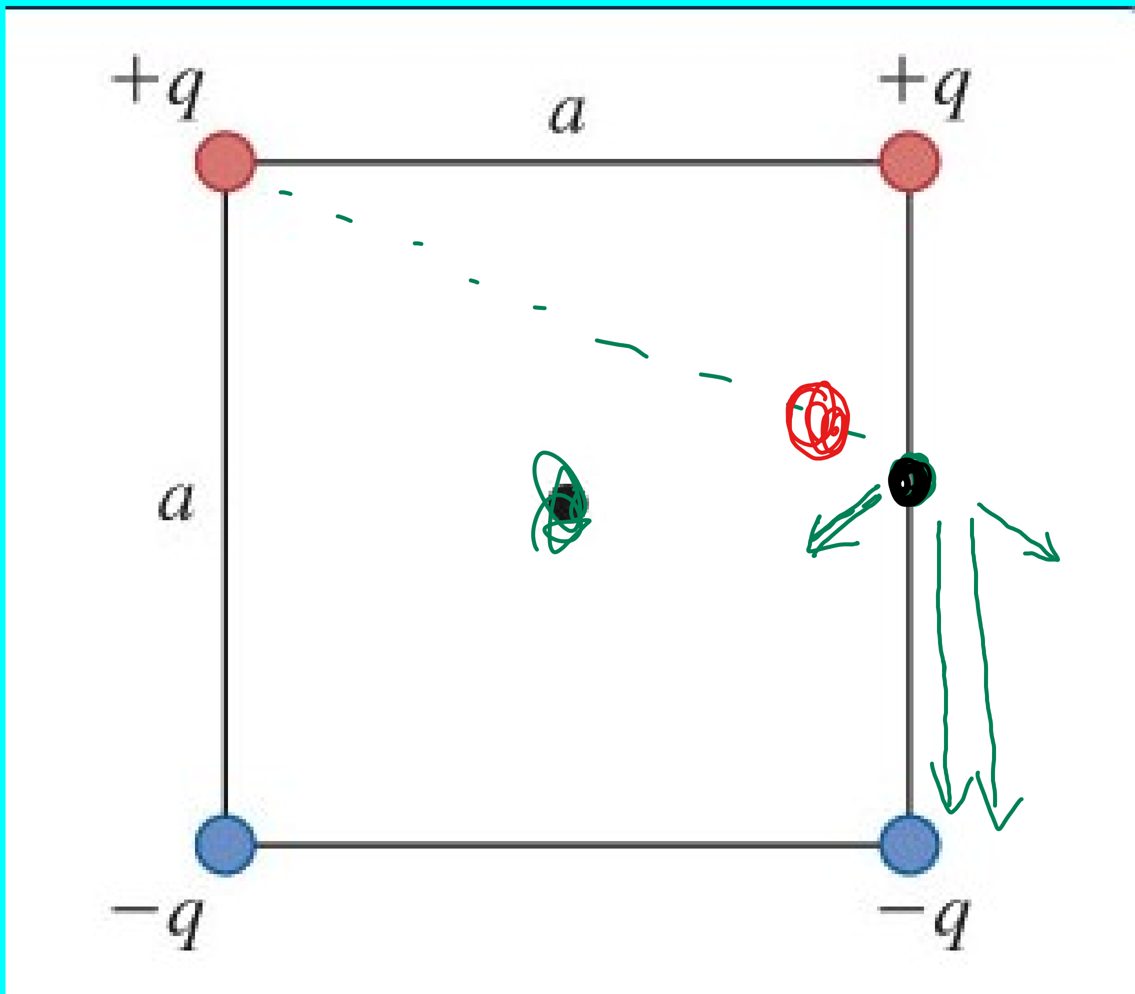
Charges are arranged on the corners of a square. The magnitude of all ‘q’s is the same.

Find the direction of the force on a positive charge in the center of the square.

