

# CHAPTER 13

## Electromagnetic Induction



**Figure 13.1** The black strip found on the back of credit cards and driver's licenses is a very thin layer of magnetic material with information stored on it. Reading and writing the information on the credit card is done with a swiping motion. The physical reason why this is necessary is called electromagnetic induction and is discussed in this chapter. (credit: modification of work by Jane Whitney)

### Chapter Outline

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#### [13.2 Lenz's Law](#)

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#### [13.4 Induced Electric Fields](#)

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**INTRODUCTION** We have been considering electric fields created by fixed charge distributions and magnetic fields produced by constant currents, but electromagnetic phenomena are not restricted to these stationary situations. Most of the interesting applications of electromagnetism are, in fact, time-dependent. To investigate some of these applications, we now remove the time-independent assumption that we have been making and allow the fields to vary with time. In this and the next several chapters, you will see a wonderful

symmetry in the behavior exhibited by time-varying electric and magnetic fields. Mathematically, this symmetry is expressed by an additional term in Ampère's law and by another key equation of electromagnetism called Faraday's law. We also discuss how moving a wire through a magnetic field produces an emf or voltage. Lastly, we describe applications of these principles, such as the card reader shown above.

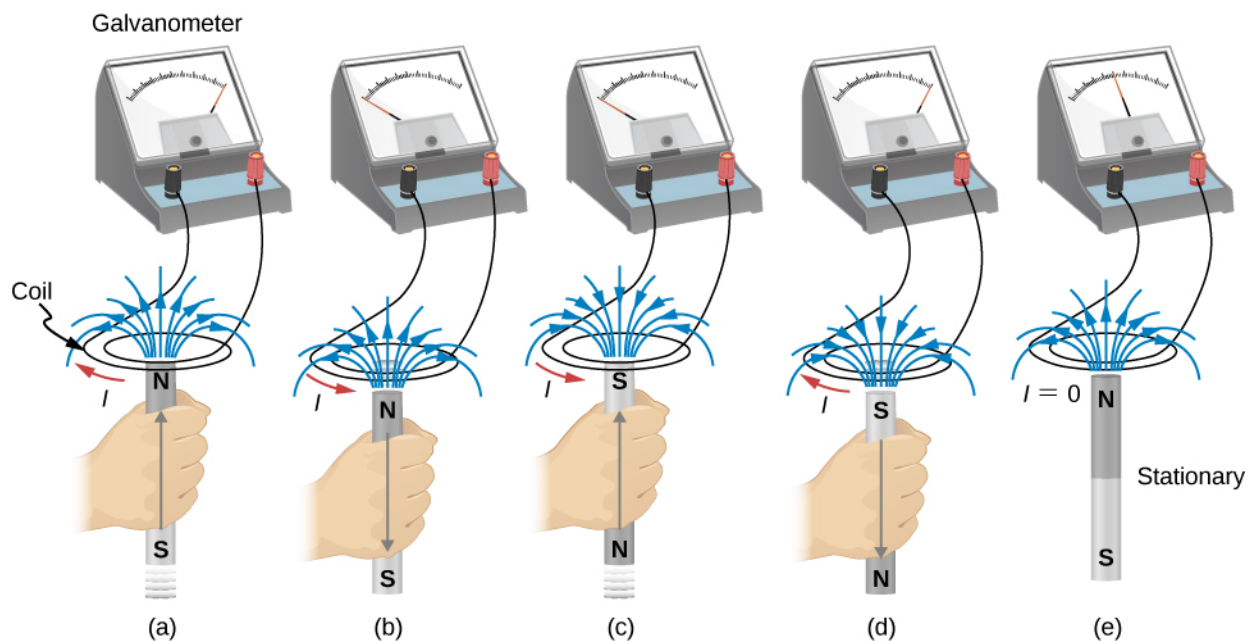
## 13.1 Faraday's Law

### Learning Objectives

*By the end of this section, you will be able to:*

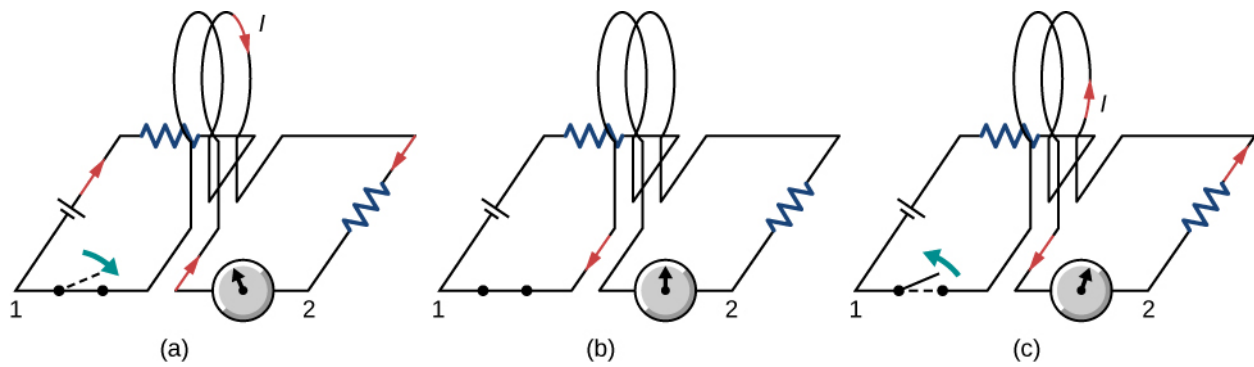
- Determine the magnetic flux through a surface, knowing the strength of the magnetic field, the surface area, and the angle between the normal to the surface and the magnetic field
- Use Faraday's law to determine the magnitude of induced emf in a closed loop due to changing magnetic flux through the loop

The first productive experiments concerning the effects of time-varying magnetic fields were performed by Michael Faraday in 1831. One of his early experiments is represented in [Figure 13.2](#). An emf is induced when the magnetic field in the coil is changed by pushing a bar magnet into or out of the coil. Emfs of opposite signs are produced by motion in opposite directions, and the directions of emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.



**Figure 13.2** Movement of a magnet relative to a coil produces emfs as shown (a–d). The same emfs are produced if the coil is moved relative to the magnet. This short-lived emf is only present during the motion. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion, as shown in (e).

Faraday also discovered that a similar effect can be produced using two circuits—a changing current in one circuit induces a current in a second, nearby circuit. For example, when the switch is closed in circuit 1 of [Figure 13.3\(a\)](#), the ammeter needle of circuit 2 momentarily deflects, indicating that a short-lived current surge has been induced in that circuit. The ammeter needle quickly returns to its original position, where it remains. However, if the switch of circuit 1 is now suddenly opened, another short-lived current surge in the direction opposite from before is observed in circuit 2.



**Figure 13.3** (a) Closing the switch of circuit 1 produces a short-lived current surge in circuit 2. (b) If the switch remains closed, no current is observed in circuit 2. (c) Opening the switch again produces a short-lived current in circuit 2 but in the opposite direction from before.

Faraday realized that in both experiments, a current flowed in the circuit containing the ammeter only when the magnetic field in the region occupied by that circuit was *changing*. As the magnet of the figure was moved, the strength of its magnetic field at the loop changed; and when the current in circuit 1 was turned on or off, the strength of its magnetic field at circuit 2 changed. Faraday was eventually able to interpret these and all other experiments involving magnetic fields that vary with time in terms of the following law:

### Faraday's Law

The emf  $\varepsilon$  induced is the negative change in the magnetic flux  $\Phi_m$  per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

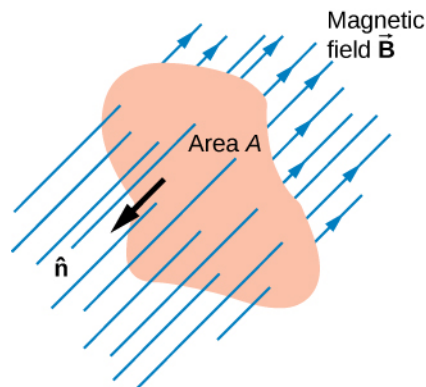
The **magnetic flux** is a measurement of the amount of magnetic field lines through a given surface area, as seen in [Figure 13.4](#). This definition is similar to the electric flux studied earlier. This means that if we have

$$\Phi_m = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA, \quad 13.1$$

then the **induced emf** or the voltage generated by a conductor or coil moving in a magnetic field is

$$\varepsilon = -\frac{d}{dt} \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = -\frac{d\Phi_m}{dt}. \quad 13.2$$

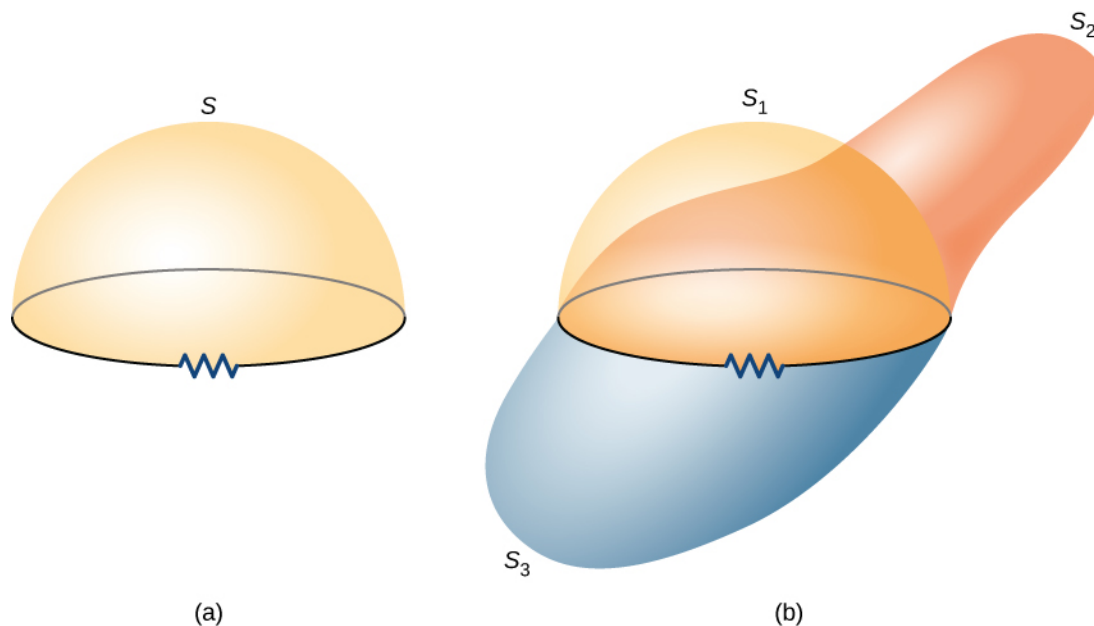
The negative sign describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with a rule known as Lenz's law, which we will discuss shortly.



**Figure 13.4** The magnetic flux is the amount of magnetic field lines cutting through a surface area  $A$  defined by the unit area vector  $\hat{\mathbf{n}}$ . If

the angle between the unit area  $\hat{n}$  and magnetic field vector  $\vec{B}$  are parallel or antiparallel, as shown in the diagram, the magnetic flux is the highest possible value given the values of area and magnetic field.

Part (a) of [Figure 13.5](#) depicts a circuit and an arbitrary surface  $S$  that it bounds. Notice that  $S$  is an *open surface*. It can be shown that *any* open surface bounded by the circuit in question can be used to evaluate  $\Phi_m$ . For example,  $\Phi_m$  is the same for the various surfaces  $S_1, S_2, \dots$  of part (b) of the figure.



**Figure 13.5** (a) A circuit bounding an arbitrary open surface  $S$ . The planar area bounded by the circuit is not part of  $S$ . (b) Three arbitrary open surfaces bounded by the same circuit. The value of  $\Phi_m$  is the same for all these surfaces.

The SI unit for magnetic flux is the weber (Wb),

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

Occasionally, the magnetic field unit is expressed as webers per square meter ( $\text{Wb}/\text{m}^2$ ) instead of teslas, based on this definition. In many practical applications, the circuit of interest consists of a number  $N$  of tightly wound turns (see [Figure 13.6](#)). Each turn experiences the same magnetic flux. Therefore, the net magnetic flux through the circuits is  $N$  times the flux through one turn, and Faraday's law is written as

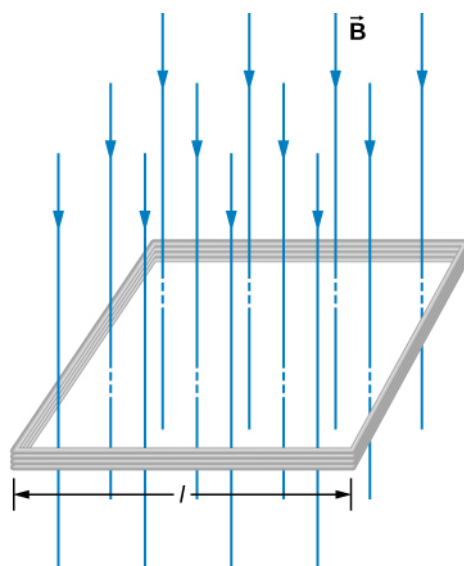
$$\varepsilon = -\frac{d}{dt}(N\Phi_m) = -N\frac{d\Phi_m}{dt}. \quad 13.3$$



### EXAMPLE 13.1

#### A Square Coil in a Changing Magnetic Field

The square coil of [Figure 13.6](#) has sides  $l = 0.25 \text{ m}$  long and is tightly wound with  $N = 200$  turns of wire. The resistance of the coil is  $R = 5.0 \Omega$ . The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate  $d\mathbf{B}/dt = -0.040 \text{ T/s}$ . (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?