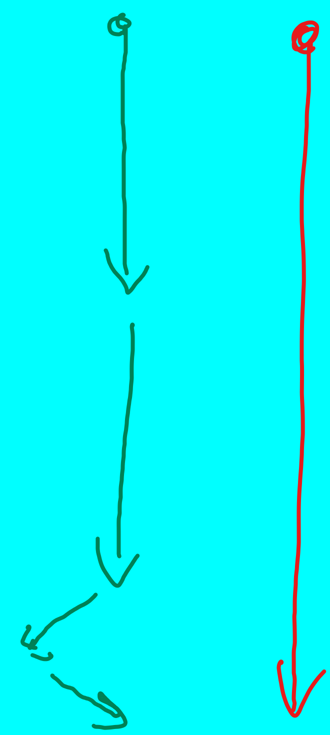
$\hat{r} \quad \hat{i} \quad \hat{j} \quad \hat{k}$

$F = k\frac{qQ}{r^2}$

Electric Field Superposition



For charges arranged as in previous question, find direction of force on a positive charge in the middle of a side.



Steps to solve a superposition problem

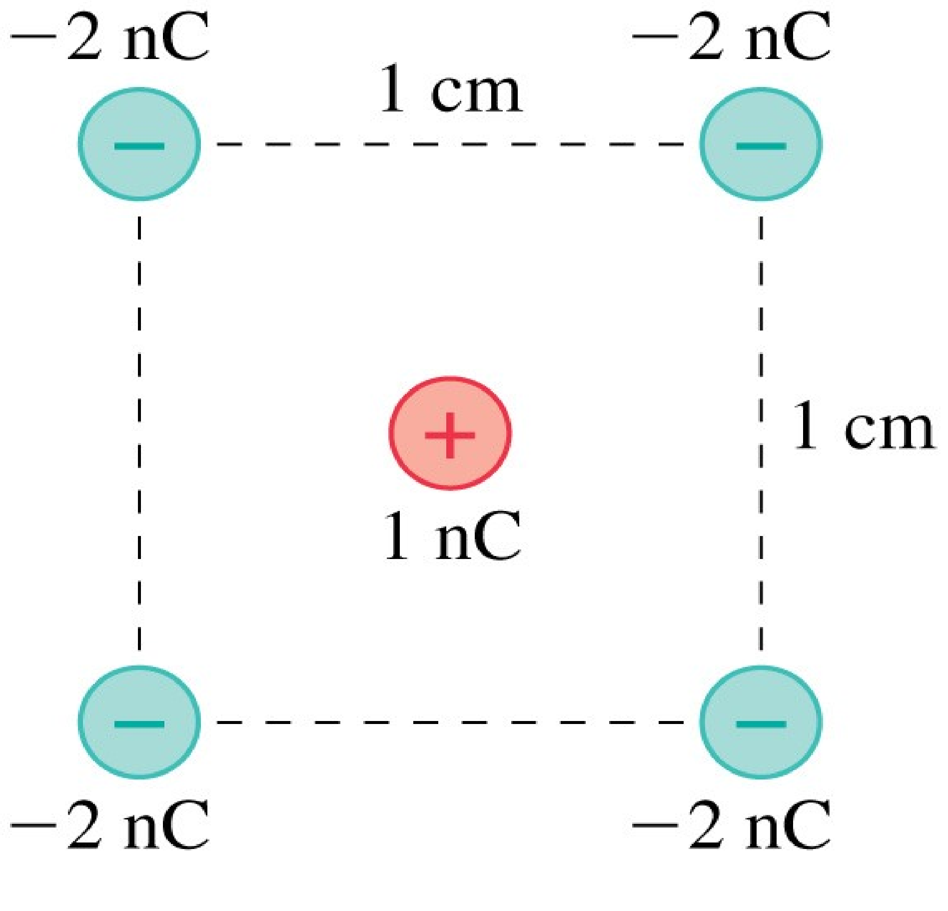
1) Identify the charge (or point P) at which you want to calculate the force (or field).

2) Draw an arrow (a vector) representing the Force Vector (or Field Vector) at the charge along a line joining it with each of the other charges.

3) Make the length of the vectors proportional to the force between the charges (shorter arrows for more distant charges)

4) Add the vectors using the tip to tail method to find the *resultant*.