**Figure 13.6** A square coil with  $N$  turns of wire with uniform magnetic field  $\vec{B}$  directed in the downward direction, perpendicular to the coil.

### Strategy

The area vector, or  $\hat{n}$  direction, is perpendicular to area covering the loop. We will choose this to be pointing downward so that  $\vec{B}$  is parallel to  $\hat{n}$  and that the flux turns into multiplication of magnetic field times area. The area of the loop is not changing in time, so it can be factored out of the time derivative, leaving the magnetic field as the only quantity varying in time. Lastly, we can apply Ohm's law once we know the induced emf to find the current in the loop.

### Solution

- a. The flux through one turn is

$$\Phi_m = BA = Bl^2,$$

so we can calculate the magnitude of the emf from Faraday's law. The sign of the emf will be discussed in the next section, on Lenz's law:

$$\begin{aligned} |\varepsilon| &= \left| -N \frac{d\Phi_m}{dt} \right| = Nl^2 \frac{dB}{dt} \\ &= (200)(0.25 \text{ m})^2 (0.040 \text{ T/s}) = 0.50 \text{ V}. \end{aligned}$$

- b. The magnitude of the current induced in the coil is

$$I = \frac{\varepsilon}{R} = \frac{0.50 \text{ V}}{5.0 \Omega} = 0.10 \text{ A}.$$

### Significance

If the area of the loop were changing in time, we would not be able to pull it out of the time derivative. Since the loop is a closed path, the result of this current would be a small amount of heating of the wires until the magnetic field stops changing. This may increase the area of the loop slightly as the wires are heated.

### ✓ CHECK YOUR UNDERSTANDING 13.1

A closely wound coil has a radius of 4.0 cm, 50 turns, and a total resistance of  $40 \Omega$ . At what rate must a magnetic field perpendicular to the face of the coil change in order to produce Joule heating in the coil at a rate of 2.0 mW?

## 13.2 Lenz's Law

### Learning Objectives

By the end of this section, you will be able to:

- Use Lenz's law to determine the direction of induced emf whenever a magnetic flux changes
- Use Faraday's law with Lenz's law to determine the induced emf in a coil and in a solenoid

The direction in which the induced emf drives current around a wire loop can be found through the negative sign. However, it is usually easier to determine this direction with **Lenz's law**, named in honor of its discoverer, Heinrich Lenz (1804–1865). (Faraday also discovered this law, independently of Lenz.) We state Lenz's law as follows:

#### Lenz's Law

The direction of the induced emf drives current around a wire loop to always *oppose* the change in magnetic flux that causes the emf.

Lenz's law can also be considered in terms of conservation of energy. If pushing a magnet into a coil causes current, the energy in that current must have come from somewhere. If the induced current causes a magnetic field opposing the increase in field of the magnet we pushed in, then the situation is clear. We pushed a magnet against a field and did work on the system, and that showed up as current. If it were not the case that the induced field opposes the change in the flux, the magnet would be pulled in produce a current without anything having done work. Electric potential energy would have been created, violating the conservation of energy.

To determine an induced emf  $\epsilon$ , you first calculate the magnetic flux  $\Phi_m$  and then obtain  $d\Phi_m/dt$ . The magnitude of  $\epsilon$  is given by  $\epsilon = |d\Phi_m/dt|$ . Finally, you can apply Lenz's law to determine the sense of  $\epsilon$ . This will be developed through examples that illustrate the following problem-solving strategy.



### PROBLEM-SOLVING STRATEGY

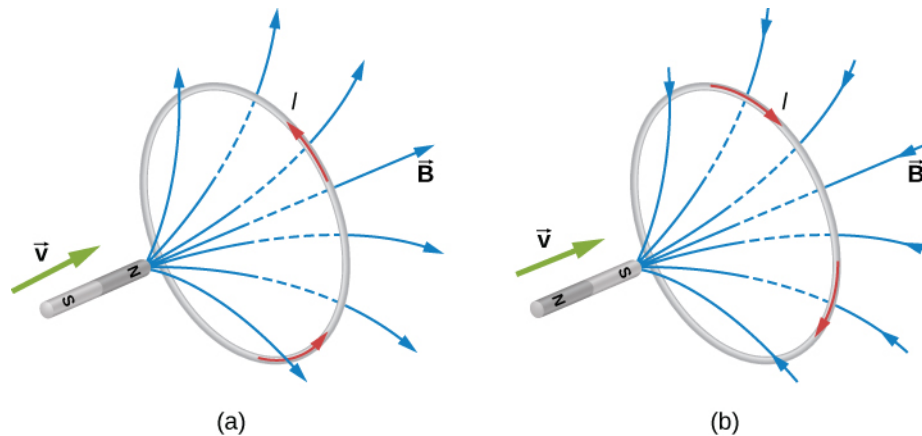
#### Lenz's Law

To use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs:

1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the applied magnetic field  $\vec{B}$ .
3. Determine whether its magnetic flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field  $\vec{B}$ . The induced magnetic field tries to reinforce a magnetic flux that is decreasing or opposes a magnetic flux that is increasing. Therefore, the induced magnetic field adds or subtracts to the applied magnetic field, depending on the change in magnetic flux.
5. Use right-hand rule 2 (RHR-2; see [Magnetic Forces and Fields](#)) to determine the direction of the induced current  $I$  that is responsible for the induced magnetic field  $\vec{B}$ .
6. The direction (or polarity) of the induced emf can now drive a conventional current in this direction.

Let's apply Lenz's law to the system of [Figure 13.7\(a\)](#). We designate the "front" of the closed conducting loop as the region containing the approaching bar magnet, and the "back" of the loop as the other region. As the north pole of the magnet moves toward the loop, the flux through the loop due to the field of the magnet increases because the strength of field lines directed from the front to the back of the loop is increasing. A current is therefore induced in the loop. By Lenz's law, the direction of the induced current must be such that its own magnetic field is directed in a way to *oppose* the changing flux caused by the field of the approaching magnet. Hence, the induced current circulates so that its magnetic field lines through the loop are directed from the back to the front of the loop. By RHR-2, place your thumb pointing against the magnetic field lines, which is toward the bar magnet. Your fingers wrap in a counterclockwise direction as viewed from the bar magnet.

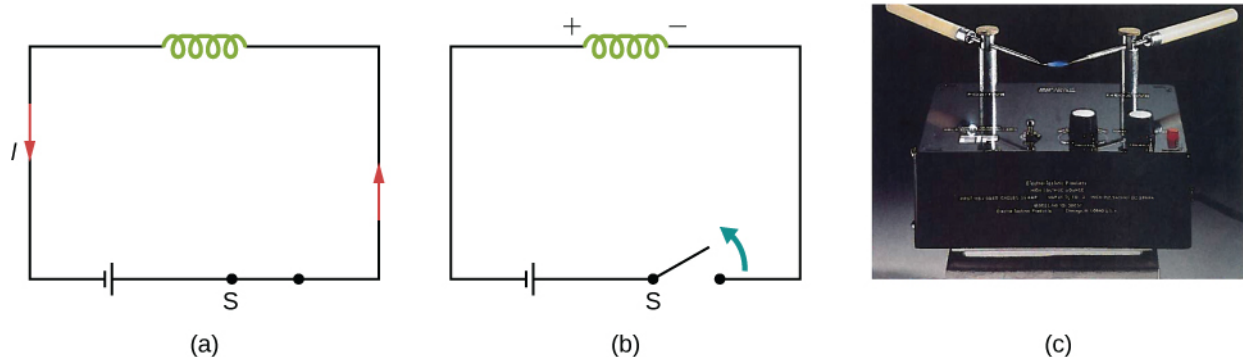
Alternatively, we can determine the direction of the induced current by treating the current loop as an electromagnet that *opposes* the approach of the north pole of the bar magnet. This occurs when the induced current flows as shown, for then the face of the loop nearer the approaching magnet is also a north pole.



**Figure 13.7** The change in magnetic flux caused by the approaching magnet induces a current in the loop. (a) An approaching north pole induces a counterclockwise current with respect to the bar magnet. (b) An approaching south pole induces a clockwise current with respect to the bar magnet.

Part (b) of the figure shows the south pole of a magnet moving toward a conducting loop. In this case, the flux through the loop due to the field of the magnet increases because the number of field lines directed from the back to the front of the loop is increasing. To oppose this change, a current is induced in the loop whose field lines through the loop are directed from the front to the back. Equivalently, we can say that the current flows in a direction so that the face of the loop nearer the approaching magnet is a south pole, which then repels the approaching south pole of the magnet. By RHR-2, your thumb points away from the bar magnet. Your fingers wrap in a clockwise fashion, which is the direction of the induced current.

Another example illustrating the use of Lenz's law is shown in [Figure 13.8](#). When the switch is opened, the decrease in current through the solenoid causes a decrease in magnetic flux through its coils, which induces an emf in the solenoid. This emf must oppose the change (the termination of the current) causing it. Consequently, the induced emf has the polarity shown and drives in the direction of the original current. This may generate an arc across the terminals of the switch as it is opened.



**Figure 13.8** (a) A solenoid connected to a source of emf. (b) Opening switch S terminates the current, which in turn induces an emf in the solenoid. (c) A potential difference between the ends of the sharply pointed rods is produced by inducing an emf in a coil. This potential difference is large enough to produce an arc between the sharp points.

### CHECK YOUR UNDERSTANDING 13.2

Find the direction of the induced current in the wire loop shown below as the magnet enters, passes through, and leaves the loop.

### ✓ CHECK YOUR UNDERSTANDING 13.3

Verify the directions of the induced currents in [Figure 13.3](#).

### ✿ EXAMPLE 13.2

#### A Circular Coil in a Changing Magnetic Field

A magnetic field  $\vec{B}$  is directed outward perpendicular to the plane of a circular coil of radius  $r = 0.50$  m ([Figure 13.9](#)). The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to  $B = (1.5\text{T})e^{-(5.0\text{s}^{-1})t}$ , where  $B$  is in teslas and  $t$  is in seconds. (a) Calculate the emf induced in the coil at the times  $t_1 = 0$ ,  $t_2 = 5.0 \times 10^{-2}$  s, and  $t_3 = 1.0$  s. (b) Determine the current in the coil at these three times if its resistance is  $10\ \Omega$ .

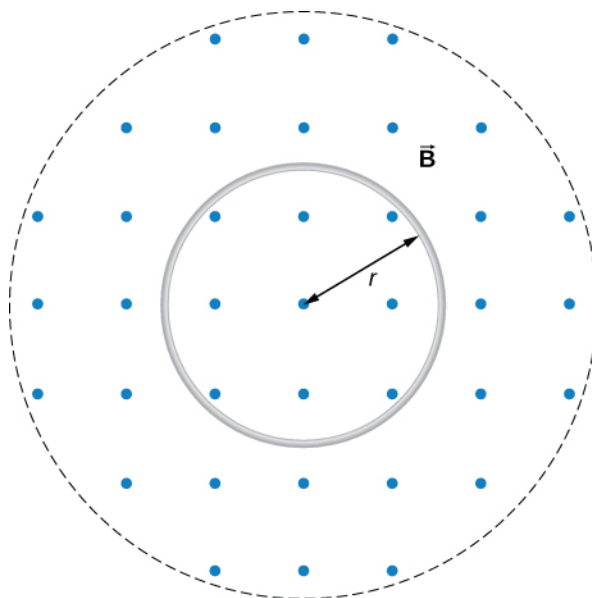


Figure 13.9 A circular coil in a decreasing magnetic field.

#### Strategy

Since the magnetic field is perpendicular to the plane of the coil and constant over each spot in the coil, the dot product of the magnetic field  $\vec{B}$  and normal to the area unit vector  $\hat{n}$  turns into a multiplication. The magnetic field can be pulled out of the integration, leaving the flux as the product of the magnetic field times area. We need to take the time derivative of the exponential function to calculate the emf using Faraday's law. Then we use Ohm's law to calculate the current.

#### Solution

- a. Since  $\vec{B}$  is perpendicular to the plane of the coil, the magnetic flux is given by

$$\begin{aligned}\Phi_m &= B\pi r^2 = (1.5e^{-5.0t} \text{ T})\pi(0.50 \text{ m})^2 \\ &= 1.2e^{-(5.0\text{s}^{-1})t} \text{ Wb}.\end{aligned}$$

From Faraday's law, the magnitude of the induced emf is

$$\varepsilon = \left| \frac{d\Phi_m}{dt} \right| = \left| \frac{d}{dt}(1.2e^{-(5.0\text{s}^{-1})t} \text{ Wb}) \right| = 6.0 e^{-(5.0\text{s}^{-1})t} \text{ V}.$$

Since  $\vec{B}$  is directed out of the page and is decreasing, the induced current must flow counterclockwise



when viewed from above so that the magnetic field it produces through the coil also points out of the page. For all three times, the sense of  $\varepsilon$  is counterclockwise; its magnitudes are

$$\varepsilon(t_1) = 6.0 \text{ V}; \quad \varepsilon(t_2) = 4.7 \text{ V}; \quad \varepsilon(t_3) = 0.040 \text{ V}.$$

b. From Ohm's law, the respective currents are

$$I(t_1) = \frac{\varepsilon(t_1)}{R} = \frac{6.0 \text{ V}}{10 \Omega} = 0.60 \text{ A};$$

$$I(t_2) = \frac{4.7 \text{ V}}{10 \Omega} = 0.47 \text{ A};$$

and

$$I(t_3) = \frac{0.040 \text{ V}}{10 \Omega} = 4.0 \times 10^{-3} \text{ A}.$$

### Significance

An emf voltage is created by a changing magnetic flux over time. If we know how the magnetic field varies with time over a constant area, we can take its time derivative to calculate the induced emf.

## EXAMPLE 13.3

### Changing Magnetic Field Inside a Solenoid

The current through the windings of a solenoid with  $n = 2000$  turns per meter is changing at a rate  $dI/dt = 3.0 \text{ A/s}$ . (See [Sources of Magnetic Fields](#) for a discussion of solenoids.) The solenoid is 50-cm long and has a cross-sectional diameter of 3.0 cm. A small coil consisting of  $N = 20$  closely wound turns wrapped in a circle of diameter 1.0 cm is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Assuming that the infinite-solenoid approximation is valid at the location of the small coil, determine the magnitude of the emf induced in the coil.

#### Strategy

The magnetic field in the middle of the solenoid is a uniform value of  $\mu_0 nI$ . This field is producing a maximum magnetic flux through the coil as it is directed along the length of the solenoid. Therefore, the magnetic flux through the coil is the product of the solenoid's magnetic field times the area of the coil. Faraday's law involves a time derivative of the magnetic flux. The only quantity varying in time is the current, the rest can be pulled out of the time derivative. Lastly, we include the number of turns in the coil to determine the induced emf in the coil.

#### Solution

Since the field of the solenoid is given by  $B = \mu_0 nI$ , the flux through each turn of the small coil is

$$\Phi_m = \mu_0 nI \left( \frac{\pi d^2}{4} \right),$$

where  $d$  is the diameter of the coil. Now from Faraday's law, the magnitude of the emf induced in the coil is

$$\begin{aligned} \varepsilon &= \left| N \frac{d\Phi_m}{dt} \right| = \left| N \mu_0 n \frac{\pi d^2}{4} \frac{dI}{dt} \right| \\ &= 20 (4\pi \times 10^{-7} \text{ T} \cdot \text{m/s}) (2000 \text{ m}^{-1}) \frac{\pi(0.010 \text{ m})^2}{4} (3.0 \text{ A/s}) \\ &= 1.2 \times 10^{-5} \text{ V}. \end{aligned}$$

### Significance

When the current is turned on in a vertical solenoid, as shown in [Figure 13.10](#), the ring has an induced emf from the solenoid's changing magnetic flux that opposes the change. The result is that the ring is fired vertically into the air.



**Figure 13.10** The jumping ring. When a current is turned on in the vertical solenoid, a current is induced in the metal ring. The stray field produced by the solenoid causes the ring to jump off the solenoid.

### INTERACTIVE

Visit this [website \(https://openstax.org/l/21mitjumping\)](https://openstax.org/l/21mitjumping) for a demonstration of the jumping ring from MIT.

## 13.3 Motional Emf

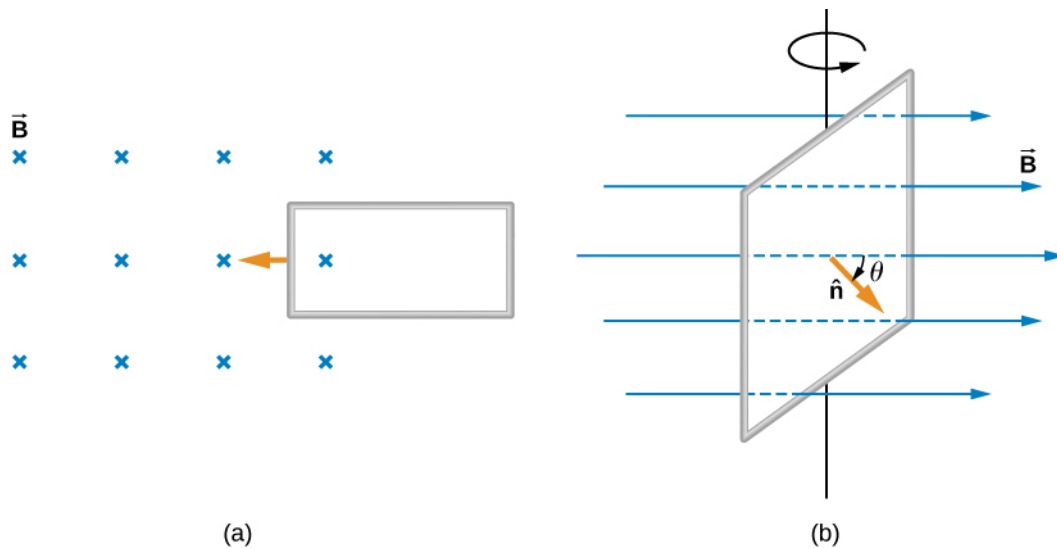
### Learning Objectives

*By the end of this section, you will be able to:*

- Determine the magnitude of an induced emf in a wire moving at a constant speed through a magnetic field
- Discuss examples that use motional emf, such as a rail gun and a tethered satellite

Magnetic flux depends on three factors: the strength of the magnetic field, the area through which the field lines pass, and the orientation of the field with the surface area. If any of these quantities varies, a corresponding variation in magnetic flux occurs. So far, we've only considered flux changes due to a changing field. Now we look at another possibility: a changing area through which the field lines pass including a change in the orientation of the area.

Two examples of this type of flux change are represented in [Figure 13.11](#). In part (a), the flux through the rectangular loop increases as it moves into the magnetic field, and in part (b), the flux through the rotating coil varies with the angle  $\theta$ .



**Figure 13.11** (a) Magnetic flux changes as a loop moves into a magnetic field; (b) magnetic flux changes as a loop rotates in a magnetic field.

It's interesting to note that what we perceive as the cause of a particular flux change actually depends on the frame of reference we choose. For example, if you are at rest relative to the moving coils of [Figure 13.11](#), you would see the flux vary because of a changing magnetic field—in part (a), the field moves from left to right in your reference frame, and in part (b), the field is rotating. It is often possible to describe a flux change through a coil that is moving in one particular reference frame in terms of a changing magnetic field in a second frame, where the coil is stationary. However, reference-frame questions related to magnetic flux are beyond the level of this textbook. We'll avoid such complexities by always working in a frame at rest relative to the laboratory and explain flux variations as due to either a changing field or a changing area.

Now let's look at a conducting rod pulled in a circuit, changing magnetic flux. The area enclosed by the circuit 'MNOP' of [Figure 13.12](#) is  $lx$  and is perpendicular to the magnetic field, so we can simplify the integration of [Equation 13.1](#) into a multiplication of magnetic field and area. The magnetic flux through the open surface is therefore

$$\Phi_m = Blx. \quad 13.4$$

Since  $B$  and  $l$  are constant and the velocity of the rod is  $v = dx/dt$ , we can now restate Faraday's law, [Equation 13.2](#), for the magnitude of the emf in terms of the moving conducting rod as

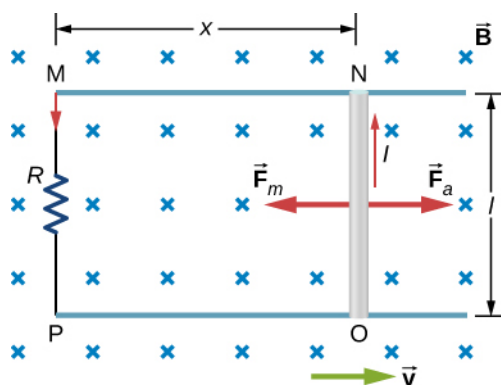
$$\varepsilon = \frac{d\Phi_m}{dt} = Bl \frac{dx}{dt} = Blv. \quad 13.5$$

The current induced in the circuit is the emf divided by the resistance or

$$I = \frac{Blv}{R}.$$

Furthermore, the direction of the induced emf satisfies Lenz's law, as you can verify by inspection of the figure.

This calculation of motionally induced emf is not restricted to a rod moving on conducting rails. With  $\vec{F} = q\vec{v} \times \vec{B}$  as the starting point, it can be shown that  $\varepsilon = -d\Phi_m/dt$  holds for any change in flux caused by the motion of a conductor. We saw in [Faraday's Law](#) that the emf induced by a time-varying magnetic field obeys this same relationship, which is Faraday's law. Thus Faraday's law *holds for all flux changes*, whether they are produced by a changing magnetic field, by motion, or by a combination of the two.



**Figure 13.12** A conducting rod is pushed to the right at constant velocity. The resulting change in the magnetic flux induces a current in the circuit.

From an energy perspective,  $\vec{F}_a$  produces power  $F_a v$ , and the resistor dissipates power  $I^2 R$ . Since the rod is moving at constant velocity, the applied force  $F_a$  must balance the magnetic force  $F_m = IlB$  on the rod when it is carrying the induced current  $I$ . Thus the power produced is

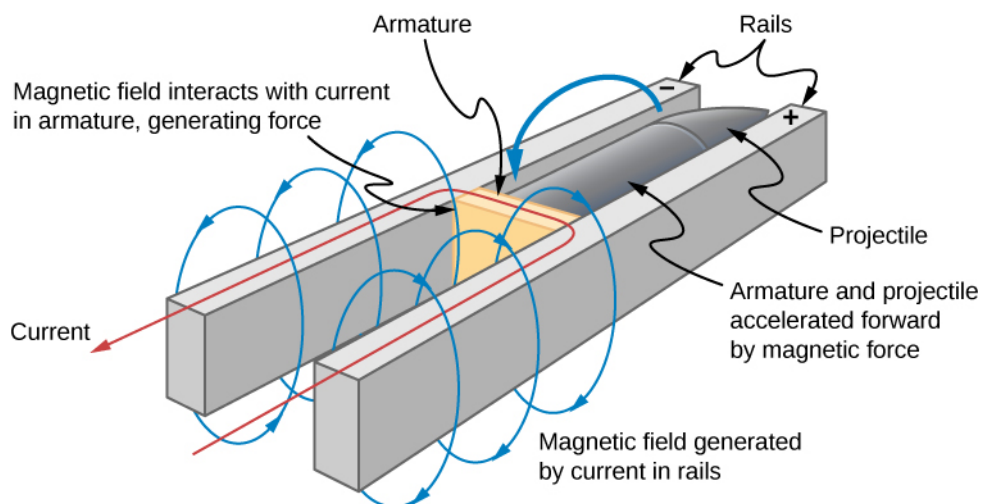
$$F_a v = IlBv = \frac{Blv}{R} \cdot lBv = \frac{l^2 B^2 v^2}{R}. \quad 13.6$$

The power dissipated is

$$P = I^2 R = \left( \frac{Blv}{R} \right)^2 R = \frac{l^2 B^2 v^2}{R}. \quad 13.7$$

In satisfying the principle of energy conservation, the produced and dissipated powers are equal.

This principle can be seen in the operation of a rail gun. A rail gun is an electromagnetic projectile launcher that uses an apparatus similar to [Figure 13.12](#) and is shown in schematic form in [Figure 13.13](#). The conducting rod is replaced with a projectile or weapon to be fired. So far, we've only heard about how motion causes an emf. In a rail gun, the optimal shutting off/ramping down of a magnetic field decreases the flux in between the rails, causing a current to flow in the rod (armature) that holds the projectile. This current through the armature experiences a magnetic force and is propelled forward. Rail guns, however, are not used widely in the military due to the high cost of production and high currents: Nearly one million amps is required to produce enough energy for a rail gun to be an effective weapon.