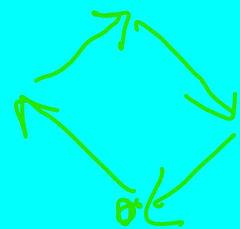
Superposition problem



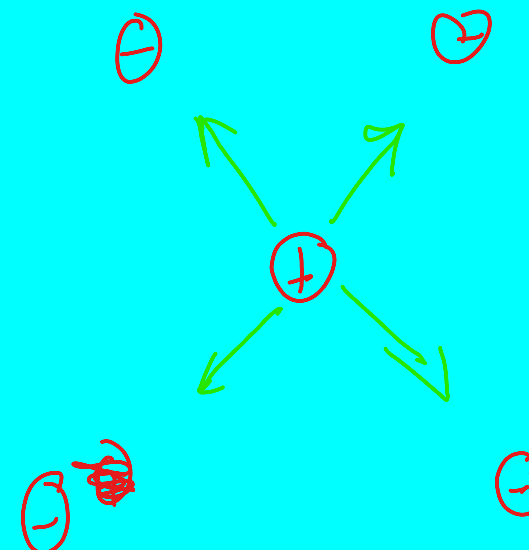
Estimate the direction of the NET force on the central charge due to the other four charges.

A) "Up"
B) Along a diagonal
C) "Left"
D) "Right"
E) What net force?

Symmetry



… is the key to easily solving otherwise difficult problems

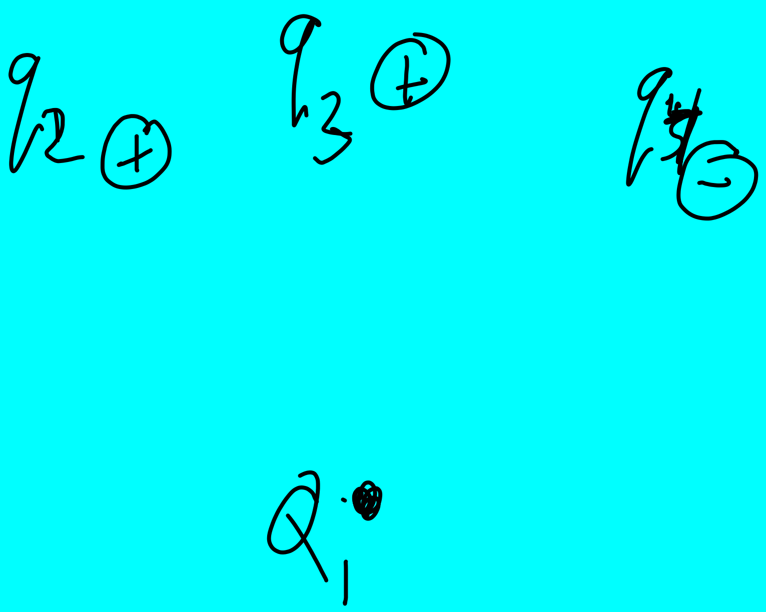


Coulomb's Law, Vector Form

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$\vec{F}_{net} = \sum_{n=1}^{N} k\frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}$$

$$\vec{F}_{net} = Q\sum_{n=1}^{N} k\frac{q_n}{r_n^2}\hat{r}_n$$



$$\vec{F}_{net\text{ on }Q} = \frac{kQq_2}{r_{12}^2}\hat{r}_{12} + \frac{kQq_3}{r_{13}^2}\hat{r}_{13} + \frac{kQq_4}{r_{14}^2}\hat{r}_{14}$$

Making friends with "r-hat"

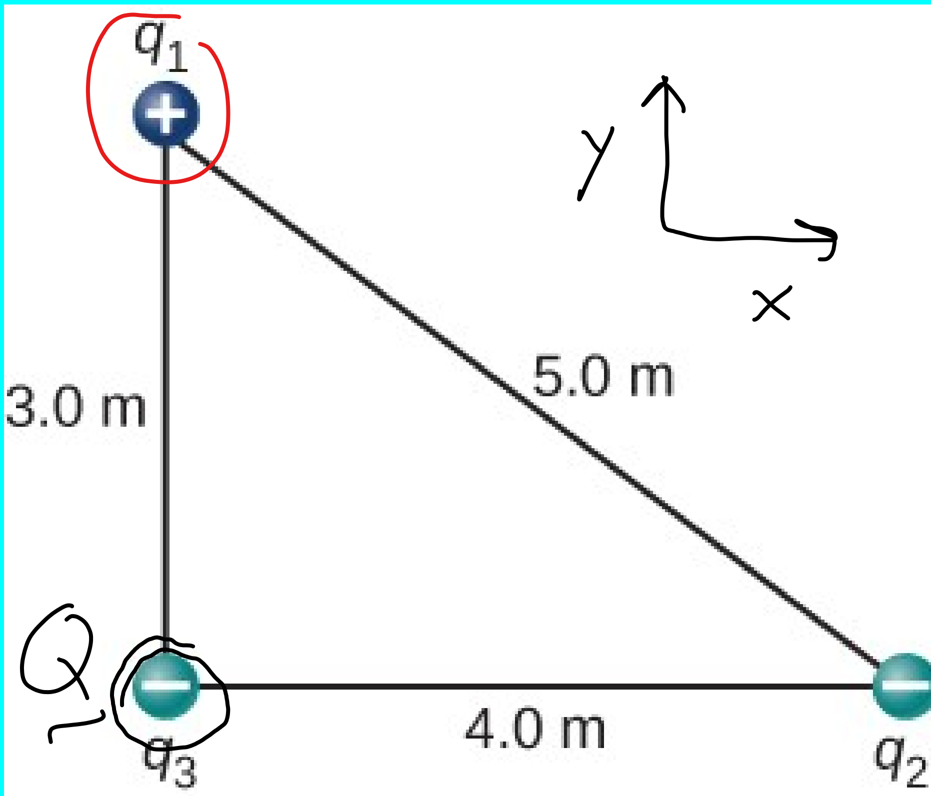
$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