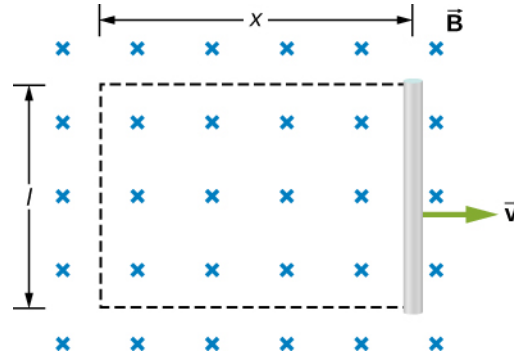
**Figure 13.13** Current through two rails drives a conductive projectile forward by the magnetic force created.

We can calculate a **motioally induced emf** with Faraday's law *even when an actual closed circuit is not present*. We simply imagine an enclosed area whose boundary includes the moving conductor, calculate  $\Phi_m$ ,

and then find the emf from Faraday's law. For example, we can let the moving rod of [Figure 13.14](#) be one side of the imaginary rectangular area represented by the dashed lines. The area of the rectangle is  $lx$ , so the magnetic flux through it is  $\Phi_m = Blx$ . Differentiating this equation, we obtain

$$\frac{d\Phi_m}{dt} = Bl \frac{dx}{dt} = Blv, \quad 13.8$$

which is identical to the potential difference between the ends of the rod that we determined earlier.

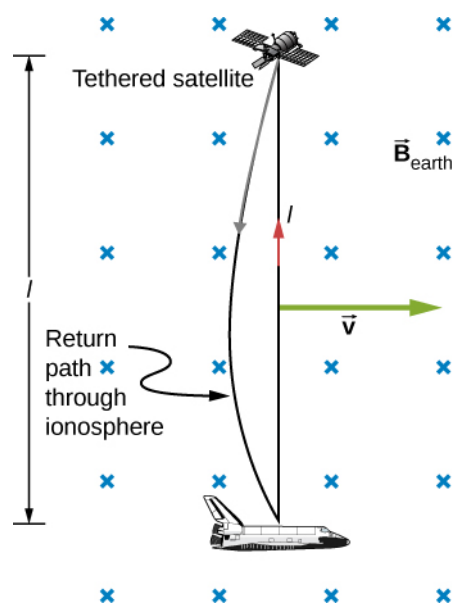


**Figure 13.14** With the imaginary rectangle shown, we can use Faraday's law to calculate the induced emf in the moving rod.

Motional emfs in Earth's weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1.0-m rod moving at 3.0 m/s perpendicular to the Earth's field gives

$$\text{emf} = B\ell v = (5.0 \times 10^{-5} \text{ T})(1.0 \text{ m})(3.0 \text{ m/s}) = 150 \mu\text{V}.$$

This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The tethered satellite was to be let out on a 20-km length of wire, as shown in [Figure 13.15](#), to create a 5-kV emf by moving at orbital speed through Earth's field. This emf could be used to convert some of the shuttle's kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path through which current could flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in [Figure 13.13](#), without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force  $F = I\ell B \sin \theta$  does the work that reduces the shuttle's kinetic and potential energy, and allows it to be converted into electrical energy. Both tests were unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. [Example 13.4](#) indicates feasibility in principle.



**Figure 13.15** Motional emf as electrical power conversion for the space shuttle was the motivation for the tethered satellite experiment. A 5-kV emf was predicted to be induced in the 20-km tether while moving at orbital speed in Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.

### EXAMPLE 13.4

#### Calculating the Large Motional Emf of an Object in Orbit

Calculate the motional emf induced along a 20.0-km conductor moving at an orbital speed of 7.80 km/s perpendicular to Earth's  $5.00 \times 10^{-5}$  T magnetic field.

##### Strategy

This is a great example of using the equation motional  $\varepsilon = B\ell v$ .

##### Solution

Entering the given values into  $\varepsilon = B\ell v$  gives

$$\begin{aligned}\varepsilon &= B\ell v \\ &= (5.00 \times 10^{-5} \text{ T})(2.00 \times 10^4 \text{ m})(7.80 \times 10^3 \text{ m/s}) \\ &= 7.80 \times 10^3 \text{ V}.\end{aligned}$$

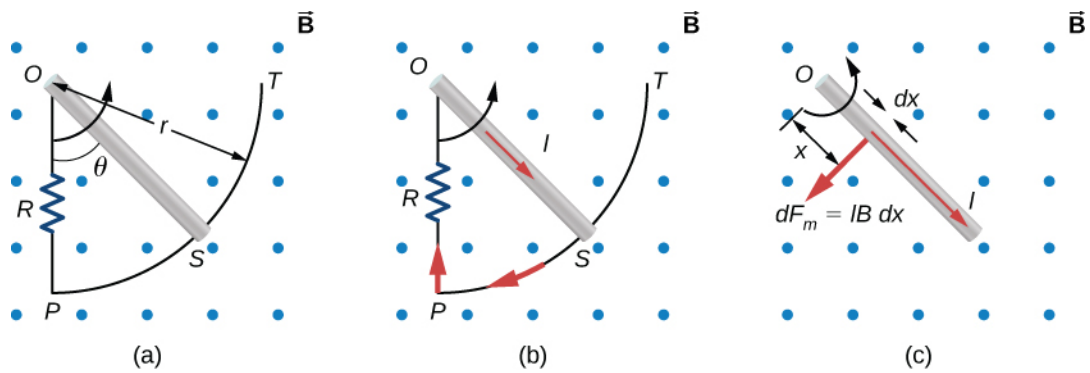
##### Significance

The value obtained is greater than the 5-kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to Earth's field. The 7.80-kV value is the maximum emf obtained when  $\theta = 90^\circ$  and so  $\sin \theta = 1$ .

### EXAMPLE 13.5

#### A Metal Rod Rotating in a Magnetic Field

Part (a) of [Figure 13.16](#) shows a metal rod  $OS$  that is rotating in a horizontal plane around point  $O$ . The rod slides along a wire that forms a circular arc  $PST$  of radius  $r$ . The system is in a constant magnetic field  $\vec{B}$  that is directed out of the page. (a) If you rotate the rod at a constant angular velocity  $\omega$ , what is the current  $I$  in the closed loop  $OPSO$ ? Assume that the resistor  $R$  furnishes all of the resistance in the closed loop. (b) Calculate the work per unit time that you do while rotating the rod and show that it is equal to the power dissipated in the resistor.



**Figure 13.16** (a) The end of a rotating metal rod slides along a circular wire in a horizontal plane. (b) The induced current in the rod. (c) The magnetic force on an infinitesimal current segment.

### Strategy

The magnetic flux is the magnetic field times the area of the quarter circle or  $A = r^2\theta/2$ . When finding the emf through Faraday's law, all variables are constant in time but  $\theta$ , with  $\omega = d\theta/dt$ . To calculate the work per unit time, we know this is related to the torque times the angular velocity. The torque is calculated by knowing the force on a rod and integrating it over the length of the rod.

### Solution

- a. From geometry, the area of the loop  $OPSO$  is  $A = \frac{r^2\theta}{2}$ . Hence, the magnetic flux through the loop is

$$\Phi_m = BA = B \frac{r^2\theta}{2}.$$

Differentiating with respect to time and using  $\omega = d\theta/dt$ , we have

$$\varepsilon = \left| \frac{d\Phi_m}{dt} \right| = \frac{Br^2\omega}{2}.$$

When divided by the resistance  $R$  of the loop, this yields for the magnitude of the induced current

$$I = \frac{\varepsilon}{R} = \frac{Br^2\omega}{2R}.$$

As  $\theta$  increases, so does the flux through the loop due to  $\vec{B}$ . To counteract this increase, the magnetic field due to the induced current must be directed into the page in the region enclosed by the loop. Therefore, as part (b) of [Figure 13.16](#) illustrates, the current circulates clockwise.

- b. You rotate the rod by exerting a torque on it. Since the rod rotates at constant angular velocity, this torque is equal and opposite to the torque exerted on the current in the rod by the original magnetic field. The magnetic force on the infinitesimal segment of length  $dx$  shown in part (c) of [Figure 13.16](#) is  $dF_m = IBdx$ , so the magnetic torque on this segment is

$$d\tau_m = x \cdot dF_m = IBx dx.$$

The net magnetic torque on the rod is then

$$\tau_m = \int_0^r d\tau_m = IB \int_0^r x dx = \frac{1}{2} IB r^2.$$

The torque  $\tau$  that you exert on the rod is equal and opposite to  $\tau_m$ , and the work that you do when the rod rotates through an angle  $d\theta$  is  $dW = \tau d\theta$ . Hence, the work per unit time that you do on the rod is

$$\frac{dW}{dt} = \tau \frac{d\theta}{dt} = \frac{1}{2} IB r^2 \frac{d\theta}{dt} = \frac{1}{2} \left( \frac{Br^2\omega}{2R} \right) Br^2\omega = \frac{B^2 r^4 \omega^2}{4R},$$

where we have substituted for  $I$ . The power dissipated in the resistor is  $P = I^2 R$ , which can be written as

$$P = \left( \frac{Br^2\omega}{2R} \right)^2 R = \frac{B^2 r^4 \omega^2}{4R}.$$

Therefore, we see that

$$P = \frac{dW}{dt}.$$

Hence, the power dissipated in the resistor is equal to the work per unit time done in rotating the rod.

### Significance

An alternative way of looking at the induced emf from Faraday's law is to integrate in space instead of time. The solution, however, would be the same. The motional emf is

$$|\varepsilon| = \int Bvdl.$$

The velocity can be written as the angular velocity times the radius and the differential length written as  $dr$ . Therefore,

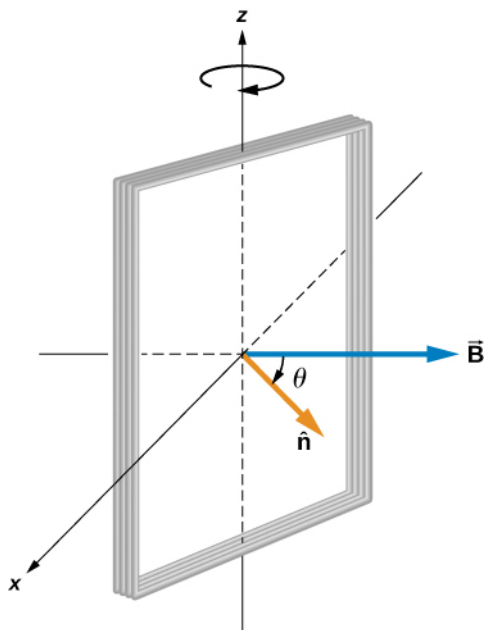
$$|\varepsilon| = B \int vdr = B\omega \int_0^l r dr = \frac{1}{2} B\omega l^2,$$

which is the same solution as before.

## EXAMPLE 13.6

### A Rectangular Coil Rotating in a Magnetic Field

A rectangular coil of area  $A$  and  $N$  turns is placed in a uniform magnetic field  $\vec{B} = B\hat{j}$ , as shown in [Figure 13.17](#). The coil is rotated about the  $z$ -axis through its center at a constant angular velocity  $\omega$ . Obtain an expression for the induced emf in the coil.



**Figure 13.17** A rectangular coil rotating in a uniform magnetic field.

### Strategy

According to the diagram, the angle between the perpendicular to the surface ( $\hat{n}$ ) and the magnetic field ( $\vec{B}$ ) is  $\theta$ . The dot product of  $\vec{B} \cdot \hat{n}$  simplifies to only the  $\cos \theta$  component of the magnetic field, namely where the magnetic field projects onto the unit area vector  $\hat{n}$ . The magnitude of the magnetic field and the area of the



loop are fixed over time, which makes the integration simplify quickly. The induced emf is written out using Faraday's law.

### Solution

When the coil is in a position such that its normal vector  $\hat{\mathbf{n}}$  makes an angle  $\theta$  with the magnetic field  $\vec{\mathbf{B}}$ , the magnetic flux through a single turn of the coil is

$$\Phi_m = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = BA \cos \theta.$$

From Faraday's law, the emf induced in the coil is

$$\varepsilon = -N \frac{d\Phi_m}{dt} = NBA \sin \theta \frac{d\theta}{dt}.$$

The constant angular velocity is  $\omega = d\theta/dt$ . The angle  $\theta$  represents the time evolution of the angular velocity or  $\omega t$ . This changes the function to time space rather than  $\theta$ . The induced emf therefore varies sinusoidally with time according to

$$\varepsilon = \varepsilon_0 \sin \omega t,$$

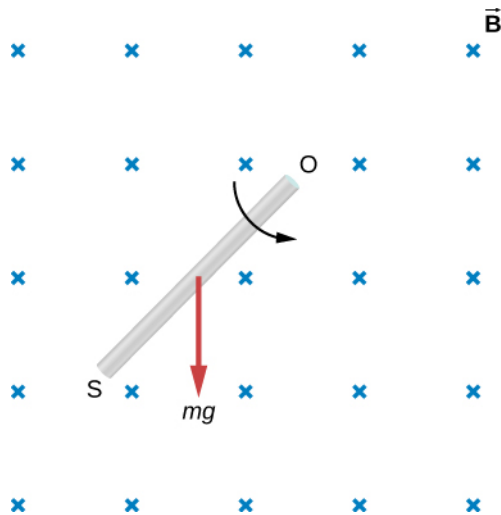
where  $\varepsilon_0 = NBA\omega$ .

### Significance

If the magnetic field strength or area of the loop were also changing over time, these variables wouldn't be able to be pulled out of the time derivative to simplify the solution as shown. This example is the basis for an electric generator, as we will give a full discussion in [Applications of Newton's Law](#).

### ✓ CHECK YOUR UNDERSTANDING 13.4

Shown below is a rod of length  $l$  that is rotated counterclockwise around the axis through  $O$  by the torque due to  $m\vec{\mathbf{g}}$ . Assuming that the rod is in a uniform magnetic field  $\vec{\mathbf{B}}$ , what is the emf induced between the ends of the rod when its angular velocity is  $\omega$ ? Which end of the rod is at a higher potential?



### ✓ CHECK YOUR UNDERSTANDING 13.5

A rod of length 10 cm moves at a speed of 10 m/s perpendicularly through a 1.5-T magnetic field. What is the potential difference between the ends of the rod?

## 13.4 Induced Electric Fields

### Learning Objectives

By the end of this section, you will be able to:

- Connect the relationship between an induced emf from Faraday's law to an electric field, thereby showing that a changing magnetic flux creates an electric field
- Solve for the electric field based on a changing magnetic flux in time

The fact that emfs are induced in circuits implies that work is being done on the conduction electrons in the wires. What can possibly be the source of this work? We know that it's neither a battery nor a magnetic field, for a battery does not have to be present in a circuit where current is induced, and magnetic fields never do work on moving charges. The answer is that the source of the work is an electric field  $\vec{E}$  that is induced in the wires. The work done by  $\vec{E}$  in moving a unit charge completely around a circuit is the induced emf  $\varepsilon$ ; that is,

$$\varepsilon = \oint \vec{E} \cdot d\vec{l}, \quad 13.9$$

where  $\oint$  represents the line integral around the circuit. Faraday's law can be written in terms of the **induced electric field** as

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt}. \quad 13.10$$

There is an important distinction between the electric field induced by a changing magnetic field and the electrostatic field produced by a fixed charge distribution. Specifically, the induced electric field is nonconservative because it does net work in moving a charge over a closed path, whereas the electrostatic field is conservative and does no net work over a closed path. Hence, electric potential can be associated with the electrostatic field, but not with the induced field. The following equations represent the distinction between the two types of electric field:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{l} &\neq 0 \text{ (induced);} \\ \oint \vec{E} \cdot d\vec{l} &= 0 \text{ (electrostatic).} \end{aligned} \quad 13.11$$

Our results can be summarized by combining these equations:

$$\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt}. \quad 13.12$$



### EXAMPLE 13.7

#### Induced Electric Field in a Circular Coil

What is the induced electric field in the circular coil of [Example 13.2](#) (and [Figure 13.9](#)) at the three times indicated?

#### Strategy

Using cylindrical symmetry, the electric field integral simplifies into the electric field times the circumference of a circle. Since we already know the induced emf, we can connect these two expressions by Faraday's law to solve for the induced electric field.

#### Solution

The induced electric field in the coil is constant in magnitude over the cylindrical surface, similar to how Ampere's law problems with cylinders are solved. Since  $\vec{E}$  is tangent to the coil,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \oint E dl = 2\pi r E.$$

When combined with [Equation 13.12](#), this gives

$$E = \frac{\epsilon}{2\pi r}.$$

The direction of  $\epsilon$  is counterclockwise, and  $\vec{\mathbf{E}}$  circulates in the same direction around the coil. The values of  $E$  are

$$\begin{aligned} E(t_1) &= \frac{6.0 \text{ V}}{2\pi (0.50 \text{ m})} = 1.9 \text{ V/m}; \\ E(t_2) &= \frac{4.7 \text{ V}}{2\pi (0.50 \text{ m})} = 1.5 \text{ V/m}; \\ E(t_3) &= \frac{0.040 \text{ V}}{2\pi (0.50 \text{ m})} = 0.013 \text{ V/m}. \end{aligned}$$

### Significance

When the magnetic flux through a circuit changes, a nonconservative electric field is induced, which drives current through the circuit. But what happens if  $d\mathbf{B}/dt \neq 0$  in free space where there isn't a conducting path? The answer is that this case can be treated *as if a conducting path were present*; that is, nonconservative electric fields are induced wherever  $d\mathbf{B}/dt \neq 0$ , whether or not there is a conducting path present.

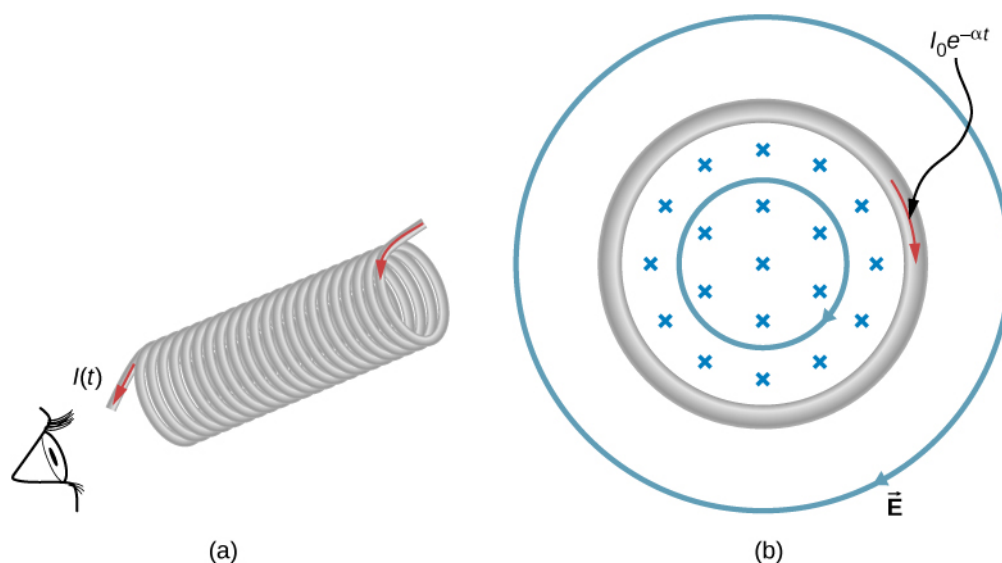
These nonconservative electric fields always satisfy [Equation 13.12](#). For example, if the circular coil of [Figure 13.9](#) were removed, an electric field *in free space* at  $r = 0.50$  m would still be directed counterclockwise, and its magnitude would still be 1.9 V/m at  $t = 0$ , 1.5 V/m at  $t = 5.0 \times 10^{-2}$  s, etc. The existence of induced electric fields is certainly *not* restricted to wires in circuits.



### EXAMPLE 13.8

#### Electric Field Induced by the Changing Magnetic Field of a Solenoid

Part (a) of [Figure 13.18](#) shows a long solenoid with radius  $R$  and  $n$  turns per unit length; its current decreases with time according to  $I = I_0 e^{-at}$ . What is the magnitude of the induced electric field at a point a distance  $r$  from the central axis of the solenoid (a) when  $r > R$  and (b) when  $r < R$  [see part (b) of [Figure 13.18](#)]. (c) What is the direction of the induced field at both locations? Assume that the infinite-solenoid approximation is valid throughout the regions of interest.



**Figure 13.18** (a) The current in a long solenoid is decreasing exponentially. (b) A cross-sectional view of the solenoid from its left end. The cross-section shown is near the middle of the solenoid. An electric field is induced both inside and outside the solenoid.

### Strategy

Using the formula for the magnetic field inside an infinite solenoid and Faraday's law, we calculate the induced emf. Since we have cylindrical symmetry, the electric field integral reduces to the electric field times the circumference of the integration path. Then we solve for the electric field.

### Solution

- a. The magnetic field is confined to the interior of the solenoid where

$$B = \mu_0 n I = \mu_0 n I_0 e^{-\alpha t}.$$

Thus, the magnetic flux through a circular path whose radius  $r$  is greater than  $R$ , the solenoid radius, is

$$\Phi_m = BA = \mu_0 n I_0 \pi R^2 e^{-\alpha t}.$$

The induced field  $\vec{E}$  is tangent to this path, and because of the cylindrical symmetry of the system, its magnitude is constant on the path. Hence, we have

$$\left| \oint \vec{E} \cdot d\vec{l} \right| = \left| \frac{d\Phi_m}{dt} \right|,$$

$$E(2\pi r) = \left| \frac{d}{dt} (\mu_0 n I_0 \pi R^2 e^{-\alpha t}) \right| = \alpha \mu_0 n I_0 \pi R^2 e^{-\alpha t},$$

$$E = \frac{\alpha \mu_0 n I_0 R^2}{2r} e^{-\alpha t} \quad (r > R).$$

- b. For a path of radius  $r$  inside the solenoid,  $\Phi_m = B\pi r^2$ , so

$$E(2\pi r) = \left| \frac{d}{dt} (\mu_0 n I_0 \pi r^2 e^{-\alpha t}) \right| = \alpha \mu_0 n I_0 \pi r^2 e^{-\alpha t},$$

and the induced field is

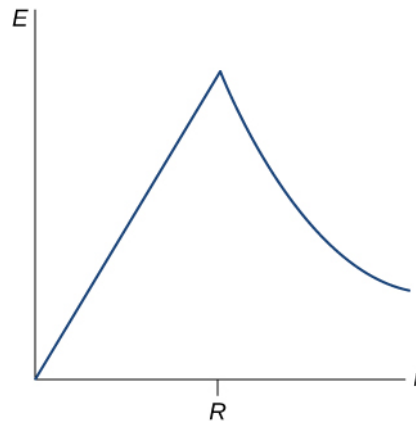
$$E = \frac{\alpha \mu_0 n I_0 r}{2} e^{-\alpha t} \quad (r < R).$$

- c. The magnetic field points into the page as shown in part (b) and is decreasing. If either of the circular paths were occupied by conducting rings, the currents induced in them would circulate as shown, in

conformity with Lenz's law. The induced electric field must be so directed as well.

### Significance

In part (b), note that  $|\vec{E}|$  increases with  $r$  inside and decreases as  $1/r$  outside the solenoid, as shown in [Figure 13.19](#).



**Figure 13.19** The electric field vs. distance  $r$ . When  $r < R$ , the electric field rises linearly, whereas when  $r > R$ , the electric field falls off proportional to  $1/r$ .

### ✓ CHECK YOUR UNDERSTANDING 13.6

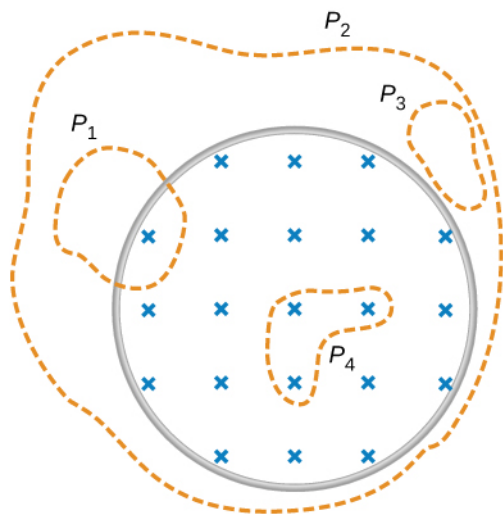
Suppose that the coil of [Example 13.2](#) is a square rather than circular. Can [Equation 13.12](#) be used to calculate (a) the induced emf and (b) the induced electric field?

### ✓ CHECK YOUR UNDERSTANDING 13.7

What is the magnitude of the induced electric field in [Example 13.8](#) at  $t = 0$  if  $r = 6.0$  cm,  $R = 2.0$  cm,  $n = 2000$  turns per meter,  $I_0 = 2.0$  A, and  $\alpha = 200$  s<sup>-1</sup>?

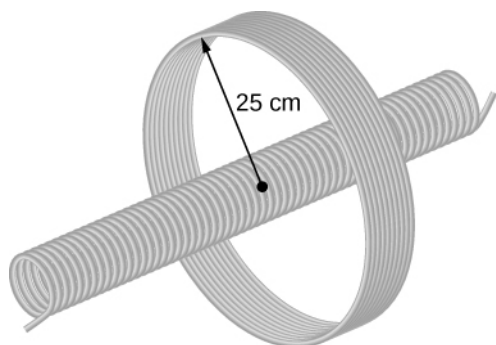
### ✓ CHECK YOUR UNDERSTANDING 13.8

The magnetic field shown below is confined to the cylindrical region shown and is changing with time. Identify those paths for which  $\varepsilon = \oint \vec{E} \cdot d\vec{l} \neq 0$ .