r-hat points from other charges to 'your' charge.

$\hat{r}$ is a unit vector like $\hat{i}$, $\hat{j}$, and $\hat{k}$

$\hat{r}$ points in different directions for different charges

Homework 5-62-ish



$q_1 = 2\times10^{-7}$ C

$q_2 = -4\times10^{-7}$ C

$q_3 = -1\times10^{-7}$ C $= Q$

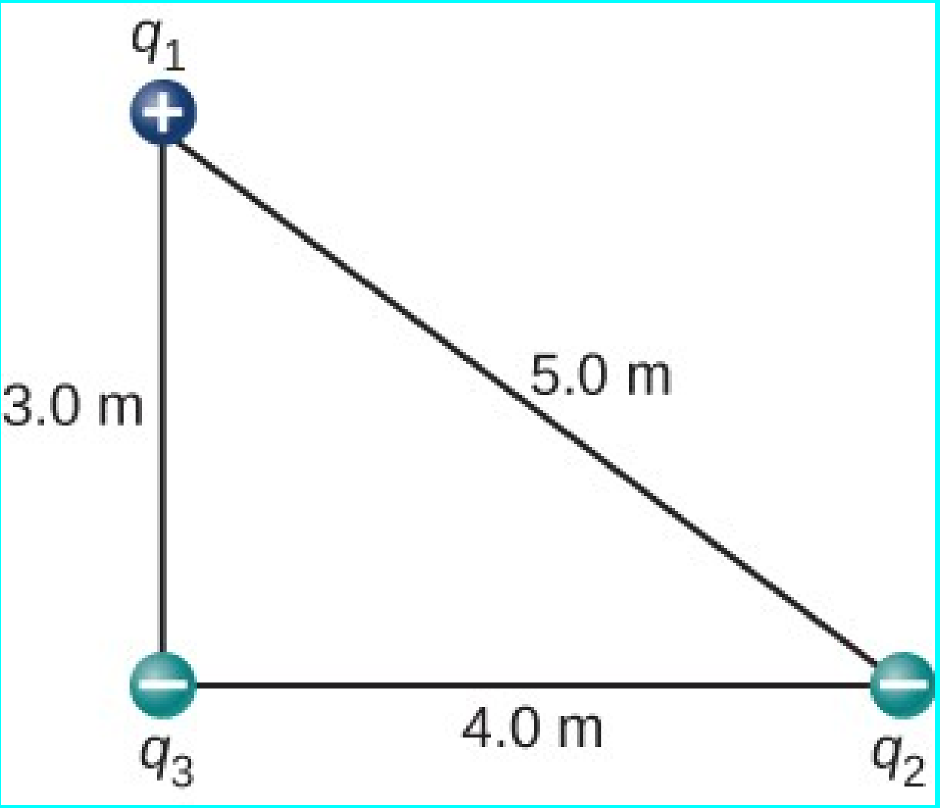
Find force on q_3

$$\vec{F} = \frac{kQq_1}{r_1^2}(-\hat{j}) - \hat{i}\frac{kQq_2}{r_2^2}$$

$$\vec{F} = (9\times10^{9})(-1\times10^{-7})\left[\frac{2\times10^{-7}}{3^2}(-\hat{j}) - \hat{i}\,\frac{(-4\times10^{-7}C)}{4^2}\right]$$

Homework 5-62-ish

$$\vec{F}_{net} = Q \sum_{n=1}^{N} k \frac{q_n}{r_n^2} \hat{r}_n$$



$q_1 = 2 \times 10^{-7}$ C

$q_2 = -4 \times 10^{-7}$ C

$q_3 = -1 \times 10^{-7}$ C

Find force on q_3

Homework 5-62-ish

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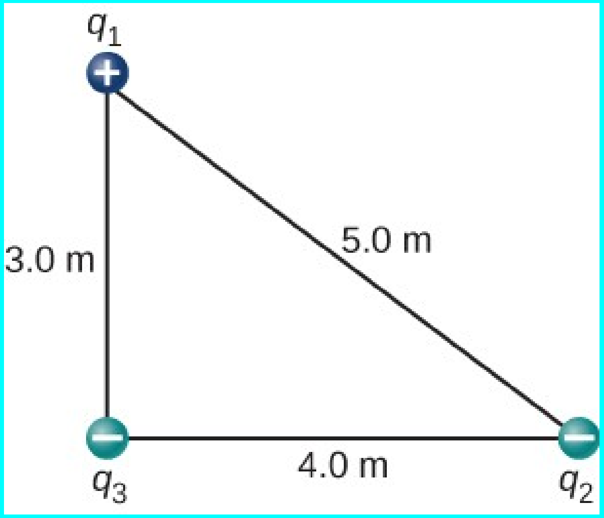

$q_1 = 2 \times 10^{-7}$ C

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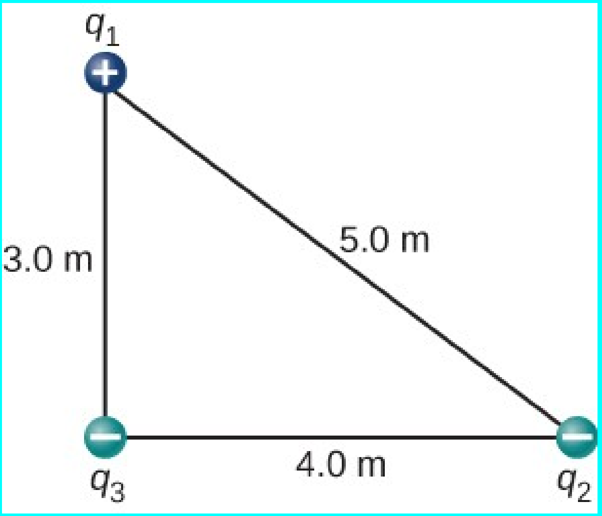
Find force on q_3

Homework 5-62-ish

$$\vec{F}_{net} = Q \sum_{n=1}^{N} k \frac{q_n}{r_n^2} \hat{r}_n$$



$q_1 = 2 \times 10^{-7}$ C

$q_2 = -4 \times 10^{-7}$ C

$q_3 = -1 \times 10^{-7}$ C

Find force on q_3

From Coulomb's Law to Electric field

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$\vec{F}_{net} = \sum_{n=1}^{N} k\frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}$$

$$\vec{F}_{net} = Q\sum_{n=1}^{N} k\frac{q_n}{r_n^2}\hat{r}_n$$

$$\vec{E} = \sum_{n=1}^{N} k\frac{q_n}{r_n^2}\hat{r}_n$$

$$\vec{F}_{net} = Q\vec{E}$$

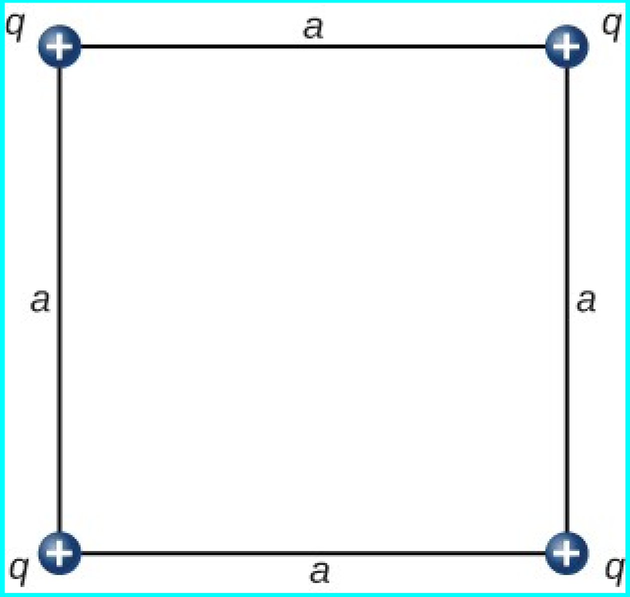
DEMOS!