### ✓ CHECK YOUR UNDERSTANDING 13.9

A long solenoid of cross-sectional area  $5.0 \text{ cm}^2$  is wound with 25 turns of wire per centimeter. It is placed in the middle of a closely wrapped coil of 10 turns and radius 25 cm, as shown below. (a) What is the emf induced in the coil when the current through the solenoid is decreasing at a rate  $dI/dt = -0.20 \text{ A/s}$ ? (b) What is the electric field induced in the coil?



## 13.5 Eddy Currents

### Learning Objectives

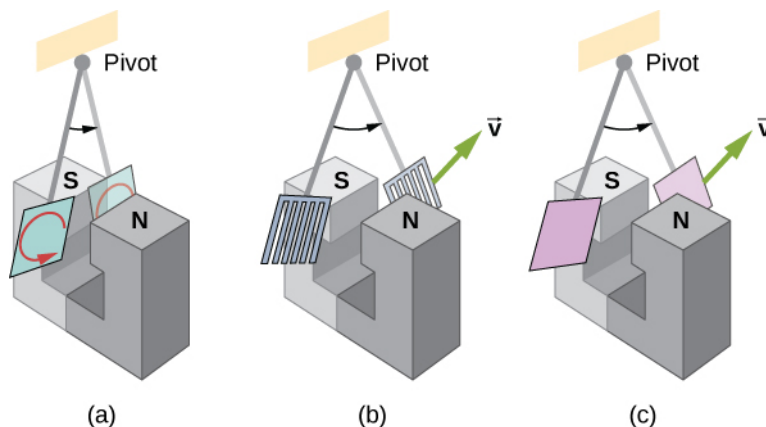
By the end of this section, you will be able to:

- Explain how eddy currents are created in metals
- Describe situations where eddy currents are beneficial and where they are not helpful

As discussed two sections earlier, a motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current in the conductor, we refer to that current as an **eddy current**.

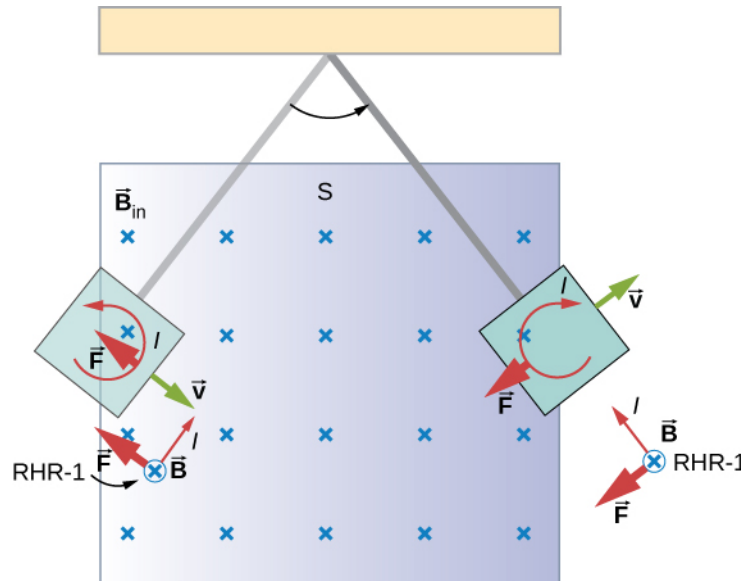
### Magnetic Damping

Eddy currents can produce significant drag, called **magnetic damping**, on the motion involved. Consider the apparatus shown in [Figure 13.20](#), which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics demonstration.) If the bob is metal, significant drag acts on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in part (b) of the figure, the magnet produces a much smaller effect. There is no discernible effect on a bob made of an insulator. Why does drag occur in both directions, and are there any uses for magnetic drag?



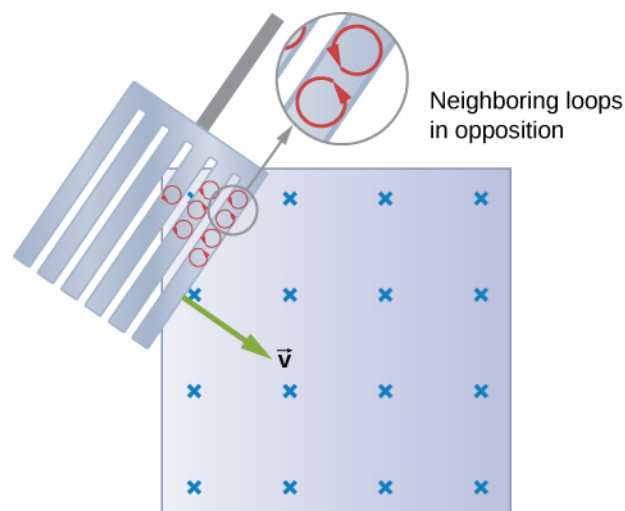
**Figure 13.20** A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

Figure 13.21 shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, setting up an eddy current (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so an unopposed force acts on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.



**Figure 13.21** A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

When a slotted metal plate enters the field (Figure 13.22), an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they must be slotted or constructed of thin layers of conducting material separated by insulating sheets.

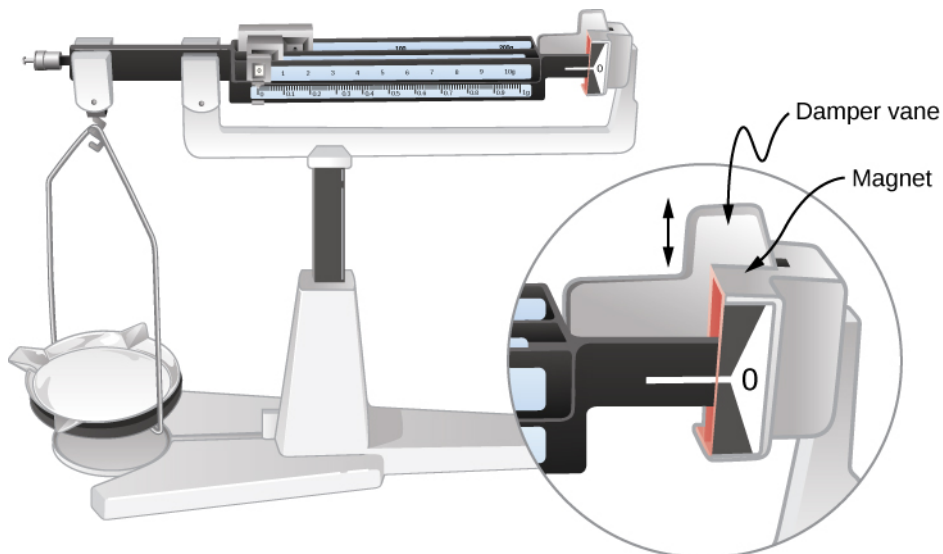


**Figure 13.22** Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to

cancel, thereby making magnetic drag almost zero.

## Applications of Magnetic Damping

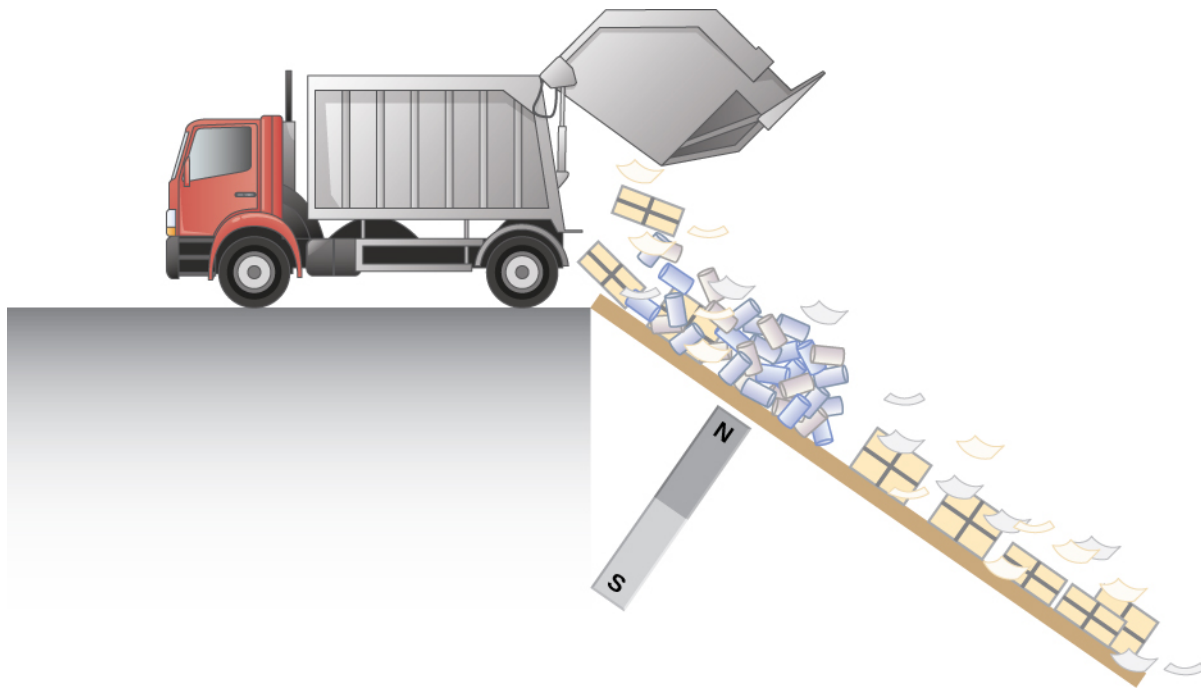
One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus, the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive (Figure 13.23). In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.



**Figure 13.23** Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.

Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals (Figure 13.24). This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.





**Figure 13.24** Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.

Other major applications of eddy currents appear in metal detectors and braking systems in trains and roller coasters. Portable metal detectors ([Figure 13.25](#)) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current is induced in a piece of metal close to the detector, causing a change in the induced current within the secondary coil. This can trigger some sort of signal, such as a shrill noise.



**Figure 13.25** A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)

Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the braking force produced decreases as speed is reduced. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare-earth magnets such as neodymium magnets are used in roller coasters. [Figure 13.26](#) shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) that pass through the magnetic field, slowing the vehicle down in much the same way as with the pendulum bob shown in [Figure 13.20](#).



**Figure 13.26** The rows of rare-earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly, producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times when the base of the pot is a conductor, such as iron or steel.

## 13.6 Electric Generators and Back Emf

### Learning Objectives

*By the end of this section, you will be able to:*

- Explain how an electric generator works
- Determine the induced emf in a loop at any time interval, rotating at a constant rate in a magnetic field
- Show that rotating coils have an induced emf; in motors this is called back emf because it opposes the emf input to the motor

A variety of important phenomena and devices can be understood with Faraday's law. In this section, we examine two of these.

### Electric Generators

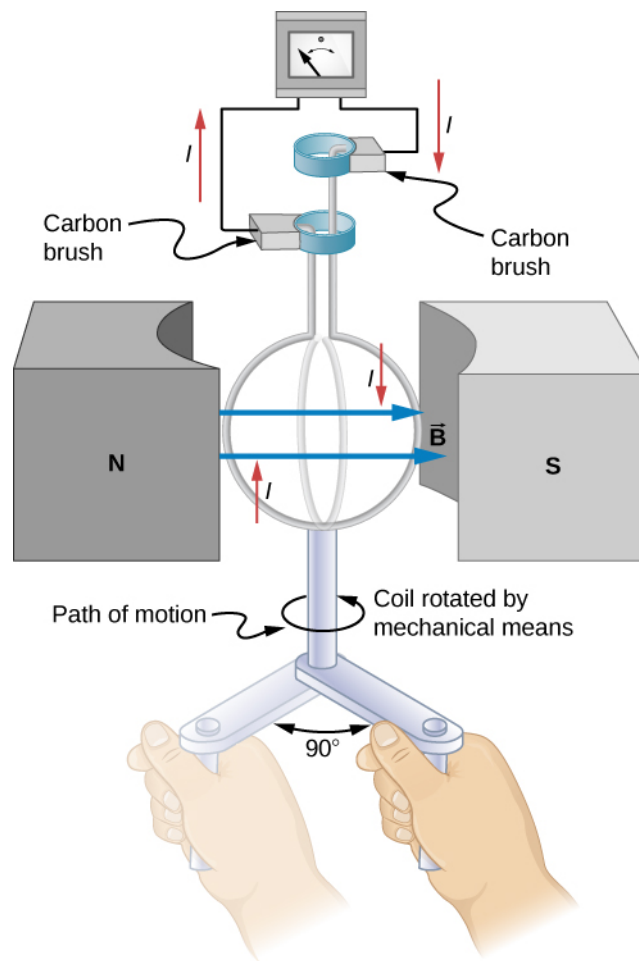
**Electric generators** induce an emf by rotating a coil in a magnetic field, as briefly discussed in [Motional Emf](#). We now explore generators in more detail. Consider the following example.