DEMOS!

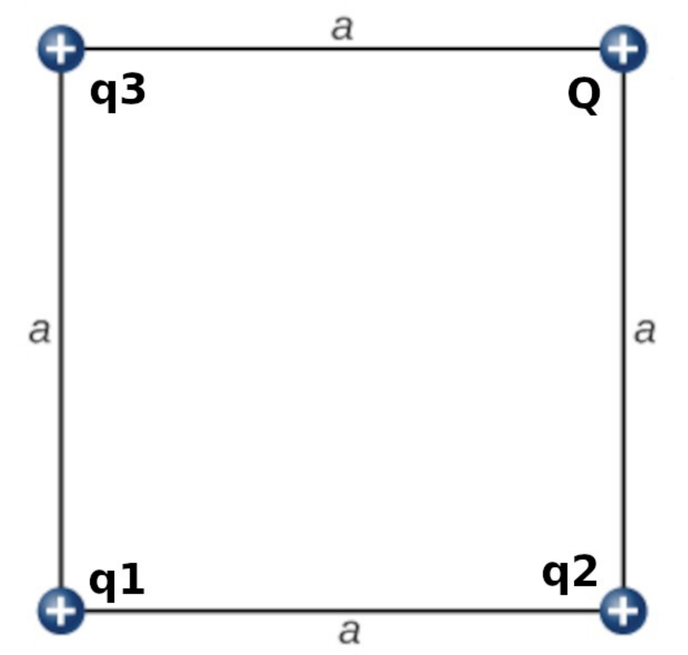
Two types of charge

Electric Field

Homework 5-63-ish



Find force on the q on top right corner



a
q3
Q
a
a
q1
q2
a

What is r_1^2?

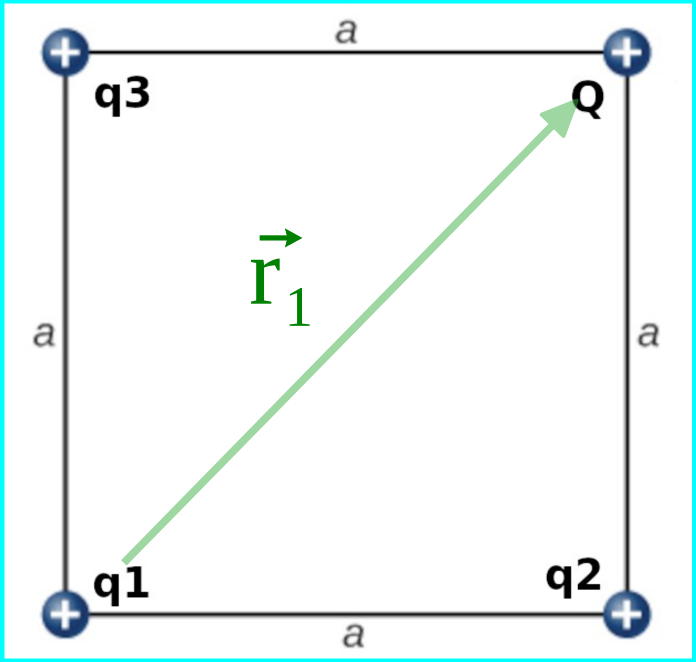
(A) $\frac{\sqrt{2}}{2}a$

(B) $\sqrt{2}a$

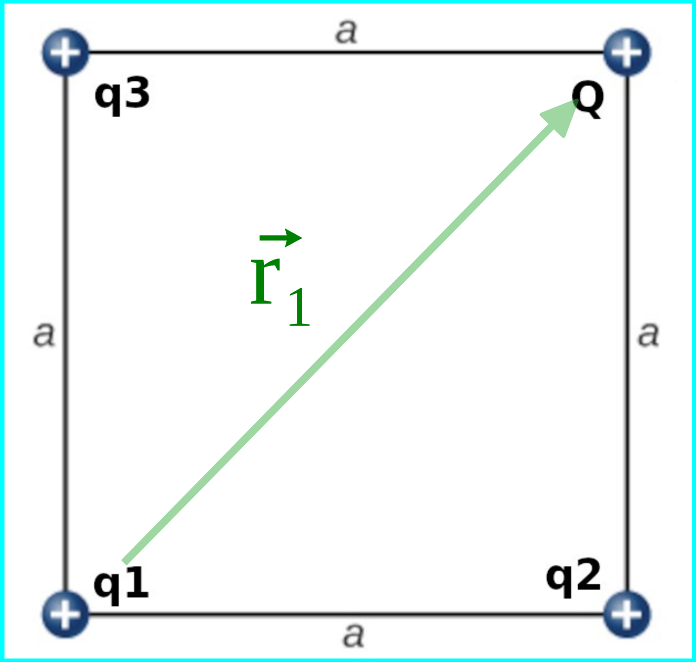
(C) $\sqrt{2}a^2$

(D) $2a$

(E) $2a^2$



What is $\vec{r}_1$?



$(A)\ \ a$

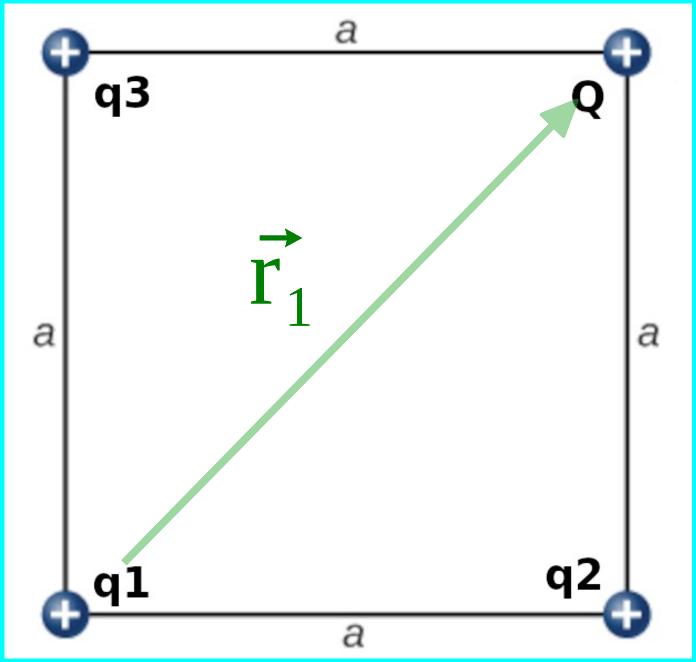
$(B)\ \ a\,\hat{i}$

$(C)\ \ a\,\hat{i} + a\,\hat{j}$

$(D)\ \ \sqrt{2}\,a\,\hat{i} + \sqrt{2}\,a\,\hat{j}$

$(E)\ \ \frac{\sqrt{2}}{2}a\,\hat{i} + \frac{\sqrt{2}}{2}a\,\hat{j}$

What is $\hat{r}_1$?



(A) a

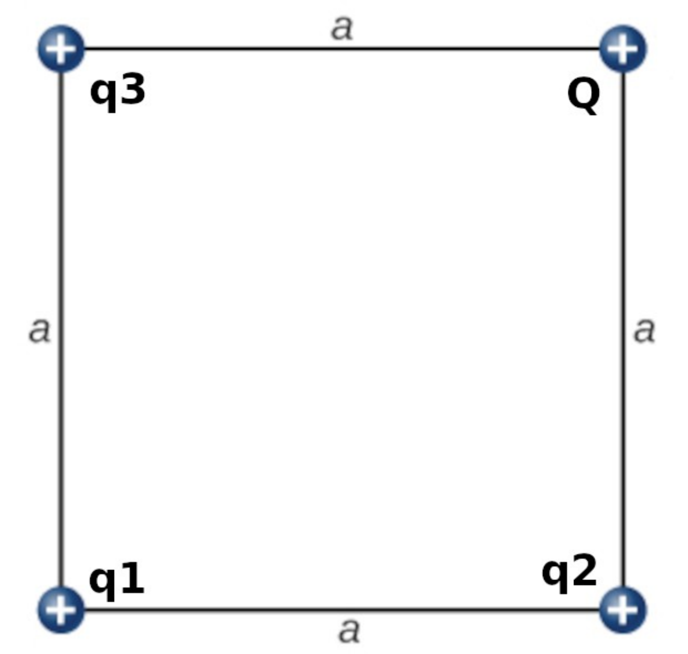
(B) $\hat{i}$

(C) $\hat{i}+\hat{j}$

(D) $\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}$

(E) $\frac{\sqrt{2}}{2}\hat{i}+\frac{\sqrt{2}}{2}\hat{j}$

Homework 5-63-ish with field



Find $\vec{E}$-field at top right corner in absence of Q

Making friends with "r-hat"