#### EXAMPLE 13.9

#### Calculating the Emf Induced in a Generator Coil

The generator coil shown in [Figure 13.27](#) is rotated through one-fourth of a revolution (from  $\theta = 0^\circ$  to  $\theta = 90^\circ$ ) in 15.0 ms. The 200-turn circular coil has a 5.00-cm radius and is in a uniform 0.80-T magnetic field. What is the emf induced?



**Figure 13.27** When this generator coil is rotated through one-fourth of a revolution, the magnetic flux  $\Phi_m$  changes from its maximum to zero, inducing an emf.

### Strategy

Faraday's law of induction is used to find the emf induced:

$$\varepsilon = -N \frac{d\Phi_m}{dt}.$$

We recognize this situation as the same one in [Example 13.6](#). According to the diagram, the projection of the surface normal vector  $\hat{n}$  to the magnetic field is initially  $\cos \theta$ , and this is inserted by the definition of the dot product. The magnitude of the magnetic field and area of the loop are fixed over time, which makes the integration simplify quickly. The induced emf is written out using Faraday's law:

$$\varepsilon = NBA \sin \theta \frac{d\theta}{dt}.$$

### Solution

We are given that  $N = 200$ ,  $B = 0.80 \text{ T}$ ,  $\theta = 90^\circ$ ,  $d\theta = 90^\circ = \pi/2$ , and  $dt = 15.0 \text{ ms}$ . The area of the loop is

$$A = \pi r^2 = (3.14)(0.0500 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2.$$

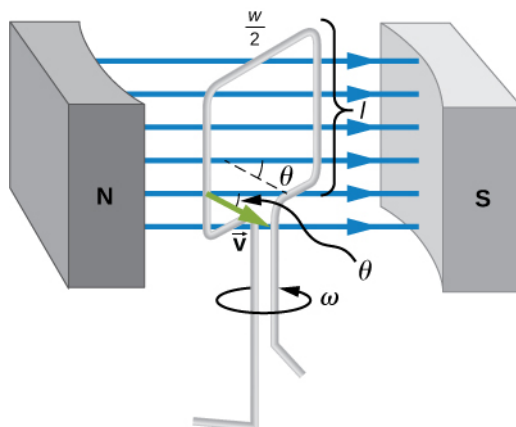
Entering this value gives

$$\varepsilon = (200)(0.80 \text{ T})(7.85 \times 10^{-3} \text{ m}^2) \sin(90^\circ) \frac{\pi/2}{15.0 \times 10^{-3} \text{ s}} = 131 \text{ V}.$$

### Significance

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in [Example 13.9](#) is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf as a function of time by considering the motional emf on a rotating rectangular coil of width  $w$  and height  $l$  in a uniform magnetic field, as illustrated in [Figure 13.28](#).



**Figure 13.28** A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be  $\epsilon = Blv$ , where the velocity  $v$  is perpendicular to the magnetic field  $B$ . Here the velocity is at an angle  $\theta$  with  $B$ , so that its component perpendicular to  $B$  is  $v \sin \theta$  (see [Figure 13.28](#)). Thus, in this case, the emf induced on each side is  $\epsilon = Blv \sin \theta$ , and they are in the same direction. The total emf around the loop is then

$$\epsilon = 2Blv \sin \theta. \quad 13.13$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity  $\omega$ . The angle  $\theta$  is related to angular velocity by  $\theta = \omega t$ , so that

$$\epsilon = 2Blv \sin(\omega t). \quad 13.14$$

Now, linear velocity  $v$  is related to angular velocity  $\omega$  by  $v = r\omega$ . Here,  $r = w/2$ , so that  $v = (w/2)\omega$ , and

$$\epsilon = 2Bl \frac{w}{2} \omega \sin \omega t = (lw)B\omega \sin \omega t. \quad 13.15$$

Noting that the area of the loop is  $A = lw$ , and allowing for  $N$  loops, we find that

$$\epsilon = NBA\omega \sin(\omega t). \quad 13.16$$

This is the emf induced in a generator coil of  $N$  turns and area  $A$  rotating at a constant angular velocity  $\omega$  in a uniform magnetic field  $B$ . This can also be expressed as

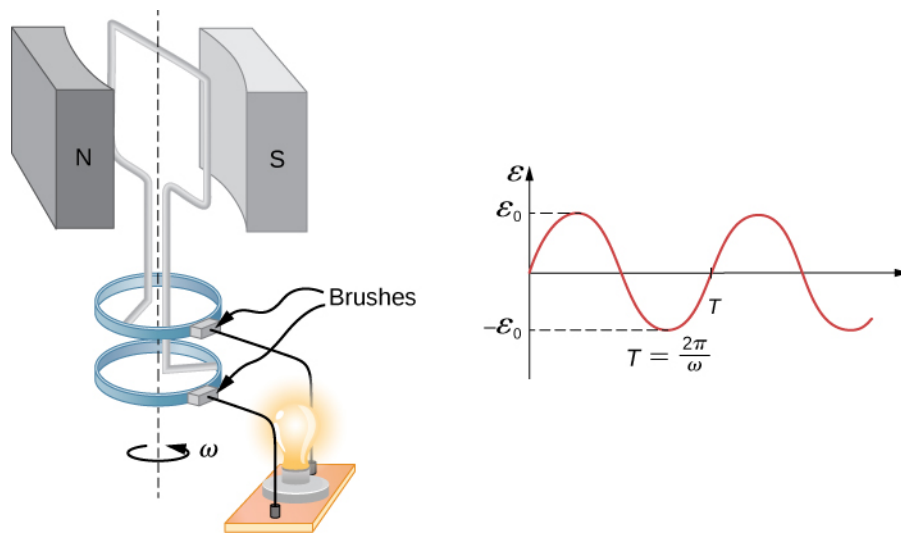
$$\epsilon = \epsilon_0 \sin \omega t, \quad 13.17$$

where

$$\epsilon_0 = NAB\omega \quad 13.18$$

is the peak emf, since the maximum value of  $\sin(\omega t) = 1$ . Note that the frequency of the oscillation is  $f = \omega/2\pi$

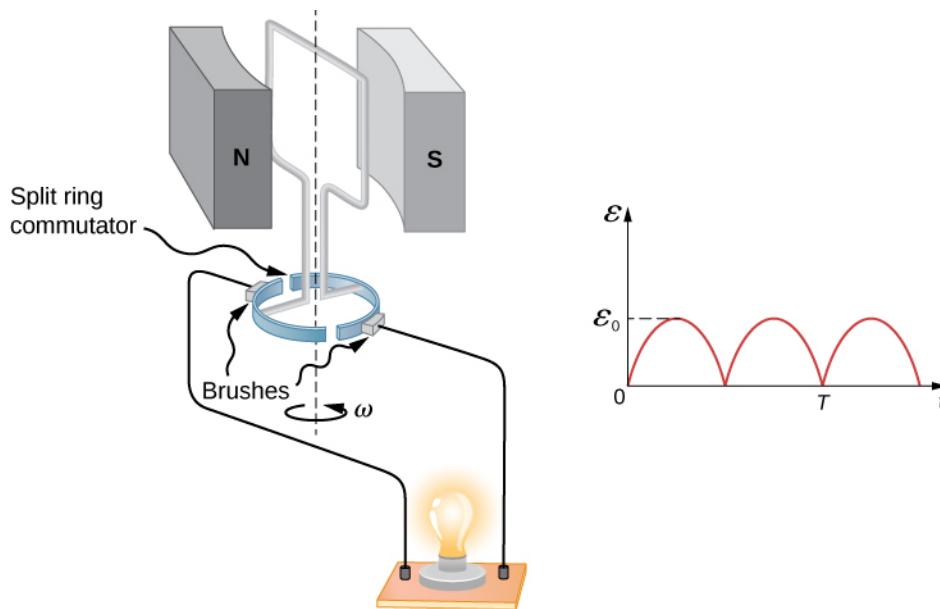
and the period is  $T = 1/f = 2\pi/\omega$ . [Figure 13.29](#) shows a graph of emf as a function of time, and it now seems reasonable that ac voltage is sinusoidal.



**Figure 13.29** The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time, where  $\epsilon_0$  is the peak emf. The period is  $T = 1/f = 2\pi/\omega$ , where  $f$  is the frequency.

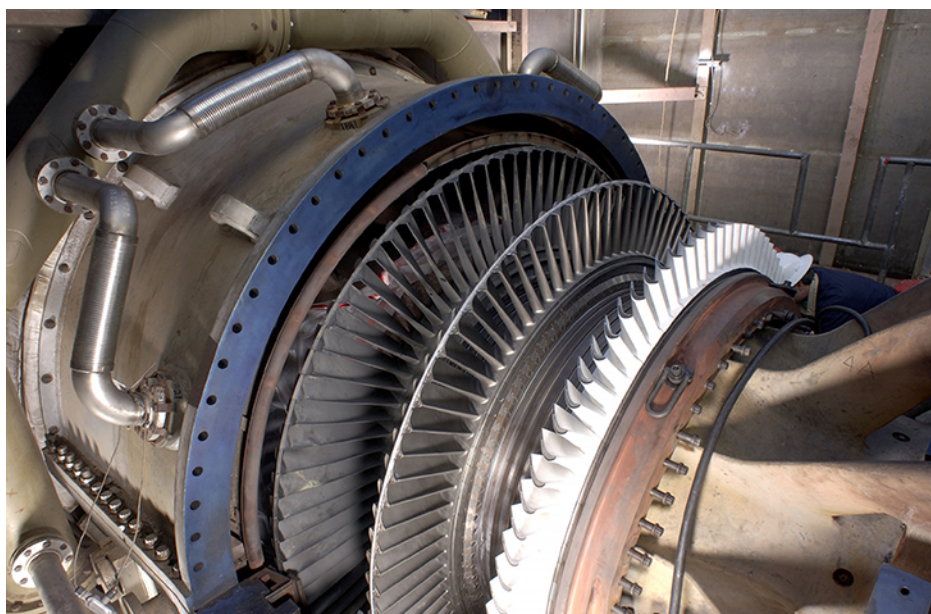
The fact that the peak emf is  $\epsilon_0 = NBA\omega$  makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater  $\omega$ ), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties.

[Figure 13.30](#) shows a scheme by which a generator can be made to produce pulsed dc. More elaborate arrangements of multiple coils and split rings can produce smoother dc, although electronic rather than mechanical means are usually used to make ripple-free dc.



**Figure 13.30** Split rings, called commutators, produce a pulsed dc emf output in this configuration.

In real life, electric generators look a lot different from the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the burning of fossil fuels, or the kinetic energy of wind. [Figure 13.31](#) shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator. The generation of electrical energy from mechanical energy is the basic principle of all power that is sent through our electrical grids to our homes.



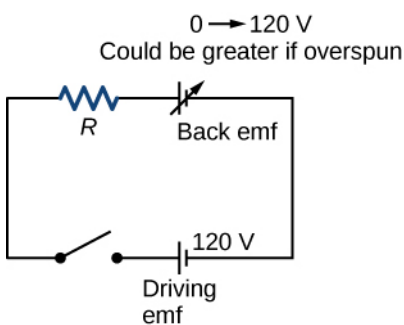
**Figure 13.31** Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft, which is connected to the generator.

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In the next section, we further explore the action of a motor as a generator.

## Back Emf

Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Thus, it is not surprising that motors and generators have the same general construction. A motor works by sending a current through a loop of wire located in a magnetic field. As a result, the magnetic field exerts torque on the loop. This rotates a shaft, thereby extracting mechanical work out of the electrical current sent in initially. (Refer to [Force and Torque on a Current Loop](#) for a discussion on motors that will help you understand more about them before proceeding.)

When the coil of a motor is turned, magnetic flux changes through the coil, and an emf (consistent with Faraday's law) is induced. The motor thus acts as a generator whenever its coil rotates. This happens whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor is opposed by the motor's self-generated emf, called the **back emf** of the motor ([Figure 13.32](#)).



**Figure 13.32** The coil of a dc motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the emf driving the motor. Back emf is zero when the motor is not turning and increases proportionally to the motor's angular velocity.

The generator output of a motor is the difference between the supply voltage and the back emf. The back emf is

zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster, the back emf grows, always opposing the driving emf, and reduces both the voltage across the coil and the amount of current it draws. This effect is noticeable in many common situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the  $IR$  drop produced in feeder lines by the large current drawn by the motor.