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

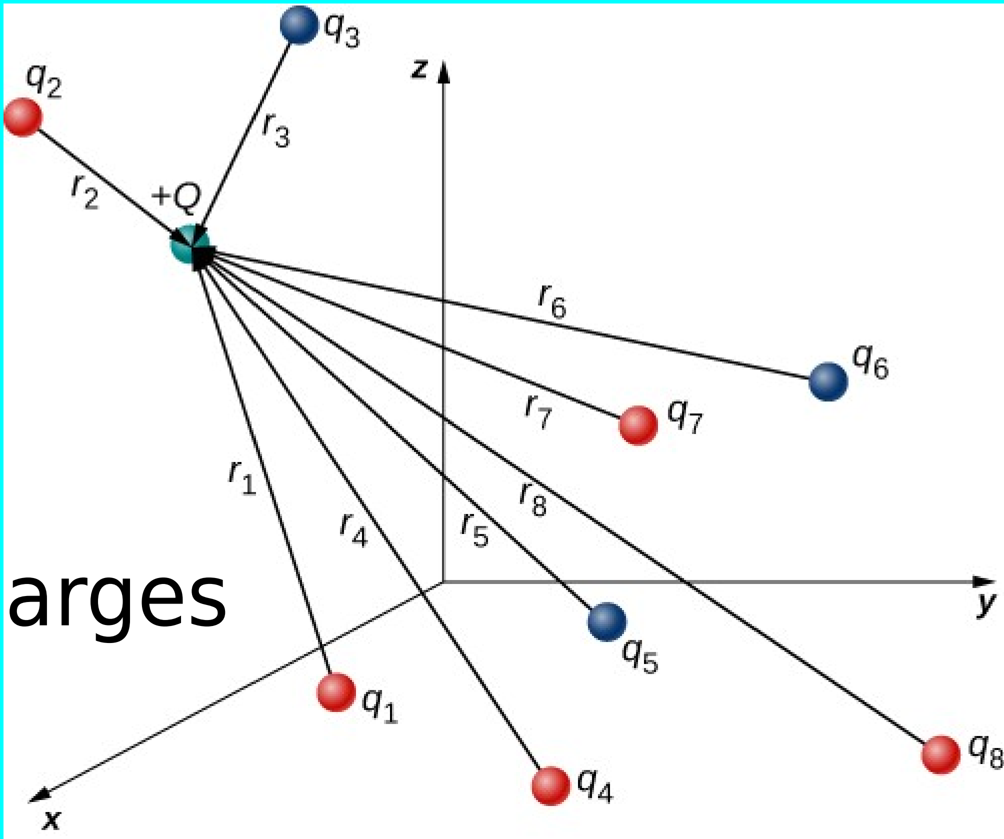
r-hat points from other charges to 'your' charge.

$\hat{r}$ is a unit vector like $\hat{i}$, $\hat{j}$, and $\hat{k}$

$\hat{r}$ points in different directions for different charges

Making friends with "r-hat"

$\hat{r}$



r-hat points from other charges to 'your' charge.

$\hat{r}$ is a unit vector like $\hat{i}$, $\hat{j}$, and $\hat{k}$

$\hat{r}$ points in different directions for different charges

“Superposition”

Key Equations

Coulomb's law	$\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$
Superposition of electric forces	$\vec{\mathbf{F}}(r) = \frac{1}{4\pi\varepsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$
Electric force due to an electric field	$\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$
Electric field at point *P*	$\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$
Field of an infinite wire	$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{z} \hat{\mathbf{k}}$
Field of an infinite plane	$\vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0} \hat{\mathbf{k}}$
Dipole moment	[redacted]

Coulomb's Law and Gravitation

$$F_E = k\frac{q_1 q_2}{r^2}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

$$k = 8.99 \times 10^9 \frac{N\,m^2}{C^2}$$

$$G = 6.674 \times 10^{-11} N\,m^2/kg^2$$

Why do masses attract?

Why do charges attract/repel?

Next Class:

More on Coulomb's law and electric field