When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil,  $P = I^2 R$ ), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it increases its angular velocity  $\omega$  until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Eddy currents in iron cores of motors can cause troublesome energy losses. These are usually minimized by constructing the cores out of thin, electrically insulated sheets of iron. The magnetic properties of the core are hardly affected by the lamination of the insulating sheet, while the resistive heating is reduced considerably. Consider, for example, the motor coils represented in [Figure 13.32](#). The coils have an equivalent resistance of  $0.400\ \Omega$  and are driven by an emf of  $48.0\ \text{V}$ . Shortly after being turned on, they draw a current

$$I = V/R = (48.0\ \text{V})/(0.400\ \Omega) = 120\ \text{A}$$

and thus dissipate  $P = I^2 R = 5.76\ \text{kW}$  of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is  $40.0\ \text{V}$ . Then at operating speed, the total voltage across the coils is  $8.0\ \text{V}$  ( $48.0\ \text{V}$  minus the  $40.0\ \text{V}$  back emf), and the current drawn is

$$I = V/R = (8.0\ \text{V})/(0.400\ \Omega) = 20\ \text{A}.$$

Under normal load, then, the power dissipated is  $P = IV = (20\ \text{A})(8.0\ \text{V}) = 160\ \text{W}$ . This does not cause a problem for this motor, whereas the former  $5.76\ \text{kW}$  would burn out the coils if sustained.



### EXAMPLE 13.10

#### A Series-Wound Motor in Operation

The total resistance ( $R_f + R_a$ ) of a series-wound dc motor is  $2.0\ \Omega$  ([Figure 13.33](#)). When connected to a  $120\text{-V}$  source ( $\mathcal{E}_s$ ), the motor draws  $10\ \text{A}$  while running at constant angular velocity. (a) What is the back emf induced in the rotating coil,  $\mathcal{E}_i$ ? (b) What is the mechanical power output of the motor? (c) How much power is dissipated in the resistance of the coils? (d) What is the power output of the  $120\text{-V}$  source? (e) Suppose the load on the motor increases, causing it to slow down to the point where it draws  $20\ \text{A}$ . Answer parts (a) through (d) for this situation.

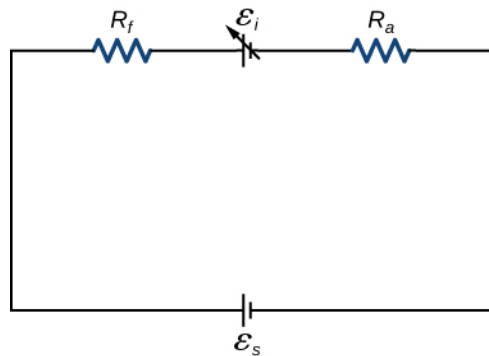


Figure 13.33 Circuit representation of a series-wound direct current motor.

#### Strategy

The back emf is calculated based on the difference between the supplied voltage and the loss from the current

through the resistance. The power from each device is calculated from one of the power formulas based on the given information.

### Solution

- a. The back emf is

$$\varepsilon_i = \varepsilon_s - I(R_f + R_a) = 120 \text{ V} - (10 \text{ A})(2.0 \Omega) = 100 \text{ V}.$$

- b. Since the potential across the armature is 100 V when the current through it is 10 A, the power output of the motor is

$$P_m = \varepsilon_i I = (100 \text{ V})(10 \text{ A}) = 1.0 \times 10^3 \text{ W}.$$

- c. A 10-A current flows through coils whose combined resistance is  $2.0 \Omega$ , so the power dissipated in the coils is

$$P_R = I^2 R = (10 \text{ A})^2 (2.0 \Omega) = 2.0 \times 10^2 \text{ W}.$$

- d. Since 10 A is drawn from the 120-V source, its power output is

$$P_s = \varepsilon_s I = (120 \text{ V})(10 \text{ A}) = 1.2 \times 10^3 \text{ W}.$$

- e. Repeating the same calculations with  $I = 20 \text{ A}$ , we find

$$\varepsilon_i = 80 \text{ V}, P_m = 1.6 \times 10^3 \text{ W}, P_R = 8.0 \times 10^2 \text{ W}, \text{ and } P_s = 2.4 \times 10^3 \text{ W}.$$

The motor is turning more slowly in this case, so its power output and the power of the source are larger.

### Significance

Notice that we have an energy balance in part (d):  $1.2 \times 10^3 \text{ W} = 1.0 \times 10^3 \text{ W} + 2.0 \times 10^2 \text{ W}$ .

## 13.7 Applications of Electromagnetic Induction

### Learning Objectives

*By the end of this section, you will be able to:*

- Explain how computer hard drives and graphic tablets operate using magnetic induction
- Explain how hybrid/electric vehicles and transcranial magnetic stimulation use magnetic induction to their advantage

Modern society has numerous applications of Faraday's law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that involve recording information using magnetic fields.

Some computer hard drives apply the principle of magnetic induction. Recorded data are made on a coated, spinning disk. Historically, reading these data was made to work on the principle of induction. However, most input information today is carried in digital rather than analog form—a series of 0s or 1s are written upon the spinning hard drive. Therefore, most hard drive readout devices do not work on the principle of induction, but use a technique known as giant magnetoresistance. Giant magnetoresistance is the effect of a large change of electrical resistance induced by an applied magnetic field to thin films of alternating ferromagnetic and nonmagnetic layers. This is one of the first large successes of nanotechnology.

Graphics tablets, or tablet computers where a specially designed pen is used to draw digital images, also applies induction principles. The tablets discussed here are labeled as passive tablets, since there are other designs that use either a battery-operated pen or optical signals to write with. The passive tablets are different than the touch tablets and phones many of us use regularly, but may still be found when signing your signature at a cash register. Underneath the screen, shown in [Figure 13.34](#), are tiny wires running across the length and width of the screen. The pen has a tiny magnetic field coming from the tip. As the tip brushes across the screen, a changing magnetic field is felt in the wires which translates into an induced emf that is converted into the line you just drew.





**Figure 13.34** A tablet with a specially designed pen to write with is another application of magnetic induction. (credit: Jane Whitney)

Another application of induction is the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape, in which a playback head reads personal information from your card.

### INTERACTIVE

Check out this [video \(https://openstax.org/l/21flashmagind\)](https://openstax.org/l/21flashmagind) to see how flashlights can use magnetic induction. A magnet moves by your mechanical work through a wire. The induced current charges a capacitor that stores the charge that will light the lightbulb even while you are not doing this mechanical work.

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Electric and hybrid vehicles also take advantage of electromagnetic induction. One limiting factor that inhibits widespread acceptance of 100% electric vehicles is that the lifetime of the battery is not as long as the time you get to drive on a full tank of gas. To increase the amount of charge in the battery during driving, the motor can act as a generator whenever the car is braking, taking advantage of the back emf produced. This extra emf can be newly acquired stored energy in the car's battery, prolonging the life of the battery.

Another contemporary area of research in which electromagnetic induction is being successfully implemented is transcranial magnetic stimulation (TMS). A host of disorders, including depression and hallucinations, can be traced to irregular localized electrical activity in the brain. In transcranial magnetic stimulation, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. The usage of TMS as a diagnostic technique is well established.

### INTERACTIVE

Check out this [Youtube video \(https://openstax.org/l/21randrelectro\)](https://openstax.org/l/21randrelectro) to see how rock-and-roll instruments like electric guitars use electromagnetic induction to get those strong beats.

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## CHAPTER REVIEW

### Key Terms

**back emf** emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor

**eddy current** current loop in a conductor caused by motional emf

**electric generator** device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field

**Faraday's law** induced emf is created in a closed loop due to a change in magnetic flux through the loop

**induced electric field** created based on the changing magnetic flux with time

**induced emf** short-lived voltage generated by a conductor or coil moving in a magnetic field

**Lenz's law** direction of an induced emf opposes the change in magnetic flux that produced it; this is the negative sign in Faraday's law

**magnetic damping** drag produced by eddy currents

**magnetic flux** measurement of the amount of magnetic field lines through a given area

**motentially induced emf** voltage produced by the movement of a conducting wire in a magnetic field

**peak emf** maximum emf produced by a generator

### Key Equations

Magnetic flux	$\Phi_m = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA$
Faraday's law	$\varepsilon = -N \frac{d\Phi_m}{dt}$
Motionally induced emf	$\varepsilon = Blv$
Motional emf around a circuit	$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_m}{dt}$
Emf produced by an electric generator	$\varepsilon = NBA \omega \sin(\omega t)$

## Summary

### 13.1 Faraday's Law

- The magnetic flux through an enclosed area is defined as the amount of field lines cutting through a surface area  $A$  defined by the unit area vector.
- The units for magnetic flux are webers, where  $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$ .
- The induced emf in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law. If there is no change in magnetic flux, no induced emf is created.

### 13.2 Lenz's Law

- We can use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs.
- The direction of an induced emf always opposes the change in magnetic flux that causes the emf,

a result known as Lenz's law.

### 13.3 Motional Emf

- The relationship between an induced emf  $\varepsilon$  in a wire moving at a constant speed  $v$  through a magnetic field  $B$  is given by  $\varepsilon = Blv$ .
- An induced emf from Faraday's law is created from a motional emf that opposes the change in flux.

### 13.4 Induced Electric Fields

- A changing magnetic flux induces an electric field.
- Both the changing magnetic flux and the induced electric field are related to the induced emf from Faraday's law.

### 13.5 Eddy Currents

- Current loops induced in moving conductors are called eddy currents. They can create significant drag, called magnetic damping.
- Manipulation of eddy currents has resulted in applications such as metal detectors, braking in trains or roller coasters, and induction cooktops.

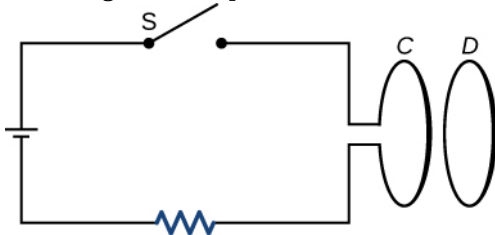
### 13.6 Electric Generators and Back Emf

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by  $\varepsilon = NBA\omega \sin(\omega t)$  where  $A$  is the area of an  $N$ -turn coil rotated at a constant angular velocity  $\omega$  in a uniform magnetic field  $\vec{B}$ .

## Conceptual Questions

### 13.1 Faraday's Law

1. A stationary coil is in a magnetic field that is changing with time. Does the emf induced in the coil depend on the actual values of the magnetic field?
2. In Faraday's experiments, what would be the advantage of using coils with many turns?
3. A copper ring and a wooden ring of the same dimensions are placed in magnetic fields so that there is the same change in magnetic flux through them. Compare the induced electric fields and currents in the rings.
4. Discuss the factors determining the induced emf in a closed loop of wire.
5. (a) Does the induced emf in a circuit depend on the resistance of the circuit? (b) Does the induced current depend on the resistance of the circuit?
6. How would changing the radius of loop  $D$  shown below affect its emf, assuming  $C$  and  $D$  are much closer together compared to their radii?



7. Can there be an induced emf in a circuit at an instant when the magnetic flux through the circuit is zero?
8. Does the induced emf always act to decrease the magnetic flux through a circuit?
9. How would you position a flat loop of wire in a changing magnetic field so that there is no induced emf in the loop?

- The peak emf of a generator is  $\varepsilon_0 = NBA\omega$ .
- Any rotating coil produces an induced emf. In motors, this is called back emf because it opposes the emf input to the motor.

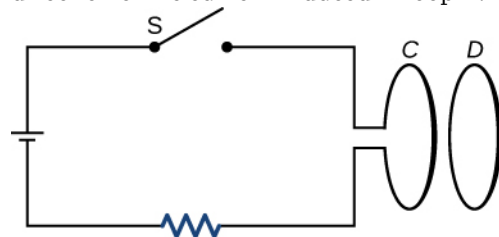
### 13.7 Applications of Electromagnetic Induction

- Hard drives utilize magnetic induction to read/write information.
- Other applications of magnetic induction can be found in graphics tablets, electric and hybrid vehicles, and in transcranial magnetic stimulation.

10. The normal to the plane of a single-turn conducting loop is directed at an angle  $\theta$  to a spatially uniform magnetic field  $\vec{B}$ . It has a fixed area and orientation relative to the magnetic field. Show that the emf induced in the loop is given by  $\varepsilon = (dB/dt)(A \cos \theta)$ , where  $A$  is the area of the loop.

### 13.2 Lenz's Law

11. The circular conducting loops shown in the accompanying figure are parallel, perpendicular to the plane of the page, and coaxial. (a) When the switch  $S$  is closed, what is the direction of the current induced in  $D$ ? (b) When the switch is opened, what is the direction of the current induced in loop  $D$ ?



12. The north pole of a magnet is moved toward a copper loop, as shown below. If you are looking at the loop from above the magnet, will you say the induced current is circulating clockwise or counterclockwise?